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STANDARD SKY CALCULATIONS FOR DAYLIGHTING DESIGN AND ENERGY PERFORMANCE PURPOSES

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Streszczenie: Daylight is important in the design of the healthy and energy efficient indoor environment. Until now the main criterion for indoor daylighting evaluation is the Daylight Factor assuming exterior overcast sky conditions. This criterion represents minimum daylight availability needed for visual tasks and work processes. However, various outdoor daylight situations can occur during any year including also sunny, cloudy, foggy states etc. The classification of sky luminance distributions that occur in nature was done and fifteen typical sky patterns were proposed for standardisation. This result was adopted and published as ISO 15469:2004/CIE S011/E:2003 Spatial distribution of daylight – CIE Standard general sky. The standard allows to calculate typical relative sky luminance distribution which can serve to predetermine sky luminances and exterior daylight illuminance in arbitrary time and locality.
The ISO/CIE Standard General Sky concept is presented together with sky luminance calculations in relative and absolute photometric units. Such an approach can orientate building physic research activities to find new criteria, methods and procedures for daylight evaluations in interiors considering more realistic daylight conditions.

Słowa kluczowe: Daylighting, sky luminance distribution, standard daylight conditions, daylighting simulations, sky luminance calculation.

1. INTRODUCTION

Daylighting in buildings is designed and evaluated after the Daylight Factor criterion representing conditions under overcast sky only [1-5]. However, time occurrences of daylight levels are permanently changing during the whole year. Evaluations in some conditions require to take into account the atmospheric properties and influences of sunlight for determining typical daylight situations. Because illuminances at ground level are calculated after sky luminance distributions these are important for exterior and interior daylighting determination.

Several methods for sky luminance calculation were published [6-9].
The presented method respects physical principles of light propagation through the atmosphere and is based on the photometric units. It serves simple formulae for the evaluation of daylighting [10]. This concept was adopted by CIE in 2003 and published by ISO in 2004, [11].

2. CALCULATION OF RELATIVE SKY LUMINANCE DISTRIBUTION

The position of the sun and of the arbitrary sky element as well as parameters *a, b, c, d, e* which describe atmospheric conditions have to be taken as input calculation quantities. The position of the sky element is defined by the zenith angle Z and the azimuth angle A_z between the sky element and the solar meridian, fig. 1.

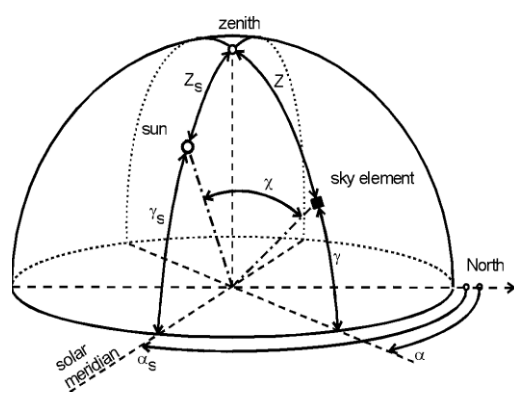

Figure 1 Sun and sky element position.

The spherical angle χ from sun to the sky element is defined by eq. (1):

$$\chi = \arccos(\cos Z_s \cos Z + \sin Z_s \sin Z \cos A_z) \qquad (1)$$

where $Az = |\alpha - \alpha_s|$.

α and α_s are azimuth angles of the sky element and sun position respectively,

Z_s is the solar zenith angle,

Z is the zenith angle of any sky element.

The ratio of the sky element luminance $L_{\gamma\alpha}$ to the zenith luminance L_z is:

$$\frac{L_{\gamma\alpha}}{L_z} = \frac{f(\chi)\,\varphi(Z)}{f(Z_s)\,\varphi(0°)} \tag{2}$$

The luminance gradation function φ relates the luminance of a sky element to its zenith angle:

$$\varphi(Z) = 1 + a \exp(b/\cos Z) \tag{3}$$

when

$0 \le Z \le \pi/2$,

and at the horizon is valid $\varphi(\pi/2) = 1$.

Its value at the zenith is:

$$\varphi(0°) = 1 + a \exp b. \tag{4}$$

The function f is expressing the scattering indicatrix which relates the relative luminance of a sky element to its spherical angular distance χ from the sun position:

$$f(\chi) = 1 + c(\exp(d\,\chi) - \exp(d\,\pi/2)) + e \cos^2 \chi \tag{5}$$

while its value at the zenith is:

$$f(Z_s) = 1 + c(\exp(dZ_s) - \exp(d\pi/2)) + e \cos^2 Z_s. \tag{6}$$

3. STANDARD PARAMETERS

A lot of tasks of daylight evaluations such window design, glare studies, energy analysis, daylight climate simulations, require predetermination of sky luminances and illluminances under various exterior conditions occurring during the whole year. Long term measurements of illuminances and luminance show that daylighting is continually changing and each day occur original direct and diffuse daylight levels and their daily courses.

For the design of daylighting in buildings and long term daylighting simulations is not possible to consider all occurring daylight situations. Similar to approaches in other branches of science also daylight science faced a problem to determine methods for the classification of real sky luminance distributions. When the sky luminance distribution and parametrisation will allow to specify daylight illuminance changes then methods for more precise daylight utilisation can be developed.

The CIE General Sky model [11] groups various in nature occurring sky luminance patterns in 15 types from those representing overcast skies to skies during foggy, cloudy as well as clear days. Both functions describe physically light transmission through atmosphere with various turbidity and cloud cover.

Table 1. Standard parameters.

Type	Gradation	Indikatrix	*a*	*b*	*c*	*d*	*e*	Description of luminance distribution
1	I	1	4,0	-0,70	0	-1,0	0,00	**CIE Standard Overcast Sky,** Steep luminance gradation towards zenith, azimuthal uniformity
2	I	2	4,0	-0,70	2	-1,5	0,15	Overcast, with steep luminance gradation and slight brightening towards the sun
3	II	1	1,1	-0,8	0	-1,0	0,00	Overcast, moderately graded with azimuthal uniformity
4	II	2	1,1	-0,8	2	-1,5	0,15	Overcast, moderately graded and slight brightening towards the sun
5	III	1	0,0	-1,0	0	-1,0	0,00	**Sky of uniform luminance**
6	III	2	0,0	-1,0	2	-1,5	0,15	Partly cloudy sky, no gradation towards zenith, slight brightening towards the sun
7	III	3	0,0	-1,0	5	-2,5	0,30	Partly cloudy sky, no gradation towards zenith, brighter circumsolar region
8	III	4	0,0	-1,0	10	-3,0	0,45	Partly cloudy sky, no gradation towards zenith, distinct solar corona
9	IV	2	-1,0	-0,55	2	-1,5	0,15	Partly cloudy, with the obscured sun
10	IV	3	-1,0	-0,55	5	-2,5	0,30	Partly cloudy, with brighter circumsolar region
11	IV	4	-1,0	-0,55	10	-3,0	0,45	White-blue sky with distinct solar corona
12	V	4	-1,0	-0,32	10	-3,0	0,45	**CIE Standard Clear Sky, low atmospheric turbidity**
13	V	5	-1,0	-0,32	16	-3,0	0,30	**CIE Standard Clear Sky, polluted atmosphere**
14	VI	5	-1,0	-0,15	16	-3,0	0,30	Cloudless turbid sky with broad solar corona
15	VI	6	-1,0	-0,15	24	-2,8	0,15	White-blue turbid sky with broad solar corona

Parameters *a*, *b* for calculation of gradation functions and *c* to *e* in equations (3) - (6) can be selected from tab. 1. It lists fifteen standard relative luminance distributions which are based on six groups of *a* and *b* values for the gradation function and six groups of *c*, *d* and *e* values for the indicatrix function.

Because calculation of the relative sky luminance $L_{\gamma\alpha} / L_z$ after eq. (2) is based on the spherical angular distance χ (between the sky element and the sun position) patterns of luminance distributions are symmetrical along solar meridian. The isolines of $L_{\gamma\alpha} / L_z$ of all fifteen CIE General Sky patterns are presented in fig. 2.

It is important to notice that the solar meridian of CIE General Sky pattern has to be orientated to the same direction as is the solar azimuth calculated for a specific locality and time. This task can occur when differently oriented house fronts will be analysed or interior daylighting will be simulated in different daylight climate.

4. CALCULATION OF SKY LUMINANCE DISTRIBUTION IN ABSOLUTE VALUES

The concept of relative sky luminance distribution calculations was applied for the determination of Daylight Factors, a criterion commonly used in daylight applications. An advantage of this approach is in the elimination of absolute daylight levels and their variability with the simple comparison of calculation reasults. Problems can occur when daylighting availability or energy consumptions in buildings during specific period have to be evaluated or supplementary artificial lighting would be designed. In these cases daylighting expressed in relative units is not applicable. Therefore new methods and criteria are expected. The formula (7) can be modified as follow:

$$L_{\gamma\alpha} = L_z \frac{f(\chi)\,\varphi(Z)}{f(Z_s)\,\varphi(0°)} \quad [\mathrm{cd.m^{-2}}] \qquad (7)$$

The unknown quantity in (7) is zenith luminance L_z. Its value can be determined from long-term measurements [12-14] or estimated by calculation. Several methods for more or less precise calculations of zenith luminance levels in cd.m^{-2} were published, e.g. [15-22]. Application of these formulae is restricted due to their relevance. For example, formulae derived only for the calculation of zenith luminance values under the overcast sky cannot be applied for predetermining zenith luminance under clear skies and vice versa.

Kittler and Darula in [10, 12-14, 23] recommended general formulae for calculation zenith luminance under

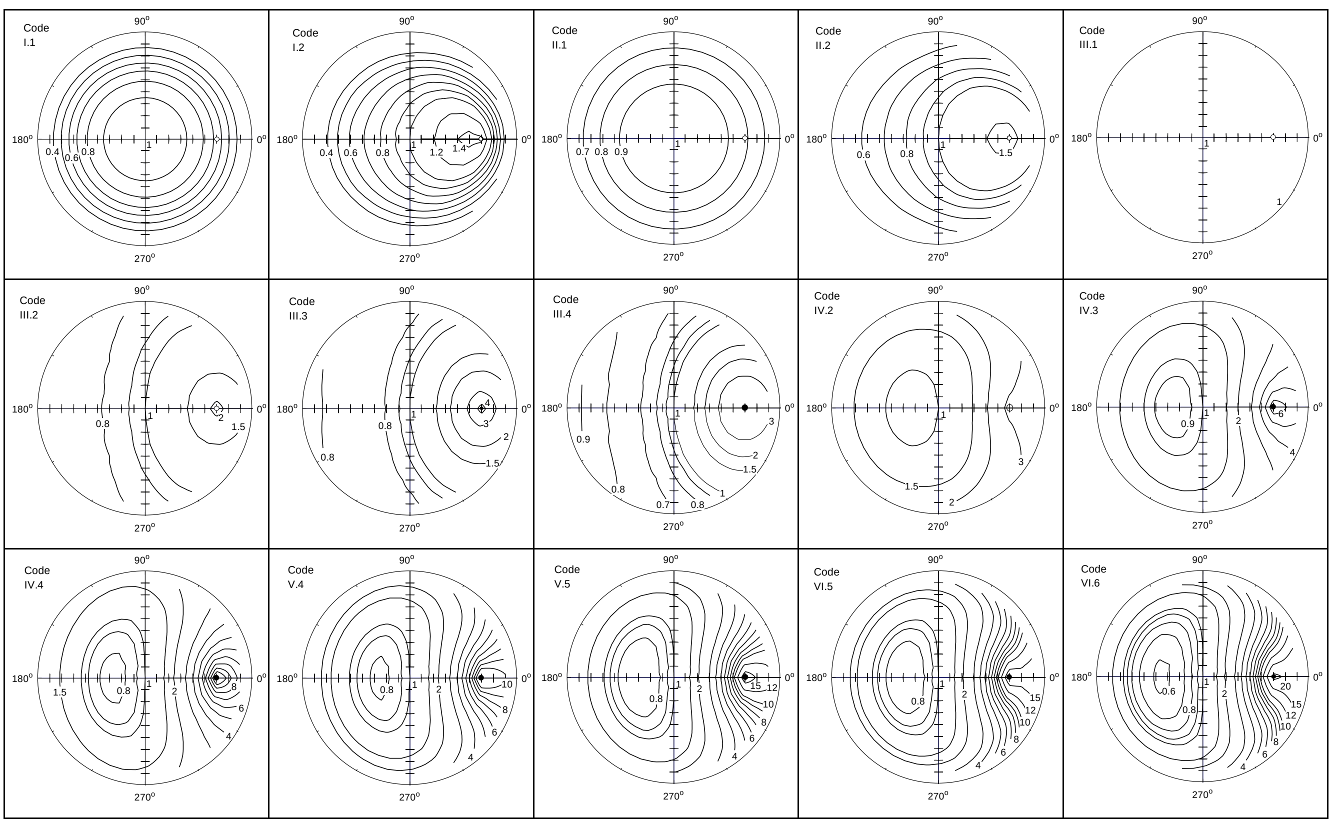

Figure 2. Relative sky luminance distributions of CIE Standard Sky for solar altitude 30°.

sunless and sunny situations. Basic input parameters are ratio D_v/E_v, i.e. normalised diffuse illuminance D_v to the horizontal extraterrestrial illuminance E_v, and luminous turbidity factor T_v, tab. 2. These parameters can be determined from regular global and diffuse measurements and typical D_v/E_v and T_v values are also published in [10, 12-14, 24, 25].

Table 2. Parameters applied for calculation of daylight descriptors in absolute units

Sky type	Sky code	Parameter					
		A1	*A2*	*B*	*C*	*D*	*E*
1	I.1	*)		54,63	1,00	0,00	0,00
2	I.2			12,35	3,68	0,59	50,47
3	II.1			48,30	1,00	0,00	0,00
4	II.2			12,23	3,57	0,57	44,27
5	III.1			42,59	1,00	0,00	0,00
6	III.2			11,84	3,53	0,55	38,78
7	III.3	0,957	1,790	21,72	4,52	0,64	34,56
8	III.4	0,830	2,030	29,35	4,94	0,70	30,41
9	IV.2	0,600	1,500	10,34	3,45	0,50	27,47
10	IV.3	0,567	2,610	18,41	4,27	0,63	24,04
11	IV.4	1,440	-0,750	24,41	4,60	0,72	20,76
12	V.4	1,036	0,710	23,00	4,43	0,74	18,52
13	V.5	1,244	-0,840	27,45	4,61	0,76	16,59
14	VI.5	0,881	0,453	25,54	4,40	0,79	14,56
15	VI.6	0,418	1,950	28,08	4,13	0,79	13,00

*) These sky types are associated with no sunlight therefore the formula in these cases is not valid.

Scattering indicatrix and gradation functions for every sky type in the proposed set [11] define the ratio of zenith luminance L_z to diffuse sky illuminance D_v:

$$\frac{L_z}{D_v}=\frac{\varphi(0°)f(Z_s)}{\int_{Z=0}^{\pi/2}\int_{\alpha=0}^{2\pi}\left[\varphi(Z)f(\chi)sinZcosZ\right]dZ\,d\alpha} \quad (8)$$

The ratio L_z / D_v can be applied as a classification parameter for the selection of CIE General Sky type from illuminance and zenith luminance measurements.
Formula (8) can be simulated by a very precise best fit approximation formula (9) valid almost to 75° of the solar altitude γ_s:

$$L_z/D_v = [\, B\,(\sin\gamma_s)^C/(\cos\gamma_s)^D + E\sin\gamma_s]/E_v \quad (9)$$

where
γ_s is solar altitude,
E_v is extraterrestrial horizontal illuminance:

$$Ev = 133{,}8\sin\gamma_s \quad [\text{klx}] \quad (10)$$

and *B*, *C*, *D* and *E* are parameters characterising each sky type, tab. 2.

Zenith luminance L_z in kcd.m^{-2} can be predetermine after the modified formula (9) to (11):

$$L_z = \frac{D_v}{E_v}\left[\frac{B(\sin\gamma_s)^C}{(\cos\gamma_s)^D} + E\sin\gamma_s\right] \quad (11)$$

The D_v/E_v ratio is influenced mainly by the overall cloudiness and turbid conditions in the direction of the sun beams which can be expressed by the luminous turbidity factor T_v. This indicates the filtering or shading effects of clouds on direct sunlight. The additional approximation of L_z in kcd.m^{-2} as a function of T_v was found and a general formula (12) is valid for all intermediate and clear situations:

$$L_Z = A\sin\gamma_S + 0{,}7(T_V+1)\frac{(\sin\gamma_S)^C}{(\cos\gamma_S)^D} + 0{,}04T_V \quad (12)$$

were
$A = (A1\ T_v + A2)$.

T_v is the luminous turbidity factor which approximates the number of ideally clean atmospheres representing an actual case. If global illuminance G_v and diffuse illuminance D_v are measured, then $P_v/E_v = G_v/E_v - D_v/E_v$ and the luminous turbidity T_v can be calculated as

$$T_v = \frac{-\ln P_v / E_v}{a_v m} \quad (13)$$

where
m is the atmospheric optical air mass,
a_v is luminous extinction coefficient.

After [26] the optical mass *m* can be calculated as:

$$m = \frac{1}{\sin\gamma_s + 0{,}50572(\gamma_s + 6{,}07995°)^{-1{,}6364}} \quad (14)$$

The luminous extinction coefficient a_v can be expressed in dependence on *m* after [17, 27]:

$$a_v = \frac{1}{9{,}9 + 0{,}043m} \quad (15)$$

5. AN EXAMPLE OF THE SKY LUMINANCE CALCULATION

The task is to estimate the luminance in the sky element $L_{\gamma\alpha}$ (azimuth α = 130° from North, vertical elevation γ = 10°), under Sky Standard IV.4 with solar altitude γ_s = 38,02° and sun azimuth α_s = 147,67°.

<u>Procedure of the calculation:</u>

The luminance distribution on the sky type IV is defined by parameters a = -1,00, b = -0,55, c = 10,00, d = -3,00 and e = 0,45 (taken from Table 1, also in [11]).

Sky element is defined by angle:
Zenith angle $Z = 90° - \gamma = 90 - 10 = 80°$

The sun zenith angle is:
$Z_s = 90° - \gamma_s = 90 - 38{,}02 = 51{,}98° = 0{,}907$ rad

Sun azimuth from the chosen element:
$A_z = |\alpha_s - \alpha| = |147{,}67 - 130| = 17{,}67°$.

Spherical angular distance of sky element from the sun position χ is:
$\cos\chi = \cos 51{,}98 \cos 80 + \sin 51{,}98 \sin 80 \cos 17{,}67 = 0{,}8462$
then $\chi = 32{,}20° = 0{,}562$ rad.

Calculation of gradation and indicatrix functions :
Gradation function for a sky element in view direction:
$\varphi(Z) = 1 - 1\exp(-0{,}55 / \cos 80) = 0{,}9579$.
Gradation function for zenith:
$\varphi(0°) = 1 + a \exp b = 1 - 1\exp(-0{,}55) = 0{,}4231$.

Indicatrix function for a sky element in view direction:
$f(\chi) = 1 + 10\,[\exp(-3\ .\ 0{,}562) - \exp(-3\,\pi/2)] + 0{,}45\ .\ 0{,}8462^2 = 3{,}0847$.
Indicatrix function for zenith:
$f(Z_s) = 1 + 10\,[\exp(-3\ .\ 0{,}907) - \exp(-3\,\pi/2)] + 0{,}45 \cos^2 51{,}98 = 1{,}7385$.

Calculation of relative luminance in the view direction normalised to zenith luminance is:
$L_{\gamma\alpha} / L_z = \varphi(Z)\, f(\chi) / ((\varphi(0°)\, f(Z_s)) = 0{,}9579\ .\ 3{,}0847/(0{,}4231\ .\ 1{,}7385) = 4{,}0175$.

If zenith luminance is known, e.g. Lvz = 4404 cd.m^{-2} (after measurements in Bratislava), then the absolute value of luminance in the investigated sky element is*:*
$L_{\gamma\alpha} = L_z\,(L_{\gamma\alpha}/L_z) = 4404\ .\ 4{,}0175 = 17693$ cd.m^{-2}

In the case that measured zenith value L_z is not available it can be estimated after formula (11) for sunny situation.
Sky standard IV.4 represents a clear sky ($A1$ = 1,440, $A2$ = -0,750, B = 24,41, C = 4,60, D = 0,72, E = 20,76) and a medium atmospheric turbidity can be assumed, i.e. T_v = 4,5.

Applying formula (12) it can be calculated:
$A = (A1\ T_v + A2) = 1{,}44\ .\ 4{,}5 - 0{,}750 = 5{,}73$

$$L_z = A \sin\gamma_s + 0{,}7(T_v + 1)\frac{(\sin\gamma_s)^C}{(\cos\gamma_s)^D} + 0{,}04 T_v \quad = \quad 5{,}73$$

$\sin(38{,}02°) + 0{,}7\,(4{,}5+1)\,(\sin(38{,}02°))^{4{,}6} / (\cos(38{,}02°))^{0{,}72} + 0{,}04\ .\ 4{,}5 = 4{,}201$ kcd.m^{-2}.

Finally the estimated luminance value of chosen sky element $L_{\gamma\alpha}$ is:
$L_{\gamma\alpha} = L_z\,(L_{\gamma\alpha}/L_z) = 4{,}201.\ 4{,}0175 = 16{,}878$ kcd.m^{-2} = 16878 cd.m^2.

In the chosen case the sky luminance $L_{\gamma\alpha}$ = 16878 cd.m^{-2} under conditions of sky luminance distribution pattern IV.4 was calculated. In a similar way it is possible to calculate relative or absolute luminances of any arbitrary sky element and represent their distribution on the whole sky, as The resulting relative sky luminance pattern is shown in Figure 3. A comparison with a fisheye photo 1, documents good similarity between the calculated luminance distribution and that on the photo. A tool SkyModeller [28] for the visualization of CIE General Sky patterns is available on <u>http://www.cadplan.com.au/</u>.

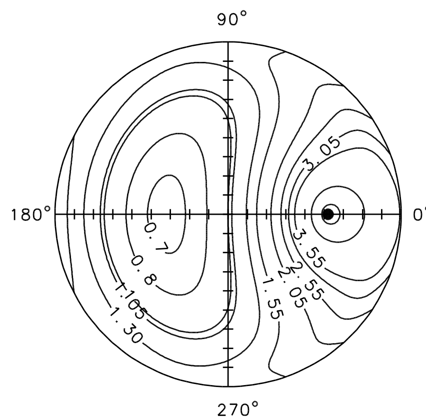

Figure 3 The relative luminance distribution representing IV.4 Sky Standard, when γ_s = 38,02°

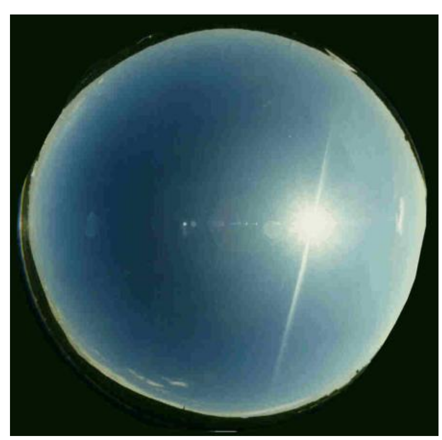

Photo 1. Fisheye picture of the clear sky

Via the integration in the solid angle of the window the sky illuminance in any point of the interior working plane can be calculated. Such online calculations are available in program MAMmodeller on the web site http://www.cadplan.com.au/.

6. CONCLUSIONS

The standard ISO 15469:2004/CIE S011/E:2003 introduced a new method for the predetermination of daylight conditions in any arbitrary locality. In spite of the standardised relative sky luminance distribution there exit possibilities for the calculation of sky luminances and daylight illuminances in physical units, i.e. in cd.m^{-2} and lux respectively.

It is important to realize, that values of sky luminances can be used only for the calculation of diffuse illuminance. In the case of sunny situations the calculation of the direct illuminance value is required. Then the global illuminance as a sum of both components, diffuse and direct, can be estimated.

The standards ISO 15469:2004 or CIE S011/E:2003 can serve not only for daylighting design purposes but also for daylight climate studies and the simulation of daylighting in general.

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