A Comparative Study on Transformer and Inductor Based LC Tanks for VCOs

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Abstract— **This paper presents a detailed comparative study of two different approaches to implement the resonator found in radio frequency (RF) voltage controlled oscillators (VCOs). An inductor LC tank VCO and a transformer LC tank VCO are compared in terms of phase noise, power consumption, tuning range and circuit area. Conclusions about the use of each approach depending on the design goals are presented.**

Index Terms—**RF; VCO; PLL**

I. INTRODUCTION

Integrated LC VCOs are a critical block in today's RF systems defining some of the important characteristics of the system. This paper presents a comparative study between inductor and transformer LC resonators for RF VCOs. Previous comparative studies focus on Q factor enhancements [1] and compare optimized transformer layout versus conventional inductor layouts neglecting several advances in the inductor design [2].

In this paper we evaluate the most efficient way to design the resonator inductive element by comparing the best possible performances for inductor and transformer. And we extend the comparison to other key design parameters such as phase noise, power consumption, tuning range and circuit area, providing a fair comparison between state of the art inductor [2] and the transformer [1] layouts.

The material of this paper is organized as follows; Section II presents the basics for the VCO design, Section III provides the theoretical support for the inductor and transformer comparison, Section IV describes the implementation of two VCO circuits with an inductor and a transformer, describing the most relevant design options and constraints of the practical circuits, Section V presents the measurement results and finally the conclusions of this work are presented in Section VI.

II. VCO BASICS

The oscillation frequency of a VCO depends only on the resonator parameters and can be approximated by:

$$f_{osc} = \frac{1}{2\pi\sqrt{L \cdot C}} \qquad (1)$$

To measure and compare the performance of tank resonators implemented with different components, a quality factor Q defined as the ratio of energy stored in each cycle by the energy dissipated in each cycle is normally used:

$$Q = 2\pi \cdot \frac{Stored\ energy\ per\ cycle}{Dissipated\ energy\ per\ cycle} \qquad (2)$$

The energy stored is given by the average magnetic and electric energy stored in the inductive and capacitive tank elements. The energy dissipated is the energy loss by the parasitics of the tank components. In the definition of the Q factor a periodical excitation is implicitly assumed. An alternative and more practical approach to obtain the Q factor is [2]:

$$Q = -\frac{W_{OSC}}{2} \cdot \frac{d\theta}{dW}\bigg|_{W=W_{OSC}} \qquad (3)$$

Where W_{OSC} is the angular oscillation frequency and θ is the phase of the tank impedance. The phase noise of a VCO at a distance *fm* from the carrier can be determined from the popular Leeson formula [3]:

$$L(f_m) = 10\log\left\{\frac{2FkT}{P}\left(1 + \left(\frac{f_{OSC}}{2Qf_m}\right)^2\right)\left(1 + \frac{f_{1/f}}{|f_m|}\right)\right\} \quad (4)$$

Where F is an empirical fitting parameter that varies with oscillator topology and must be measured. T is the absolute temperature, P is the power of the oscillator signal, f_{OSC} is the oscillator frequency and Q is the loaded quality factor of the resonator, $f_{1/f}$ is the corner frequency of the $1/f$ noise of the active devices.

Due to the tank parasitics, the active devices need to supply the power to compensate the tank losses. The amount of power necessary to sustain the oscillation can be determined using [2]:

$$P_{Loss} = \frac{R_p}{2 \cdot L^2 \cdot W_{OSC}^2} \cdot V_{Peak}^2 \qquad (5)$$

To control the oscillator frequency it is normally used a voltage controlled capacitor (varactor). The tuning range provided by this method depends on how much the varactor capacitance changes with the applied voltage and the relative

weight of the varactor when compared to the inductor. TABLE I presents a summary of the design goals based on several optimization criteria's such as Power, Noise and Tuning Range. The optimization goals described in TABLE I where derived based on (1) thru (5).

TABLE I. SUMMARY OF RESONATOR PARAMETER OPTIMIZATION BASED ON SEVERAL CRITERIA.

Parameter	Low Power	Low Noise	Tuning Range
L	Maximize	Maximize	Minimize
C	Minimize	Minimize	Maximize
Rp	Minimize	Minimize	n.d.
Vpeak	Minimize	Maximize	n.d.

III. TRANSFORMER VS INDUCTOR RESONATOR

Two alternatives to implement the inductive element of a LC tank are considered, the use of an inductor and the use of a transformer. Fig. 1 shows the electrical model of the inductor resonator and transformer resonator. In both cases the tank losses are represented by a loss resistor R_p. Other sources of losses and couplings have been neglected since their effects are less noticeable. In the model of the transformer resonator it is assumed that the same area of the inductor is used and thus each of the transformer windings has an $L/2$ inductance and a loss resistance of $R_p/2$.
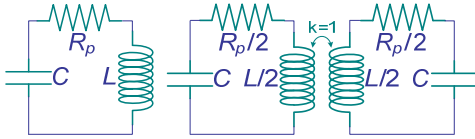

Fig. 1. Inductor and transformer model.

Using the model of Fig. 1 to obtain the impedance of the inductor resonator results in:

$$Z_{LC} = \left(\frac{\dfrac{R_p}{(W_{OSC} \cdot C)^2}}{R_p^2 + \left(W_{OSC} \cdot L - \dfrac{1}{W_{OSC} \cdot C} \right)^2} \right.$$
$$\left. - j \frac{\dfrac{R_p^2}{W_{OSC} \cdot C} + \dfrac{L}{C}\left(W_{OSC} \cdot L - \dfrac{1}{W_{OSC} \cdot C} \right)}{R_p^2 + \left(W_{OSC} \cdot L - \dfrac{1}{W_{OSC} \cdot C} \right)^2} \right) \qquad (6)$$

At resonance the imaginary part of the tank impedance is null, setting the imaginary part of (6) to zero and solving for W_{OSC} we obtain:

$$W_{OSC} = \sqrt{\frac{1}{LC} - \frac{R_p^2}{L^2}} \qquad (7)$$

Eq. (7) differs from the ideal case of (1) due to the presence of the parasitic resistor. However for the typical values of L, C and R, this deviation is small and can be neglected. The tank impedance at resonance is:

$$Z_{LC}(W_{OSC}) = \frac{L}{R_p \cdot C} \qquad (8)$$

The Q factor of the LC resonator is determined using (3) and θ of (6), resulting:

$$Q = (L/R_p) \cdot W_{OSC} \qquad (9)$$

This procedure can also be used to obtain the resonant frequency, impedance at oscillation and the Q factor for the transformer. Due to the presence of two inductances there will also be two resonant frequencies. For the case of having the primary inductance and the secondary inductance equal to $L/2$ and the primary and secondary tuning capacitance equal to C we obtain:

$$W_{OSC1}^2 = 1/[(L/2 + M)C] \qquad (10)$$
$$W_{OSC2}^2 = 1/[(L/2 - M)C] \qquad (11)$$

where M is the mutual inductance $M = k\sqrt{(L/2)^2}$. The impedance for each resonance frequency is:

$$Z_{Trans}(W_{OSC1}) \approx (L/2 + M)/(R_p \times C) \qquad (12)$$
$$Z_{Trans}(W_{OSC2}) \approx (L/2 - M)/(R_p \times C) \qquad (13)$$

From (12) and (13) we can see that for $k=1$, the impedance at W_{OSC1} will tend to $L/(2R_p \times C)$ and at W_{OSC2} will tend to zero. For the typical values of k found in integrated transformers (0.6 to 0.8) the impedance at resonant frequency W_{OSC1} will always be higher that at resonant frequency W_{OSC2}. As a result the Q factor at W_{OSC1} is always higher that at W_{OSC2}. Given the higher Q factor at W_{OSC1} the oscillation amplitude at this frequency will also be larger when compared to W_{OSC2}. The Q factor at W_{OSC1} is:

$$Q = \frac{(L/2 + M) \cdot W_{OSC1}}{R_p/2} \qquad (14)$$

For $k=1$ (14) becomes:

$$Q = (2 \cdot L \cdot W_{OSC1})/R_p \qquad (15)$$

This is twice what we obtain in the case of an inductor resonator. TABLE II presents a comparison of the parameters discussed for the inductor resonator and transformer resonator for the case of ideal transformer coupling $k=1$.

TABLE II. COMPARISON BETWEEN INDUCTOR RESONATOR AND TRANSFORMER RESONATOR WITH IDEAL COUPLING FACTOR $K=1$.

Parameter	Inductor	Transformer
W_{OSC}	$\sqrt{1/LC}$	$\sqrt{1/LC}$
$Z(W_{OSC})$	$L/(R_p \times C)$	$L/(R_p \times C)$
Q	$L \cdot W_{OSC}/R_p$	$2 \cdot L \cdot W_{OSC}/R_p$

From TABLE II we can see that the transformer resonator has the same oscillation frequency and the same resonance impedance, but achieves a greater Q factor which benefits the

phase noise of the oscillator. The main disadvantage of the transformer resonator resides in the fact that it requires two tuning capacitors and thus a slightly greater area.

IV. TRANSFORMER AND INDUCTOR VCOS

To validate the results of the previous section two VCO circuits with inductor and transformer resonator are design and fabricated. The VCO topology is presented in Fig. 2. It consists of two complementary cross coupled differential pairs. This VCO topology provides better isolation from the voltage supply and thus better noise performance. Also due to the current reuse it is possible to obtain twice the transconductance from the active devices for a given bias current. The main disadvantage of this circuit is the large number of stacked transistors which require a large voltage head room. To compare the performance of the inductor and transformer resonator all active devices and bias are the same in both circuits and the only difference is in the inductive element in the LC tank.
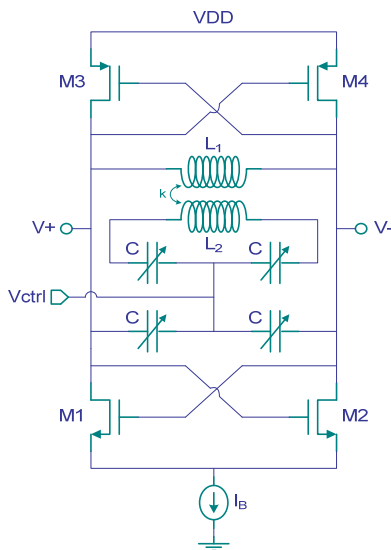

Fig. 2. Transformer VCO.

For the oscillations to start it is required that the active device transconductances compensate for the LC tank losses. This condition translates to:

$$gm \geq 1/[Z(W_{OSC})] \qquad (16)$$

Our goal is to determine the most efficient way to design the resonator inductive element by comparing the best possible performances for inductor and transformer. The performance of the inductive devices is evaluated through an electromagnetic simulator (FAST Henry) [4]. The test chip is implemented in a $0.35\mu m$ process with 4 metal layers and no thick top metal. The inductive elements are designed in a differential mode and use the 3 top metal layers connected in parallel. Fig. 3 shows the fabricated inductive devices, the inductor and transformer area is $0.040mm^2$ and $0.046mm^2$ respectively. To assert the best inductor performance the conventional inductor layout is compared to the modified layout in [2]. The simulation results are presented in Fig. 4 and Fig. 5 and confirm the advantages of using wider tracks for the

outer inductor windings. This technique presents the best trade-off between inductor area and winding resistance. The modified layout of [2] is also applied to the transformer and compared to the conventional layout approach. However in this case, despite the improving effect of the winding resistance reduction, there was also a decrease in the k coefficient of the transformer and the overall performance of the modified transformer layout presented a slightly decrease as it is shown in Fig. 6 and Fig. 7. As a result two VCO circuits where implemented with a modified inductor layout and a conventional transformer layout, TABLE III summarizes the simulation results of the devices.
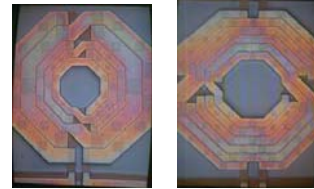

Fig. 3. Inductor and Transformer photographs.

TABLE III. PASSIVE DEVICES PARAMETERS.

Parameter	Inductor	Transformer
Inductance	2.44nH	1.42nH
Parasitic R	5.072Ω	4.687Ω
Q@3.16GHz	9.57	10.59

TABLE III shows that the implemented transformer differs from the ideal case presented in TABLE II. This is due to the smaller k coefficient of 0.775 obtained in the practical device. Also the parasitic resistance of the transformer is larger than the ideal case because of the larger number of windings and crossings that are impossible to avoid in the transformer layout. Despite all these effects the transformer resonator still presents a larger Q factor than the inductor resonator. Fig. 5 and Fig. 6 show that due to non ideal effects, the transformer resonator has a smaller resonance impedance that the inductor. From (16) we can see that the transformer resonator requires larger gm but $gm \propto \sqrt{I_{Bias}}$ and thus:

$$\frac{gm_{Trans}}{gm_{Ind}} = \frac{\sqrt{I_{Trans}}}{\sqrt{I_{Ind}}} \qquad (17)$$

Combining (16) and (17) and solving for the transformer bias current results:

$$I_{Trans} = \left(\frac{Z_{LC}(W_{OSC})}{Z_{Trans}(W_{OSC1})}\right)^2 \times I_{Ind} \qquad (18)$$

(18) represents the power tradeoff between using an inductor and a transformer resonator to obtain higher Q factors. Finally the simulated phase noise performance of the two VCO is compared in Fig. 8. Since the Q of both circuits is very similar, only very small differences in the phase noise performance are observed. The transformer resonator provides better phase noise at lower frequency offsets and the inductor resonator at larger offsets.
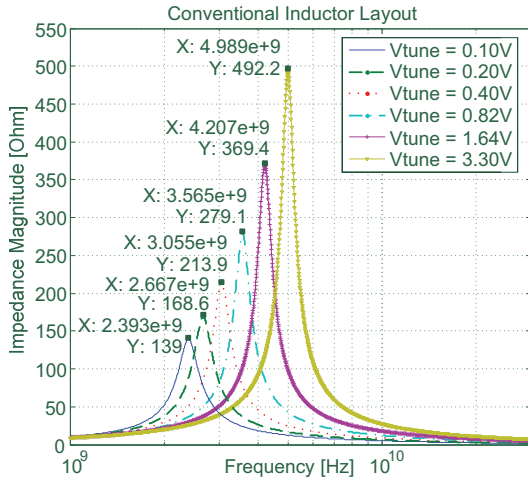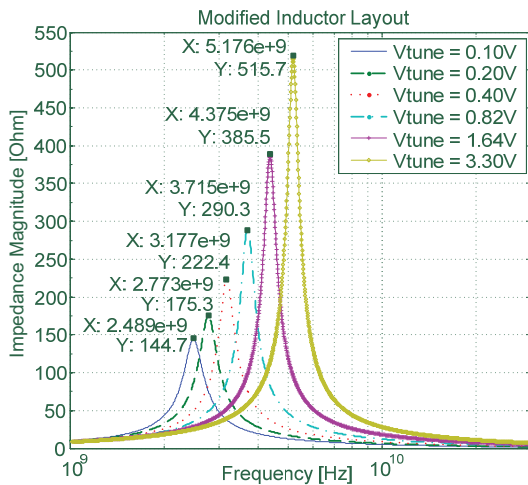
Fig. 4. Conventional inductor layout.
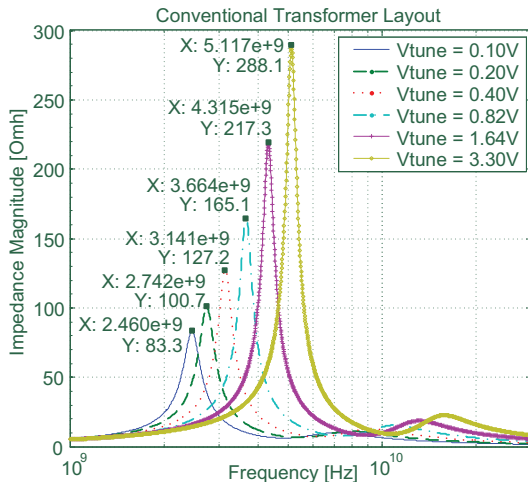

Fig. 5. Modified inductor layout.

Fig. 6. Conventional transformer layout.

V. MEASUREMENT RESULTS

To verify the validity of the concepts presented in this work the test chip with the two VCOs was mounted on a PCB board with SMA connectors. The circuits were characterized using a spectrum analyzer with phase noise measurement capabilities. The phase noise measurements were evaluated in single end mode (measurements taken from one VCO terminal only) and in two distinct situations, with the tuning terminal set to the minimum possible voltage and with the tuning terminal set to the maximum possible voltage. Fig. 9 and Fig. 10 shows the phase noise plots obtained for the inductor VCO for the maximum and minimum tuning voltage. The inductor VCO has a frequency range from 1.355GHz to 2.202GHz corresponding to a tuning range of 47.61%. When compared to the resonance plots of Fig. 5 the oscillation frequency is significantly lower since Fig. 5 only simulates the LC tank and does not include the extra parasitic capacitances added by the active devices and the output buffer.
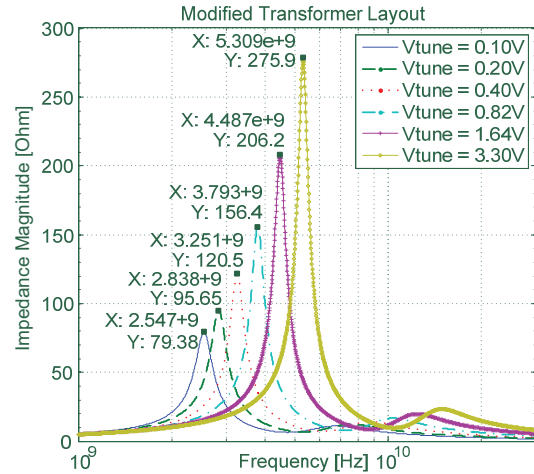
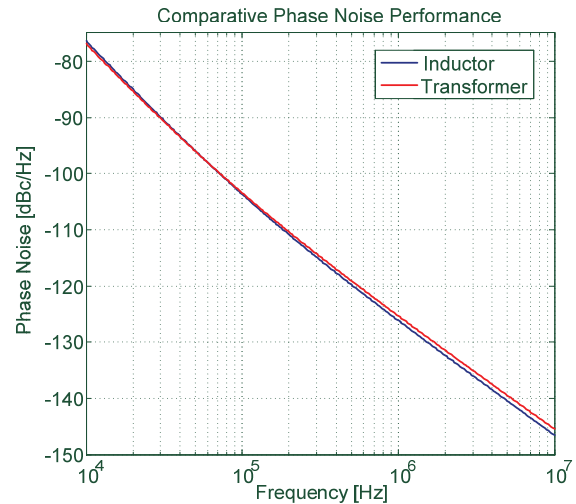

Fig. 7. Modified transformer layout.

Fig. 8. Inductor and transformer phase noise.

Fig. 11 and Fig. 12 present the phase noise plots for the transformer VCO. Due to the lower resonance impedance of the transformer VCO the phase noise plot for the minimum tuning voltage was obtained using a bias current according to (18). The transformer VCO has a frequency range from 1.790GHz to 2.280GHz corresponding to a tuning range of 24.08%.

When compared to Fig. 8 we can see that for offsets greater than 100kHz the inductor and transformer VCO present very

close agreement with the simulated phase noise. Some of the differences can be attributed to the difference in the setups, Fig. 8 was obtained with differential signals and the simulation results do not present the frequency drift errors that arise when testing a VCO in open loop.
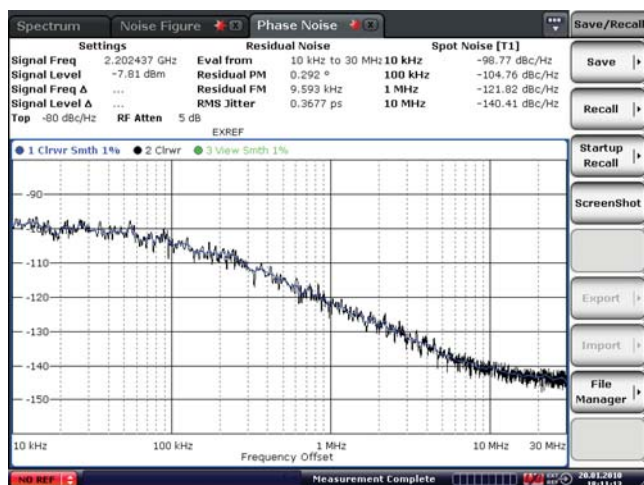

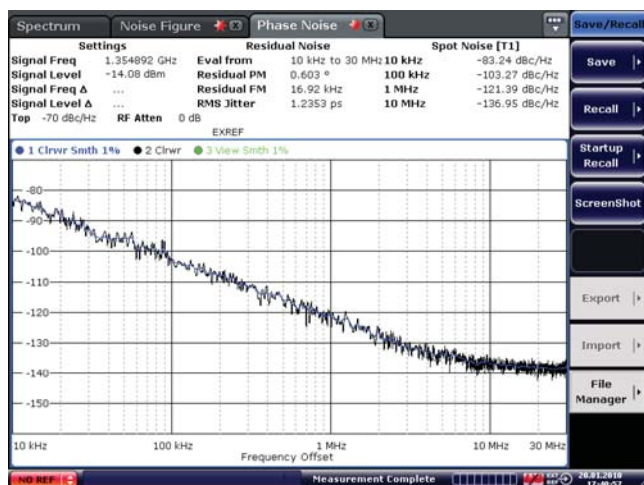Fig. 9. Inductor VCO with tuning voltage set to the maximum.

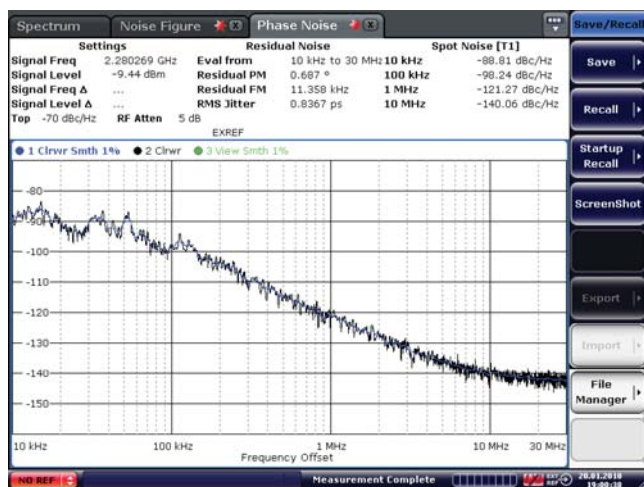Fig. 10. Inductor VCO with tuning voltage set to the minimum.

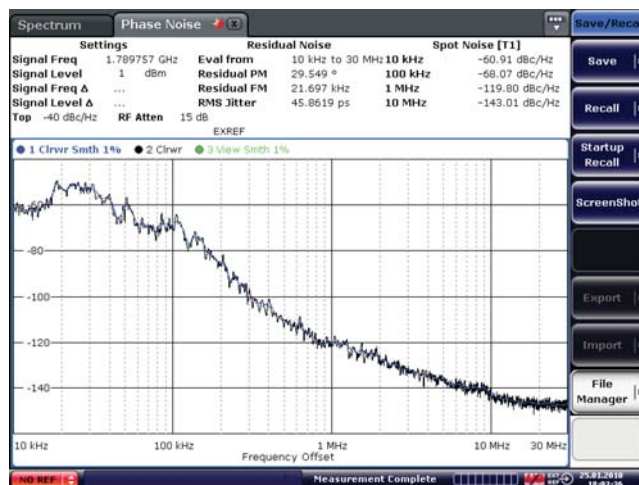Fig. 11. Transformer VCO with tuning voltage set to the maximum.

Fig. 12. Transformer VCO with the tuning voltage set to the minimum.

VI. CONCLUSIONS

A transformer is an alternative method to obtain the inductive element in a LC resonator. In this paper it is showed that in the ideal case the transformer resonator would benefit from twice the Q factor of an inductor resonator for approximately the same circuit area. The higher Q factor of the transformer provides better phase noise results without additional power consumption. However practical integrated transformers with 3 metal layers have low coupling factors leading to smaller gains in Q factor, and smaller resonance impedance leading to higher power consumption. Another disadvantage is the requirement of two tuning capacitors when compared with just one of the inductor resonator. Despite these disadvantages an integrated transformer can still provide higher Q factors than inductors, and with a more advanced technology, with more metal layers available, a larger k factor can be obtained which would approach the transformer resonator to the ideal case and reduce the power penalty for using the transformer.

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