LARGE-ARRAY SIGNAL PROCESSING IN LONG-RANGE UNDERWATER CHANNELS: COHERENCE EFFECTS AND ADAPTIVE APPROACH TO THE MULTIMODE SIGNAL CONTROL

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The signal coherence degradation in spatial domain is an intrinsic scenario for large-array signal processing if the array size is greater as compared with the coherence length. For underwater sound, study of this issue is physically motivated by the fact that the discrete spectrum signal-carrying modes lose the mutual covariance at long ranges, so the signal coherence loss at the array inputs can be considerable. This is a case of comparative study of the large-array beamformers, optimal one included, aimed at realistic estimations of the coherence-induced effects on their performance. In this paper, we address the other aspect of the problem, namely, the source array synthesis in a random-inhomogeneous multimode channel under the criteria specified for the signal processor in remote receiving array. The criteria of the output SNR and the array SNR gain are considered to be a reasonable choice. An adaptive approach to the multimode signal control by the use of vertical source array is then formulated as an iterative algorithm for the source excitations and the corresponding modal spectrum of the signal wavefield. Computer simulation is performed for the vertical source and receiving arrays operating in the shallow-water channel from the Barents Sea.

INTRODUCTION

Recent developments of the array signal processing have been stimulated in part by numerous applied problems of the signal detection and estimation in random-inhomogeneous and nonstationary transmission channels with linear arrays. The most actuality of these problems in the last decades was found with application to low-frequency sonars [1] due to a basic feature of long-range underwater sound in the upper ocean, namely, its multimode propagation. From the point of view of a general signal processing theory, the subject is array processing of the discrete spatial spectrum signal against the noise background which also contains a discrete spectrum component. The most interesting case here is considered to be the case of large arrays, where the term “large array” emphasizes a possibility of (at least, partial) spatial resolution of the discrete spectrum modes and, therefore, a possibility to use any differences in modal spectra of the signal and the noise interference to be processed. Moreover, a principal issue of large arrays in random-inhomogeneous channels is the signal coherence, or its loss, which is a natural and well known scenario of long-range signal propagation in underwater channels.

Thus, two specific features of large-array signal processing in long-range underwater channels are of the most importance: (i) multimode discrete spreading of the signal spatial spectrum and (ii) loss of the signal coherence in space, which results from multiple signal scattering by random inhomogeneities of the channel or/and its random surface. The both features motivate an adequate study of the model-based signal processing. In this context, comparative study of the array processor techniques focused on realistic estimations of the array processor performances under specific environmental conditions is considered to be one of the key issues. Such an analysis would give an effective prognosis for the coherence-induced performance degradation.

In this paper, we start with a short overview of our recent research in the field outlined above. Our approach is based on a general model of the acoustic signal and noise fields of the discrete spatial spectrum and theoretical development of the small-signal detection problem with application to long-range underwater sound (Sec. II). Then, in Sec. III, we exploit an interrelated consideration of the source array synthesis and array signal processing in the particular case when the both arrays operate in a random-inhomogeneous underwater channel and their mutual distance is large enough for the signal coherence loss to be considerable. The optimal source excitation and optimal beamforming are shown to be obtained by the use of a similar technique of the algebraic eigenvalue-eigenvector decomposition associated with a statistical model of signal propagation through the channel. We then show that this consideration leads to formulation of an adaptive approach to the source array synthesis and specific algorithms which realize this approach. We emphasize that excitation of the source array is varied in direct dependence on the statistical effects of multimode signal propagation and the chosen criterion of signal processing at the channel “output” so the algorithms developed are naturally adaptive ones. Next, we present in Sec. IV some numerical example to demonstrate the efficiency of this approach. Our simulations are performed here for the vertical source and receiver arrays arranged in a shallow-water sound channel typical for the summer season in the Barents Sea.

1. LARGE-ARRAY BEAMFORMERS IN MULTIMODE CHANNELS

Generally, the problem of array signal processing is to detect a signal source and/or to estimate unknown source or transmission parameters. In this respect, the maximum-likelihood (ML) processor is well known to be of fundamental importance because it is optimal for a variety of detection and estimation criteria. Conventional beamformers such as those used for plane-wave detection or bearing estimation in radar are not generally an effective choice in realistic underwater channels. The multimode signal propagation leads to the fact that (i) spatial components of the signal and ambient noise fields are not plane waves but normal modes of a channel (the plane waves can be only a particular case of horizontal arrays), and (ii) the signal coherence loss is caused by range-dependent degradation of the cross-modal covariance due to statistical effects of multiply sound scattering. The last specific feature leads to the quadratic scheme of optimal array beamforming which is considered to be a generalization of the well-known linear (weight-and-sum) scheme under the conditions of the coherence-reduced signal against the noise background [2-4]. Incorporation of the discrete spectrum signal model to comparative study of the linear and quadratic array processors, optimal ones included, with emphasis on cross-modal covariance effects on large-array performance was first made in [5,6] and then developed in [7,8].

As is well known, signal wavefield in a multimode transmission channel far enough from a source can be generally represented as a superposition of the discrete spectrum modes. Thus, the signal and additive noise interference have a similar vector representation at an N-element array inputs:

$$\mathbf{s} = \sum_{\mathrm{m}=1}^{M} a_{\mathrm{m}}\mathbf{u}_{\mathrm{m}} \ , \qquad \mathbf{n} = \mathbf{n}_0 + \sum_{\mathrm{m}=1}^{M} b_{\mathrm{m}}\mathbf{u}_{\mathrm{m}} \ , \tag{1}$$

where the vector $\mathbf{u}_{\mathrm{m}}$ represents the m-th modal shape at the array inputs, or modal vector ($m = 1,2, \ldots, M$); $a_1, a_2, \ldots, a_M$ are the signal modal amplitudes; similarly, $b_1, b_2, \ldots, b_M$ are the ambient noise modal amplitudes; M is a total number of the discrete spectrum modes; $\mathbf{n_0}$ is the additive non-modal noise component, array noise included. We suppose that **s** and **n** are random and statistically independent vectors, **n** is a Gaussian vector, and $\mathbf{n_0}$ is a spatially-white isotropic noise. Following a general idea of spatial-temporal processing factorization, we restrict ourselves to the study of array signal processing in the space domain only. This means that all the considerations refer to some fixed frequency of the signal and noise wavefields.

The $(N \times N)$ spatial covariance matrices of the signal and noise wavefields at the array input are given by

$$\mathbf{M}_{\mathrm{ss}} = <\mathbf{s}^*\mathbf{s}^{\mathrm{T}}> = \mathbf{U}^*\mathbf{R}_{\mathbf{ss}}\mathbf{U}^{\mathrm{T}}, \quad \mathbf{M}_{\mathrm{nn}} = <\mathbf{n}^*\mathbf{n}^{\mathrm{T}}> = \mathbf{I} + \mathbf{U}^*\mathbf{R}_{\mathrm{nn}}\mathbf{U}^{\mathrm{T}}, \tag{2}$$

where the "internal" matrices $\mathbf{R}_{\mathrm{ss}}$ and $\mathbf{R}_{\mathrm{nn}}$ are the $(M \times M)$-dimensional cross-modal covariance matrices of the signal and noise, respectively, which are determined by the mutual correlations of the modal amplitudes, the $(N \times M)$-dimensional matrix **U** is the matrix of modal structure, each m-th column of which is simply the modal vector $\mathbf{u}_{\mathrm{m}}$, and **I** is the unit matrix corresponding to the spatially-white noise under the assumption that the input powers of the modal signal and noise are both normalized to its power. Hereinafter the superscripts "*", "$^{\mathrm{T}}$", and "$^+$" denote complex conjugation, transpose, and complex conjugate transpose, respectively.

For the array processor performance we use the output signal-to-noise ratio (SNR) defined as the deflection ratio, or detection index [2-4]:

$$q = \frac{<P(\mathbf{s}+\mathbf{n})> - <P(\mathbf{n})>}{[<P^2(\mathbf{n})> - <P(\mathbf{n})>^2]^{1/2}} \tag{3}$$

where $P(\cdot)$ is the power in the processor output (detection statistic). It is known that maximum of the output SNR (3) leads to the small-signal asymptotic of the optimal ML processor, however, it can be realized for an arbitrary (non-Gaussian) signal when the ML approach is not generally applicable.

The array gain G is then defined as the output SNR q normalized to the input SNR q_0 which is determined by spatially-averaged input signal and noise intensities:

$$G = \frac{q}{q_0}, \quad q_0 = \frac{\mathrm{Tr}(\mathbf{M}_{\mathrm{ss}})}{\mathrm{Tr}(\mathbf{M}_{\mathrm{nn}})} \ . \tag{4}$$

The SNR performance (3) was effectively used to compare the array gain (4) and its coherence-induced loss for the set of array beamformers, linear and quadratic ones included, in different physics-based scenarios of the signal and noise inputs, which were motivated by long-range low-frequency underwater sound [3-8].

The intrinsic factors, the cross-modal covariance and modal resolution (formally, orthogonal properties of the modal vectors from Eq. (1)), were shown [5,6] to affect mutually array beamforming scheme and the array gain. The most dramatic situation arises if the signal-carrying modes are weakly correlated and the array length is large enough to resolve all the modes. In this particular case, the signal rank *R* (namely, rank of the signal covariance matrix from Eq. (2)) is about *M*, and the optimal beamformer can be physically interpreted as incoherent combination of *M* partial linear beamformers which are spatial filters of the partial signal modes. In practice, however, only the largest signal eigenvalues and an "effective" signal rank R_{eff} are of real importance for the SNR performance examination [4,6,7]. Both the first (largest) eigenvalue and the "effective" rank are determined by the array length as compared to the signal coherence length, so can be *a priori* estimated from the model-based calculations or directly measured by the array as was made in Ref. [9]. In a more general case of partial cross-modal covariance and/or partial modal resolution by the receiving array, the signal rank increases (i.e. the signal coherence length increases for a fixed array size), and the optimal quadratic processor contains much less number of the partial linear beamformers, $R_{eff} << M$. Realistic estimates can give, however, the case of $R_{eff} >> 1$ if the total number of signal-carrying modes $M >> 1$. For example, this is a real case of long-range underwater sound in the upper ocean [7].

Moreover, the beamforming techniques very differ from the practical point of view of environmental and/or system robustness. The conventional plane-wave beamformer demonstrates a poor performance as compared to an upper SNR limit realized by the optimal beamformer. From the other side, a clear disadvantage of the optimal scheme is its considerable sensitivity to mismatch effects, first of all, to modal mismatch which occur if any (environmental or system) errors in the modal shape estimates are not negligible [10]. A physical origin of such an increased sensitivity to modal mismatch is mainly cross-modal phase shifts of the signal modes, which are also incorporated to array signal processing to realize the optimal SNR performance.

2. ADAPTIVE APPROACH TO THE MUTIMODE SIGNAL CONTROL

We now turn to the signal propagation through an underwater sound channel from an N_s-element vertical source array (from the channel "input") to an *N*-element vertical receiving array (to the channel "output"). Taking into account our simulation aimed at the shallow-water sound we assume a water channel of some depth, in which the speed of sound is a function of depth, and statistical effects of multiply sound scattering is caused mainly by the statistically rough surface perturbed by the developed wind waves.

The source array is characterized by the N_s-dimensional vector $\mathbf{f}$; its entries f_j are the source excitation coefficients ($j = 1, 2, ..., N_S$) for fixed source coordinates. The received signal vector $\mathbf{s}$ from Eq. (1) is, respectively, the *N*-dimensional vector; its entries s_k are originated from the source excitations and propagation conditions and given in a general form by

$$s_k = \sum_{j=1}^{N_s} f_j G_{jk}, \quad \mathbf{s} = \sum_{j=1}^{N_s} f_j \mathbf{G}_j \;, \tag{5}$$

where the matrix entry G_{jk} is the channel Green's function for the *j*-th source and the *k*-th receiver. The vector $\mathbf{G}_j$ can be nominated, therefore, as "the Green's vector" produced at receiving array inputs by the *j*-th source with the unit excitation. Statistical properties of the entries G_{jk} (also to the entries of the signal matrix $\mathbf{M}_{ss}$ (2)) are fully determined by the random channel inhomogeneities. Similarly to the previous formulations, all the quantities here and below correspond to some fixed frequency of the Fourier-transformed data.

The quantity of our ultimate interest is the signal covariance matrix at the receiving array input, which is given by the use of Eq. (5) in a general form as

$$\mathbf{M}_{ss} = \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} f_i^* f_j < \mathbf{G}_i^* \mathbf{G}_j^T > , \tag{6}$$

where the (N x N)-dimensional matrices $< \mathbf{G}_i^* \mathbf{G}_j^T >$ are produced by the second moments of the channel Green's function. Physically, Eq. (6) describes the correlations of the signals produced by the *j*-th and *i*-th sources and received by the *k*-th and *l*-th sensors, respectively.

Criteria and adaptive algorithms of the source array optimization

From the point of view of effective signal processing in the random channel "output", particular problem of the source array optimization is to maximize the processor performances, the SNR or the SNR gain. If the receiving array is located at a long distance from the sources, the signal coherence loss leads to essential spreading of the signal eigenvalue spectrum and, therefore, to the SNR performance degradation which can be compensated in part by the use of quadratic array beamformer. Taking into account the fact that the signal eigenvalues "are formed" by the different groups of signal-carrying modes [5,6], we can vary the source excitation vector **f** in such a way to focus the sources to the modes which then form the first (largest) signal eigenvalue. This focusing should lead to sharp increasing of the first eigenvalue and essential narrowing of the eigenvalue spectrum, so we can exploit the basic scheme of linear (weight-and-sum) beamformer as a proper choice and find its optimal weight vector to realize the maximum SNR and gain.

For an arbitrary linear beamformer characterized by an arbitrary *N*-dimensional array weight vector **w**, the output SNR (3) is given by the well-known ratio of the signal and noise quadratic forms which are, namely, the output powers of the signal and noise, respectively:

$$q = \frac{\mathbf{w}^+ \mathbf{M}_{ss} \mathbf{w}}{\mathbf{w}^+ \mathbf{M}_{nn} \mathbf{w}} . \tag{7}$$

With the help of Eq. (6) the output signal power (the numerator in Eq. (7)) can be expressed as a function of both the array vectors, one of which is the source excitation vector **f** and the second one is the beamformer weight vector **w**:

$$P_s = \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} f_i^* f_j \mathbf{w}^+ < \mathbf{G}_i^* \mathbf{G}_j^{\mathrm{T}} > \mathbf{w} = \mathbf{f}^+ \mathbf{P} \mathbf{f} \,, \tag{8a}$$

Equation (8a) gives us the output signal power as a matrix quadratic form of the $(N_s \times N_s)$-dimensional matrix **P**, generated by the source vector **f**. Its physical meaning is clear if we rewrite the matrix entries as

$$P_{ij} = < y_i^* y_j > , \qquad y_j = \mathbf{w}^{\mathrm{T}} \mathbf{G}_j \,. \tag{8b}$$

We see that each entry y_j is the beamformer response to the single j-th source with the unit excitation, and this response could be physically interpreted as the partial (for each partial source) output signal. The full set of these signals form the N_s-dimensional vector **y** at the receiving array output. The matrix **P** is, therefore, the covariance matrix of this output signal **y**, and its diagonal elements are exactly the output powers corresponding to the unit sources.

Thus, we obtain finally the output SNR (7) as the function of both the source and beamformer vectors:

$$q = \frac{\mathbf{w}^+ \mathbf{M}_{\mathrm{ss}}(\mathbf{f}) \mathbf{w}}{\mathbf{w}^+ \mathbf{M}_{\mathrm{nn}} \mathbf{w}} = \frac{\mathbf{f}^+ \mathbf{P} \mathbf{f}}{\mathbf{w}^+ \mathbf{M}_{\mathrm{nn}} \mathbf{w}} \,. \tag{9}$$

The first expression here is effective to state a conventional problem of optimal beamforming for a given signal wavefield at the array inputs (namely, for a given source vector **f**). In its turn, the second expression is effective to state an "inverse" problem of optimal source synthesis for a given remote beamformer (namely, for a given weight vector **w**). Statistical properties of the transmission channel are very essential in both the statements and give the origin for the covariance matrices $\mathbf{M}_{\mathrm{ss}}$ and **P**, respectively.

From the general formulations outlined above, the following iterative scheme of combined optimization of both the source and receiver arrays in a random-inhomogeneous channel could be evidently developed.

As the first step, we take the starting source vector **f**(0), the role of which is "to train" the array system in the transmission channel. This vector could be chosen in a rather simple way; for example, it could consist of uniform entries corresponding to the equal source excitations. However, more complicated *a priori* considerations could be also involved. One of the reasonable choices is to derive the starting vector **f**(0) from the source synthesis problem stated to maximize the input signal power at the array receivers. Such a problem is quite similar to the synthesis problem developed by Talanov for a regular multimode channel [11]. In any case, we obtain for the receiving array (i) the starting covariance matrix $\mathbf{M}_{\mathrm{ss}}(\mathbf{f}(0))$ at the receiving array inputs and (ii) the weight vector **w**(0) and the output SNR $q(0)$. The last two terms are given from the following algebraic eigenvalue-eigenvector problem (for more details, see Refs. [4-8]):

$$\lambda_p \mathbf{v}_p = \mathbf{M}_{\mathrm{nn}}^{-1} \mathbf{M}_{\mathrm{ss}} \mathbf{v}_p \,, \quad p = 1, 2, ..., \mathrm{rank}(\mathbf{M}_{\mathrm{ss}}) \,. \tag{10}$$

The largest eigenvalue λ_1 gives us the maximum SNR $q(0)$, and the eigenvector $\mathbf{v}_1$, the optimal beamformer **w**(0).

As the second step, we reform the source vector **f**(0) in such a way to maximize the SNR $q(\mathbf{f})$ from Eq. (9) for the given **w**(0). This procedure is derived from a similar eigenvalue-eigenvector problem, but, however, for the matrix **P** from Eq. (8b):

$$\mu_p \mathbf{f}_p = \mathbf{P}\,\mathbf{f}_p\,, \quad p = 1, 2, ..., \mathrm{rank}(\mathbf{P})\,. \tag{11}$$

Here, the largest eigenvalue μ_1 gives us the new value of the SNR, denoted as $q(1)$, by

$$q(1) = \frac{\mu_1}{\mathbf{w}(0)^+ \mathbf{M}_{\mathrm{nn}}\, \mathbf{w}(0)}\,, \tag{12}$$

and the eigenvector $\mathbf{f}_1$ gives us the new source vector **f**(1).

These two steps are together the first cycle of the adaptive procedure of the source–receiver system optimization. Its result is, therefore, the source vector **f**(1), the output SNR $q(1)$, and beamformer weight vector **w**(0). The second and subsequent cycles are the same as the first one: for each j-th cycle we optimize the weight vector $\mathbf{w}(j-1)$ for the given (obtained from the previous cycle) source vector $\mathbf{f}(j-1)$ and then correct the source excitations to enhance the output SNR $q(j)$. Such an iterative scheme of combined step-by-step optimization gives us, therefore, the adaptive control of the signal wavefield in a random channel. We use the term "adaptive" to emphasize a direct dependence of the source array vector and the corresponding modal spectrum of the signal wavefield from both the environmental conditions (which give the channel Green's function and the signal covariance matrix at a far distance) and specific criterion of the array signal processing to be stated for the receiving system. We use here the maximum SNR criterion but this is not generally a single choice.

To develop a similar approach to maximize the array gain G instead of the SNR performance we should take into account that the input SNR q_0 (4) is also a function of the source excitation due to its dependence upon the signal intensity (averaged over the array elements). For our study, it is useful to represent the array gain as the product of two factors, the "signal gain" and the "noise gain", which show directly the processor efficiency for the desired signal against the modal noise background:

$$G = \frac{\mathbf{w}^+ \mathbf{M}_{\mathrm{ss}}(\mathbf{f})\,\mathbf{w}}{\mathrm{Tr}(\mathbf{M}_{\mathrm{ss}}(\mathbf{f}))} \times \frac{\mathrm{Tr}(\mathbf{M}_{\mathrm{nn}})}{\mathbf{w}^+ \mathbf{M}_{\mathrm{nn}}\,\mathbf{w}}\,. \tag{13}$$

We see that the first ratio, which is actually the signal gain, is the function of the source vector **f** and, therefore, could be enhanced by its iterations as was shown above. The second ratio, or the noise gain, does not formally and physically depend on the sources and could be enhanced only by the array weight vector. It is easy to show that the signal gain is the following ratio of the quadratic forms generated by the source vector:

$$G_{\mathrm{s}} = \frac{\mathbf{w}^+ \mathbf{M}_{\mathrm{ss}}(\mathbf{f})\,\mathbf{w}}{\mathrm{Tr}(\mathbf{M}_{\mathrm{ss}}(\mathbf{f}))} = \frac{\mathbf{f}^+ \mathbf{P}(\mathbf{w})\,\mathbf{f}}{\mathbf{f}^+ \mathbf{T}\,\mathbf{f}}\,. \tag{14}$$

Here, the entries of the new $(N_{\mathrm{s}} \times N_{\mathrm{s}})$-dimensional matrix **T** are given by

$$T_{ij} = \sum_{k=1}^{N} < G_{ik}^* G_{jk} >

and show the cross-source correlations of the Green's vectors at the receiving array input.

To maximize the array gain G (13) as a function of the array weight vector $\mathbf{w}$ for a given source vector $\mathbf{f}$ and, therefore, for a given signal wavefield, we need to maximize the output SNR q from Eq. (9). The problem of our particular interest is, however, to maximize the gain as the function of the source vector $\mathbf{f}$, so we obtain another eigenvalue-eigenvector basis derived from Eq. (14):

$$\nu_p \mathbf{f}_p = \mathbf{T}^{-1}\, \mathbf{P}\, \mathbf{f}_p\,, \qquad p = 1, 2, ..., \mathrm{rank}(\mathbf{P})\,. \tag{15}$$

Similarly to Eqs. (10)-(12), the largest eigenvalue ν_1 is the signal gain factor, and the resulted array gain is given by the use of Eq. (13) as

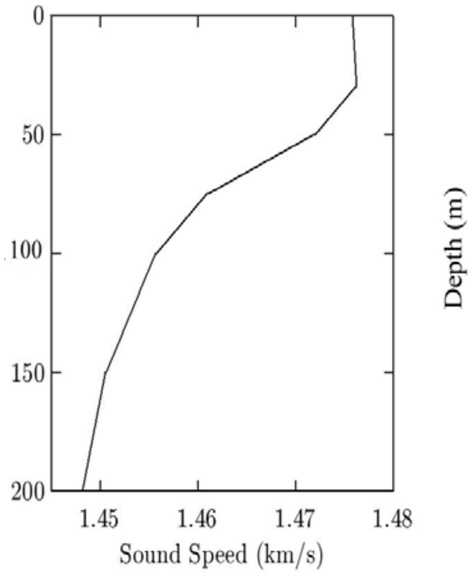
$$G = \nu_1 \frac{\mathrm{Tr}(\mathbf{M}_{\mathrm{nn}})}{\mathbf{w}^{+}\, \mathbf{M}_{\mathrm{nn}}\, \mathbf{w}}\,. \tag{16}$$

In its turn, the first eigenvector $\mathbf{f}_1$ from Eq. (15) gives us the optimal source excitation vector under this criterion. To avoid any singularities caused by possible degeneration of the matrix $\mathbf{T}$, we probably need to add a weak full-rank matrix, for example, simplest unit matrix. This additional matrix is formally similar to a weak isotropic component in the noise covariance matrix to be inverted in Eq. (10) to obtain the optimal beamformer in all the cases, even if the modal noise covariance is not a full-rank matrix.

The step-by-step iterations of the source vector (15), the array weight vector (10), and the array gain (16) lead us to another adaptive technique of the multimode signal wavefield control as is compared with the previous derivations under the SNR criterion. A general procedure is, however, the same. It starts from some training source vector and then focuses on the consequent maximization of the array gain as the result of step-by-step reforming the both array vectors.

3. COMPUTER SIMULATION

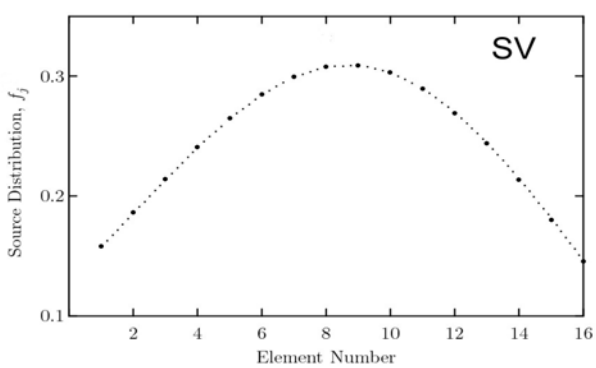
In this section we outline the results of numerical examinations of the adaptive source array synthesis presented above. The simulations are performed for the vertical $\lambda/2$-arrays of the acoustic sources and receivers in a realistic shallow-water channel. All numerical predictions are demonstrated for a typical sound-speed profile from the summer Barents Sea (the water depth is chosen to be 200 m) under proper assumptions on the rough sea surface. The surface is assumed to be perturbed by the wind waves, and the well-known Pierson–Moskowitz spectrum is used for the surface autocorrelation function. This model is exploited to calculate the channel Green's functions that give us the entries of the signal covariance matrix for a given set of the source and receiver coordinates and the other source/environmental parameters. The same model was used effectively in our previous studies focused on the signal coherence and its effects on array beamforming in realistic shallow-water environments [8,9].

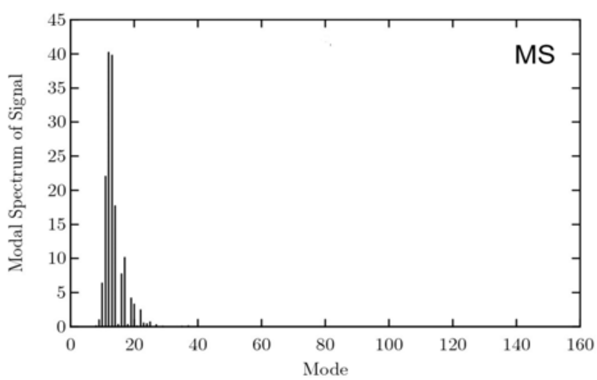


A short comment concerns the noise covariance matrix which is also required for examination of the output SNR and the array gain. The noise interference from Eq. (2) is physically assumed to be the sum of relatively weak spatially white (full-rank) noise and the anisotropic modal interference. For the last component we incorporate the well-adopted model of the surface-generated ambient sea noise developed by Kuperman and Ingenito [12] and use their formulae to calculate the noise cross-modal covariance matrix $\mathbf{R}_{nn}$. The ratio of the white noise intensity to the modal noise intensity (averaged over the array elements) is fixed at the small level of –23 dB.

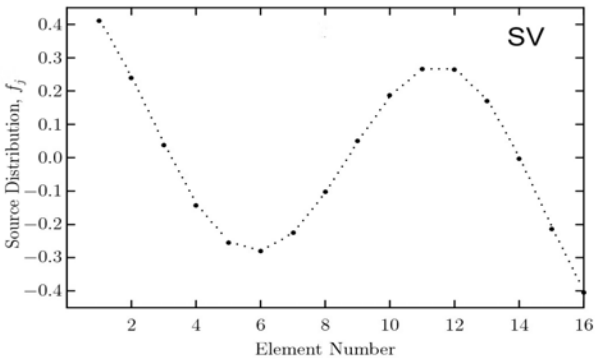
The following set of the parameters is used in simulations: the signal-carrying frequency (1000 Hz), the midpoint depth of the source array (100 m), number of elements in the source and receiver arrays (N_s = 16 and N = 64, respectively), depth of the first receiver (3 m), and the distance between the arrays (100 km). The environmental parameter which is required to predict the signal covariance matrix and the signal coherence degradation at a fixed distance is the speed of wind (10 m/s).

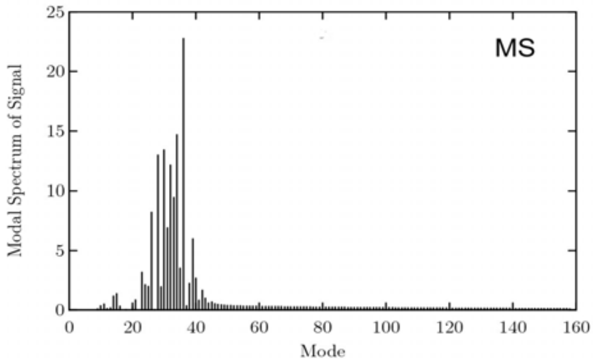
The simulation results are presented in a following way. First, we show the set of figures corresponding to the starting iteration as it was formulated above. The starting source vector is calculated to maximize the source power transformation to the total signal intensity at the receiving array inputs as was mentioned above. The left figure shows the first eigenvector of the source synthesis problem, which is exactly the starting source vector (SV). We note that the number of the largest eigenvalues (their spectrum is not shown here), or effective rank of the matrix governing this synthesis problem, is small in comparison with the total source number N_s. Therefore, the number of essential components in the corresponding orthogonal basis is also small and restricts the choice of the source excitations to be used for effective focusing of the source power to the discrete spectrum modes. We see also from the left figure that the starting source vector exhibits rather smooth spatial distribution of its entries, so the starting iteration for the source vector could be chosen to be the simplest one, for example, the vector with equal entries (i.e., the uniform source excitations). The modal spectrum (MS) of the radiated signal is also shown to illustrate the starting iteration (the right figure). We see here that number of the signal-carrying modes is much less then the total number of the discrete spectrum modes in the channel (for the chosen frequency of 1000 Hz).





Next, we show analogous figures for the first iteration under the criterion of the maximum SNR gain as was formulated in Sec. III. In this case, the eigenvalues and eigenvectors are calculated directly from Eq. (15). The adaptive corrections of the source vector (SV) and the excited modal spectrum (MS) are demonstrated here to illustrate how the algorithm of the signal wavefield control works.

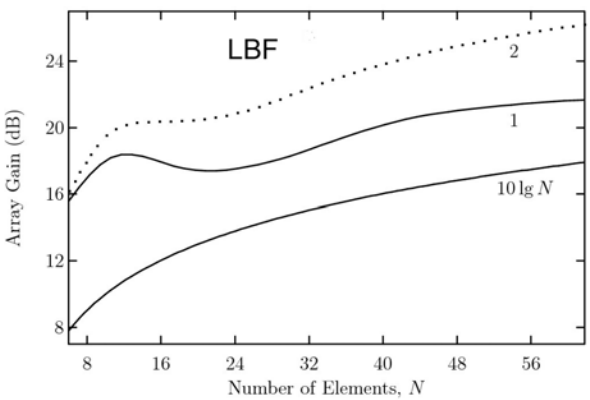


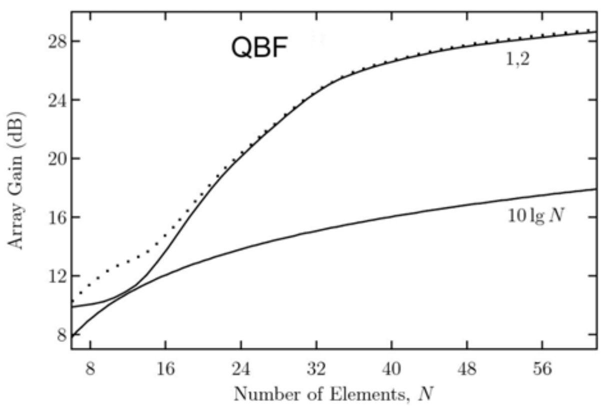


Finally, we show the array gain G as a function of the receivers number N for the optimal linear (LBF) and quadratic (QBF) beamformers calculated for the starting and the first iterations of the source vector (the curves 1 and 2, respectively). The figures illustrate the effect of adaptive source correction on the output beamformer performance, which is anticipated to be mainly an enhancement of the LBF gain. Some important comments on these figures should be given.

First, for both the optimal beamformers, LBF and QBF, the array gain is shown to be considerably high in comparison with the number of array elements, even in the case of starting iteration in spite of the coherence-induced loss of the signal gain (the first factor in Eq. (13)). Note that the simulated signal correlations over the array (from its midpoint to the periphery) vary from 1.0 to ~0.2, so this is really the case of coherence-reduced signal. Such an optimistic situation for the array gain can be realized only due to rather effective modal noise cancellation (i.e., to the second factor in Eq. (13)) which, in its turn, is possible if the signal and ambient sea noise are localized in the different groups of modes. Calculations from the ambient noise model of Kuperman and Ingenito [12] show its maximum modal intensities near the modal numbers of 40-50, whereas the signal occupies low-order modes near the numbers of 10-20.

The second feature is an essential gain difference between the beamformers for the starting iteration (compare the curves 1), which is about 8 dB for the full-size array (for N = 64). This is exactly the effect of the signal coherence degradation as was shown in details in our previous papers [6-8]. A pronounced feature is also a practical vanishing of this “gap” between the LBF and QBF performances after the first adaptive iteration of the source vector. As we see from the left figure, this is the result of essential adaptive increase of the LBF gain.





We also emphasize that the second and subsequent iterations almost does not lead to any enhancement of the gain. For example, for the full-size array the LBF gain values versus the number of the adaptive iterations are the following: *G(0)* = 21.7 dB, *G(1)* = 28.5 dB, and *G(2)* = 28.6 dB. This means that the adaptive algorithm presented is rather effective and provides the fast convergence to the optimal (for the given set of parameters) array gain performance.

The extended simulations performed for the other set of parameters demonstrate that the effect of adaptive source iterations on the array gain depends critically on the signal-carrying frequency and the source array location in the channel. Physically, these dependencies are originated from the very different conditions for the signal mode excitation if the frequency and/or source depths are changed, and from the frequency-dependent signal coherence. In all the cases examined the resulted gain enhancement is achieved by adaptive control of the signal modal spectrum against the modal noise background. The signal intensity and coherence over the receiving array also depend on the modes excited at the channel "input". So the signal modal spectrum control by the sources is the key point of the approach discussed. Moreover, in real environmental conditions of shallow-water sound propagation we should take into account the effect of range-dependent evolution of the modal spectrum which is a combined effect of the bottom-induced modal stripping and surface-induced sound scattering [9].

4. CONCLUSIONS

A fundamental feature of the source synthesis problem in a regular multimode channel, as was shown by Talanov [11], is a finite number M of the propagating normal modes which form the signal wavefield far enough from sources. This means that the problem has a double discreteness restricted by a total number of the sources and a total number of the normal modes, and the problem itself reduces to the interrelated eigenvalue–eigenvector problems in the N_s-dimensional "source domain" and the M-dimensional "mode domain". As a result, we have a general technique (i) to estimate the efficiency of an arbitrary source array in a channel from the point of view of effective exciting the given set of propagating modes, and (ii) to derive the optimal source vector to maximize this efficiency. Our further development in this theory concerns a more complicated case of a random-inhomogeneous multimode channel, or, in the other words, of a large distance for which we should take into account the coherence loss. The optimal source excitation was obtained by using a general technique of the eigenvalue–eigenvector expansion associated with the physics-based model of multimode signal propagation in a random channel. The particular criteria of the source array optimization can be different. For example, the practically useful formulations are to maximize the output SNR or the SNR gain derived for an N-element receiving array. Thus, the basic idea is to adaptively control the signal wavefield in such a way to achieve the source-induced optimization of the processor performance against the ambient noise background at a large distance from the sources.

Following this idea we presented the iterative algorithms of the signal wavefield control in random-inhomogeneous underwater channel by the use of vertical source array. Our numerical examinations performed for realistic shallow-water channel from the Barents Sea exhibit distinctly the adaptive technique as an effective tool for the multimode wavefield control at long ranges. An important conclusion is the fact that significant SNR gain enhancement can be achieved almost from the first adaptive iteration of the source array. In particular, the LBF gain increase was shown to be up to 5-10 dB depending critically on the source parameters and the distance.

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