DATA PROCESSING OF BROAD-BAND SIGNALS RECEIVED BY MULTI-ELEMENT CYLINDRICAL TRANSDUCER

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The sonar based on cylindrical transducer compared with linear transducer has few advantages. Particularly its omni-directional central symmetry enables simplification of the echo signal Direction of Arrival (DOA) estimation. When the number of transducer staves is the power of two the efficient Fast Fourier Transformation can be used to calculate spatial convolution.

The article presents theoretical and technological principles of the real time digital data processing of the wideband echo signals received by cylindrical hydroacoustic transducer. Described procedures enable high resolution underwater target location and precise data display. The implementation of effective broadband pulse compression technique with high range resolution will be presented. The results of signal processing modeling and examples of application will be also discussed.

INTRODUCTION

The cylindrical transducer is composed of many piezoelectric elements regularly spaced on the circumference of cylinder. Each element of the transducer complies with specified direction of signal arrival. The hydroacoustic wave incident at piezoelectric element induces voltage. Its level is proportional to momentary acoustic pressure near the transducer surface. This acoustic pressure meets the wave equation:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)p(\vec{r},t) = 0 \quad (1)$$

In this equation c means sound speed in sea water medium, $\vec{r}$ is the vector from any point of defined coordinates. For single frequency it is possible to separate variables in equation (1) by following replacement:

$$p(\vec{r},t) = p(\vec{r})e^{2\pi j \cdot f \cdot t} \quad (2)$$

This substitution leads to Hemholtz equation:

$$\left(\nabla^2 + \frac{4\pi^2 f^2}{c^2}\right)p(\vec{r}) = 0 \quad (3)$$

In the case of cylindrical transducer it is possible to obtain numerical solution of this equation e.g. using finite element methods (FEM). Professional literature concerning to designing process of the sonar receiver - particularly DOA estimation describe the functional algorithms. They are applicable for multi-elements transducers, where the steering vector is defined as known function of the angle of incidence wave.

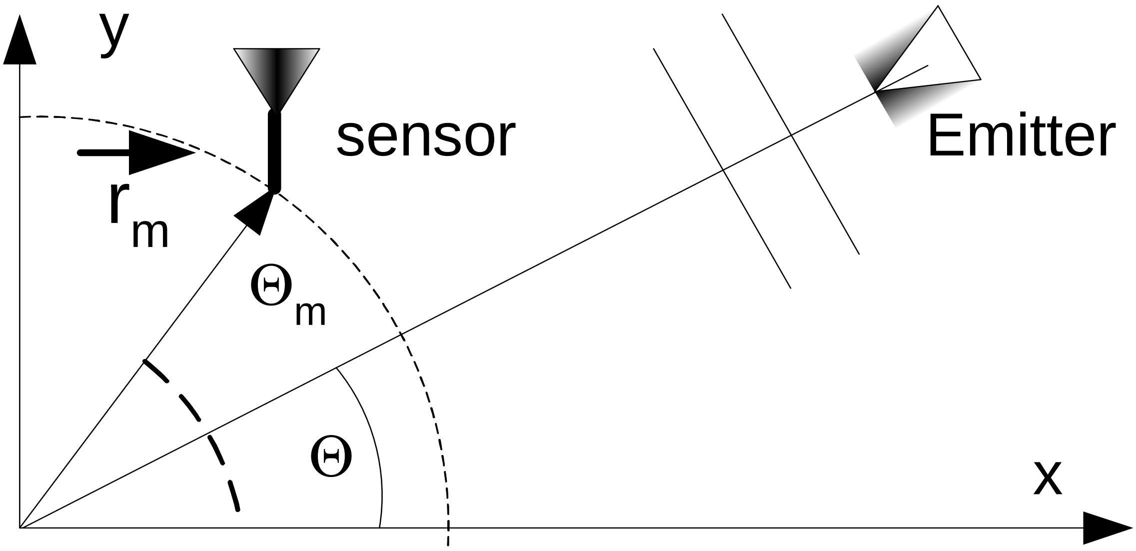

Fig.1 Two – dimensional array geometry

On the assumption that acoustic wave is a plane one in far field the Hemholtz equation can be transform to function:

$$p_f(\vec{r}_m, \Theta) = a_{f,m}(\Theta) \cdot e^{2\pi j \cdot f \cdot \tau_m(\Theta)} \quad (4)$$

For equation (4) the steering vector is defined as mapping of the single transducer element number $m \in 0..L-1$ (where L is the number of total transducer elements) into the function of incident wave angle:

$$\vec{a}(\Theta) = m \mapsto (\Theta \mapsto p_f(\vec{r}_m, \Theta)) \quad (5)$$

The coefficient a_{fm} defines the directional attenuation beampattern of m element and coefficient τ_m is the time delay resulting from distance differences between individual transducer elements and sound source.

The conventional narrow-band beamformer boils down to creation the signal replicas for the set of chosen directions. Therefore the complex weighting consists in delay

compensation and amplitude weighting. It is done for defined analytically, numerically or experimentally point source of the acoustic wave.
In case of cylindrical transducer due to its azimuth symmetry it is possible to use the periodical replica of signal instead of finite replica vector of signal. Its formula is following:

$$p_f = m \mapsto (\Theta \mapsto p_f(\vec{r}_m, \Theta) \quad (6)$$

$$p_f(\vec{r}_{m+L}, \Theta) = p_f(\vec{r}_m, \Theta) \qquad (7)$$

This function can be treated as the periodic function of the single variable due to evenly distribution of the transducer elements. This variable is the angle difference between location of the chosen transducer element θ_m and selected direction of estimation θ.

$$p_f(\vec{r}_m, \Theta) = p_f(\vec{r}_0, \Theta - \Theta_m) \quad (8)$$

In case of wideband signals the natural expansion of the matched filtration to direction of signal arrival is formation the time-spatial matrix of echo signal replica for point targets. The next step of calculation is finding the time-spatial convolution maximum of the received signal segment with calculated analytically, numerically or measured conjugate matrix of signal replica. Figure 2 shows the module map at the input and output of the time-spatial convolution algorithm for the selected segment.

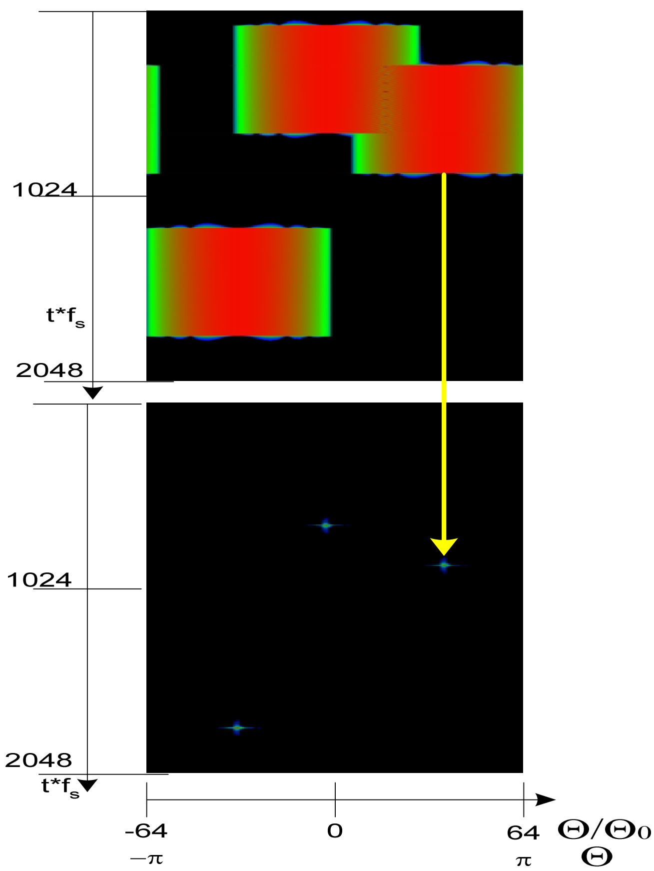

Fig.2 The convolution effect of the received signal and time-spatial matrix replica

The high quality picture results from wideband signal sampling according to Nyquist's criteria. In time domain the sampling frequency is:

$$f_s > 2 \cdot B \qquad (9)$$

Where B is the frequency bandwidth.
The sampling for angle of signal arrival is equivalent to the matching of the distance between transducer adjacent elements:

$$d \le \frac{\lambda}{2} \qquad (10)$$

Fast Fourier Transform (FFT) is the most efficient method to calculate time-spatial convolution if the number of directions and time samples in the segment are the power of 2.

1. MATRIX OF THE WIDEBAND SIGNAL REPLICA FOR CYLINDRICAL TRANSDUCER

The plane wave incident on the m-element of the transducer can be treated as the function depending only on the time delay related to m-element position.

$$p_i = m \mapsto \left(t \mapsto p(t+\tau_m)\right) (11)$$

This function is the product of complex amplitude and spectral line carrier.

$$p_i(m,t) = s(t+\tau_m)e^{2\pi j f_0 (t+\tau_m)} \qquad (12)$$

The complex amplitude function can be expanded into Fourier series limited only to non-zero spectral lines (Nyquist).

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k \in 0..N-1} S[k] e^{2\pi j \cdot f_s \cdot \frac{k \cdot t}{N}} \qquad (13)$$

Inserting the dependence (13) into equation (12), leads to following expansion:

$$p_i(m,t) \sim \sum_k S[k] e^{2\pi j \cdot f_k \cdot (t+\tau_m)} \qquad (14)$$

$$f_k = f_0 \pm f_s \frac{k}{N} \qquad (15)$$

This function is a sum of many signals expressed by formula (2). In this way the solution of Hemholtz equation (fig. 3) for every frequency f_k leads to amplitude modulation near the cylindrical transducers surface. Its form is following:

$$p_0(m,t) \sim \sum_k S[k] \cdot a_{f_k, m} \cdot e^{2\pi j \cdot \tau_m \cdot f_k} \cdot e^{2\pi j \cdot f_k t} \qquad (16)$$

$$p_0(m,t) \sim \sum_k S[k] w[m,k] \cdot e^{2\pi j \cdot \frac{k}{N} f_s t} \cdot e^{2\pi j \cdot f_0 \cdot t} \qquad (17)$$

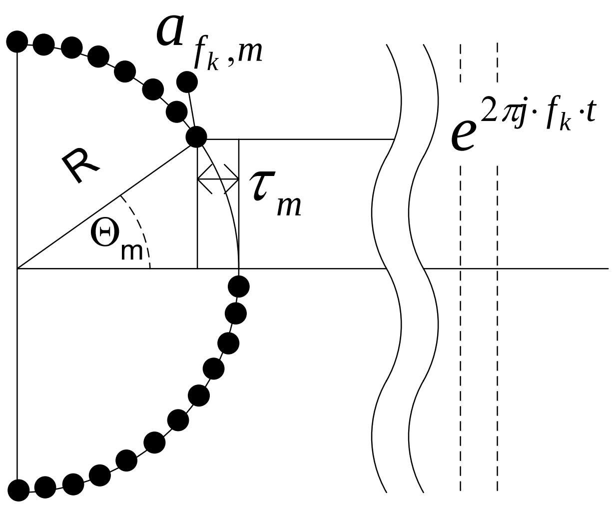

Fig.3 Helmholtz equation parameters

2. CALCULATION OF SPATIAL CONVOLUTION USING DFT

The real signal arriving at the Uniform Cylendrical Array (UCA) sensor of the element number m is denoted as x(m,t). This signal is multiply by $e^{-2\pi j\cdot f_0\cdot t}$ factor for quadrature detection. Next the signal is sampled with frequency f_s. The final operation on the signal is two dimensional Fourier (2DFFT) transformation. As the result of these conversions the following formula is obtained:

$$(n,k)\mapsto X[n,k]=DFT_{L,N}\left((m,t\cdot f_s)\mapsto x(m,t)e^{-2\pi j\cdot f_0 t}\right) \quad (18)$$

Similar transformation realized on the matrix of signal replica leads to relation:

$$(n,k)\mapsto P[n,k]=DFT_L\left(m\mapsto S[k]w[m,k]\right) \quad (16)$$

It's necessary to notice that knowledge of the Helmoltz equation solution for every frequency and complex signal envelope is sufficient to acquire two dimensional spectrum of the replica matrix. Calculation of two dimensional time-spatial convolutions can be replaced by inverse DFT of the spectrum product.

$$(m,n)\mapsto |B[m,n]| = DFT_{L,N}^{-1}((n,k)\mapsto X[n,k]\cdot P[n,k])| \quad (19)$$

The scheme of this processing method is shown on the figure 4.

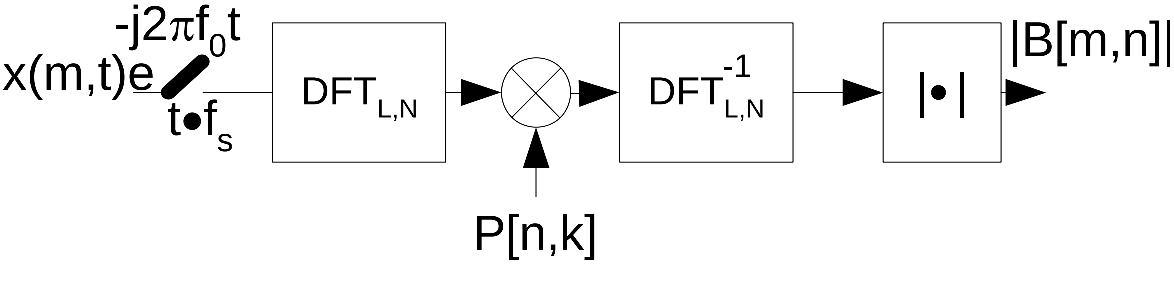

Fig.4 Beamformer processing block diagram

When DFT can be replaced by FFT the processing time is shorter due to access to calculation effective technique.

3. TECHNICAL ASPECTS OF THE SIGNAL PROCESSING REALIZATION

Signal processing algorithm described in above point leads to high angular and range resolution sonar display. The range resolution is inversely proportional to signal bandwidth. On assumption that (10) equation is satisfied the angular resolution depends on signal central frequency. The data bandwidth stream processed in real time is about tens of MB/s. The matched filtration in time domain requires operation for segments longer than sounding pulse length.

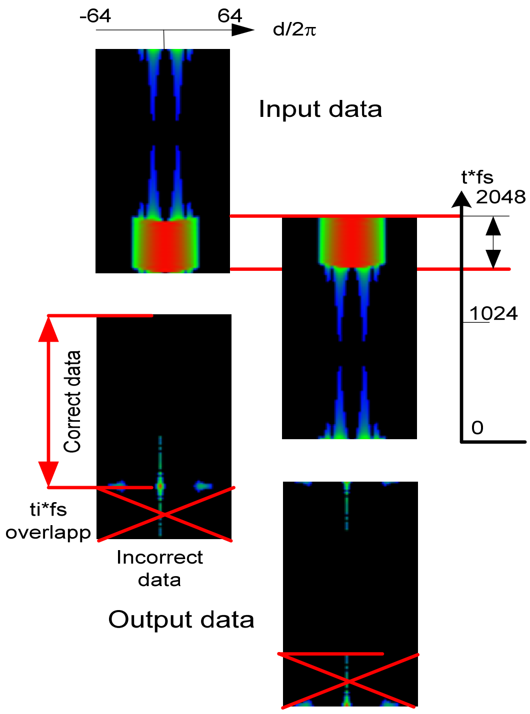

Fig.5 Overlaps application

The figure 5 shows the decomposition of the received long signal divided into smaller segments. Application of overlaps for the input segments arise from necessity to reject part of output segments results. Therefore the presented on the figure 4 beamformer processing block diagram should be supplemented by the segments forming block. This block provides to the calculation processor the segments stream with frequency a bit higher than data stream. The calculating processor needs to complete computation in time shorter than N/f_s-ti. The figure 6 presents the formal scheme of the echo signal processing taking into account input and output processors performing the segmentation process and recombination of the data stream.

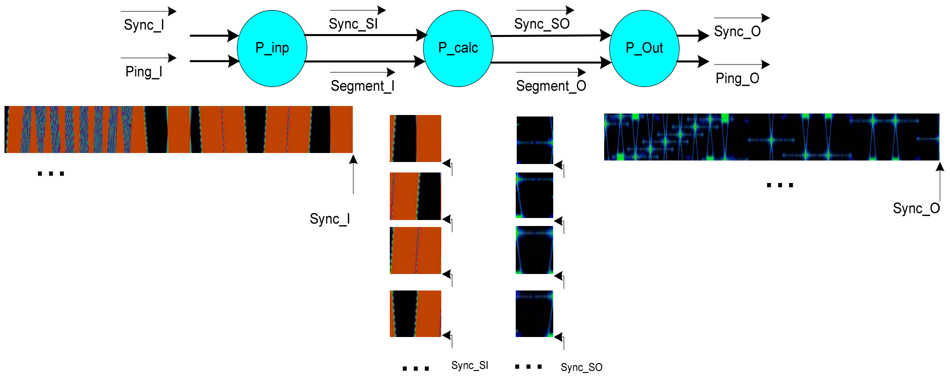

Fig.6 Processing scheme with segmentation

The critical parameter to evaluation the single segment processing is time required for FFT calculation. The most advantageous version of computation block realization is pipe processing. In this case following operations should be performed parallel during (N/f_s)-ti:

- N times L points FFT,
- L times N points FFT,
- N*L multiplications of the signal and replica complex spectrums,
- N times L points FFT^{-1},
- L times N points FFT^{-1},
- N*L computation of the complex number module.

The input stream lasting $1/f_s$ provide to the beamformer more then L complex values. Each of FFT processors should continuously process data with frequency higher then $L*f_s$. The most perspective SOC (System-On-Chip) solutions take advantage of FPGA technology. Well-known solutions are base on cascade connection of two or three RADIX blocks as the figure 4 shows. Such block simultaneously convert quadruple pipe data stream. While the input stream is written to input buffer with frequency f_{clock} each RADIX-4 block carry out 4 iterations. For example, in case of 256 points FFT within 256 clock cycles each block performs 4 iterations on 4 streams of 64 complex numbers. FFT which has more then 256 points the iterations are transferred to subsequent RADIX-4 block. As the result if this operation the linear complexity of the FFT algorithm is obtained. It enables on-line processing of the incoming data stream without clock frequency increase. On the base of catalogue data for VIRTEX-4 device manufactured by Xilinx company the clock frequency is above 400MHz what corresponds to the data stream 1.6 GB/s.

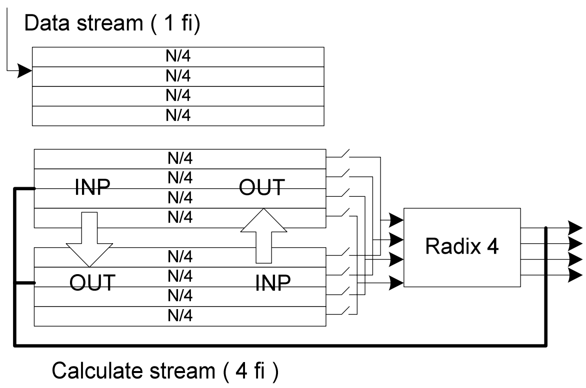

Fig.7 The idea of the pipe FFT

4. CONCLUSSIONS

The omnidirectional symmetry in azimuth axis of the cylindrical array enables directly application of DFT algorithms. Usage of overlaps allows to solve the problem of the non-periodical signal property in time domain. The problem of two dimensional replica matrix spectrum acquisitions for point target echo signal is sorted out theoretically or by measurements for artificial target. The FPGA hardware technologies are sufficient for fast signal processing realization. The acquisition of large amount of parallel samples is technologically the most difficult element to realization.

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