APPLICATION OF NON-UNIFORM B-SPLINES OF 2-ND ORDER FOR DESCRIPTION VERTICAL DISTRIBUTION OF SOUND SPEED IN WATER

ARTUR MAKAR

Polish Naval Academy
69, Śmidowicza Str., 81-103 Gdynia, Poland
Artur.Makar@amw.gdynia.pl

Modeling of the vertical distribution of the sound speed in water is useful during determination of the depth on the basis of the acoustic method, modeling refraction phenomena and determination the trajectory of the acoustic wave. For modeling 1D functions (vertical distribution of the sound speed) and 2D (surface of the sea bottom for hydrography) uniform B-Splines was used.

In this paper description of uniform B-Splines and description and application of non-uniform B-Splines have been presented

INTRODUCTION

For description the vertical distribution of the sound speed in water was used uniform B-Splines [8, 10, 11, 12, 17, 18, 19]. Normalized B-Spline of the 1-st order can be describer in following form:

$$\Phi_i(h) = \frac{1}{m^2}\begin{cases} 0 & \text{for} \quad h \le h_0 + (i-1)m \\ h - h_0 + (i-1)m & \text{for} \quad h_0 + (i-1)m \le h \le h_0 + im \\ h_0 + (i+1)m - h & \text{for} \quad h_0 + im \le h \le h_0 + (i+1)m \\ 0 & \text{for} \quad h \ge h_0 + (i+1)m \end{cases}, \quad i \in \overline{0, N} \qquad (1)$$

where $N < n$ will be the set of points $h_i = h_0 + im$, $m = \frac{h_n - h_0}{N}$ dividing the interval $[h_0, h_n]$ on N subintervals $\Delta_n : h_0 < \ldots < h_N$.

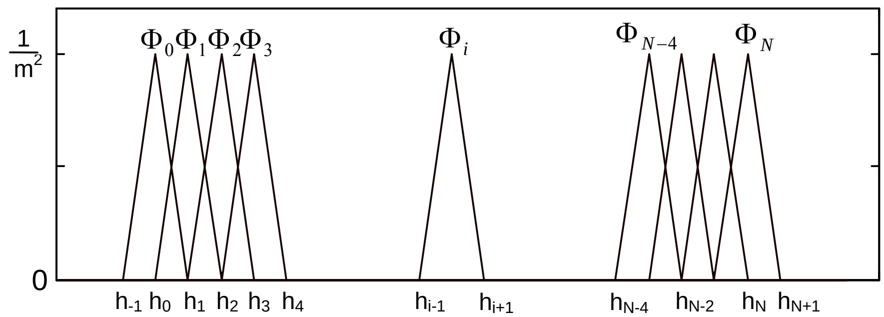

Fig.1. Basis B-Splines of the 1-st order on the interval $[h_0, h_n]$.

For example, B-Spline of the 3-rd order can be described in following form:

$$\Phi_{i,3}(h) = \frac{1}{3! m_h^4} \begin{cases} 0 & h \in (-\infty, h_{i-2}] \\ (h - h_{i-2})^3 & h \in [h_{i-2}, h_{i-1}] \\ m_h^3 + 3m_h^2 (h - h_{i-1}) + 3m_h (h - h_{i-1})^2 - 3(h - h_{i-1})^3 & h \in [h_{i-1}, h_i] \\ m_h^3 + 3m_h^2 (h_{i+1} - h) + 3m_h (h_{i+1} - h)^2 - 3(h_{i+1} - h)^3 & h \in [h_i, h_{i+1}] \\ (h_{i+2} - h)^3 & h \in [h_{i+1}, h_{i+2}] \\ 0 & h \in [h_{i+2}, \infty) \end{cases} \quad (2)$$

with nodes $h_i = x_0 + ig_h$, $m_h := \frac{h_N - h_0}{N}$, $i \in \overline{-1, N+1}$.

1. NON-UNIFORM B-SPLINES

Let $H = \{h_0, \ldots h_m\}$ be a nondecreasing sequence of real numbers, i.e., $h_i \le h_{i+1}$, *i=0,…,m-1*. The h_i, are called *knots,* and H is the *knot vector*. The *i*-th B-spline basis function of *p*-degree *(order p+1),* denoted by $N_{i,p}(h)$, is defined as

$$N_{i,0}(h) = \begin{cases} 1 & \text{if } h_i \le h \le h_{i+1} \\ 0 & \text{otherwise} \end{cases},$$

$$N_{i,p}(h) = \frac{h - h_i}{h_{i+p} - h_i} N_{i,p-1}(h) + \frac{h_{i+p+1} - h}{h_{i+p+1} - h_{i+1}} N_{i+1,p-1}(h). \quad (3)$$

Note that

- $N_{i,0}(h)$ is a step function, equal to zero everywhere except on the half-open interval $h \in [h_i, h_{i+1})$,
- for p > 0, $N_{i,p}(h)$ is a linear combination of two *(p* - l)-degree basis functions,
- computation of a set of basis functions requires specification of a knot vector, *H,* and the degree, *p,*
- Equation (3) can yield the quotient %; we define this quotient to be zero,
- the $N_{i,p}(h)$ are piecewise polynomials, defined on the entire real line; generally only the interval $[h_0, h_m]$ is of interest,

the half-open interval, $[h_i, h_{i+1})$, is called the *i*th *knot span;* it can have zero length, since knots need not be distinct.

2. EXAMPLE

Let $H = \{h_0 = 0, h_1 = 0, h_2 = 0, h_3 = 1, h_4 = 1, h_5 = 1\}$ and $p = 2$. B-spline functions of degrees 0, 1 and 2 can be written in following form:

$$\begin{gathered} N_{0,0} = N_{1,0} = 0 \quad -\infty < \mathrm{h} < \infty \\ N_{2,0} = \begin{cases} 1 & 0 \le h < 1 \\ 0 & \text{otherwise} \end{cases} \\ N_{3,0} = N_{4,0} = 0 \quad -\infty < \mathrm{h} < \infty \end{gathered} \tag{4}$$

$$\begin{aligned} N_{0,1} &= \frac{h-0}{0-0} N_{0,0} + \frac{0-h}{0-0} N_{1,0} = 0 \qquad -\infty < h < \infty \\ N_{1,1} &= \frac{h-0}{0-0} N_{1,0} + \frac{1-h}{1-0} N_{2,0} = \begin{cases} 1-h & 0 \le h < 1 \\ 0 & \text{otherwise} \end{cases} \\ N_{2,1} &= \frac{h-0}{1-0} N_{2,0} + \frac{1-h}{1-1} N_{3,0} = \begin{cases} h & 0 \le h < 1 \\ 0 & \text{otherwise} \end{cases} \\ N_{3,1} &= \frac{h-1}{1-1} N_{3,0} + \frac{1-h}{1-1} N_{4,0} = 0 \qquad -\infty < h < \infty \end{aligned} \tag{5}$$

$$\begin{aligned} N_{0,2} &= \frac{h-0}{0-0} N_{0,1} + \frac{1-h}{1-0} N_{1,1} = \begin{cases} (1-h)^2 & 0 \le h < 1 \\ 0 & \text{otherwise} \end{cases} \\ N_{1,2} &= \frac{h-0}{1-0} N_{1,1} + \frac{1-h}{1-0} N_{2,1} = \begin{cases} 2h(1-h) & 0 \le h < 1 \\ 0 & \text{otherwise} \end{cases} \\ N_{2,2} &= \frac{h-0}{0-0} N_{0,1} + \frac{1-h}{1-0} N_{1,1} = \begin{cases} h^2 & 0 \le h < 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{6}$$

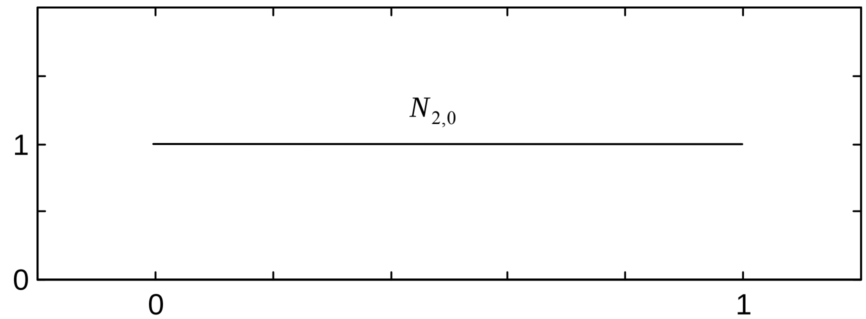

Fig.2. The nonzero zeroth-degree basis functions, $H = \{0,0,0,1,1,1\}$

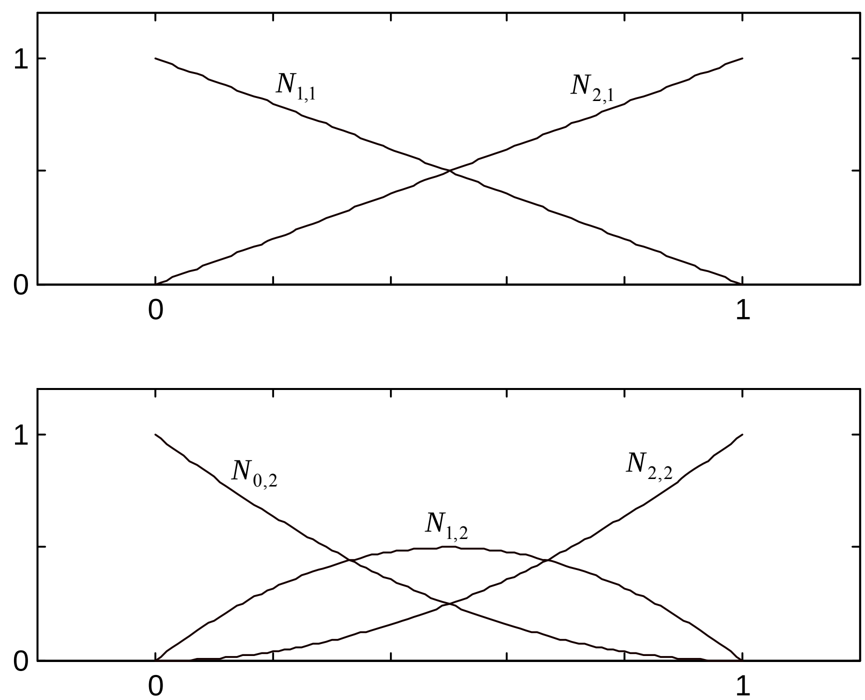

Fig.3. The nonzero first-degree (top) and second (bottom) basis functions, $H = \{0,0,0,1,1,1\}$

$N_{i,2}$, restricted to the interval $h \in [0,1]$, are the quadratic Bernstein polynomials. For this reason, the B-spline representation with a knot vector of the form

$$h = \{\underbrace{0,\ldots,0}_{p+1}, \underbrace{1,\ldots,1}_{p+1}\}$$

is a generalization of the Bézier representation.

3. B-SPLINE CURVE

A *p*-th degree B-spline curve is defined by

$$\mathbf{C}(h) = \sum_{i=0}^{n} N_{i,p}(h) w_i \mathbf{P}_i \qquad a \le h \le b \tag{7}$$

where the $\{\mathbf{P}_i\}$ are the *control* points (forming a *control polygon*), the $\{w_i\}$ are the *weights* and the $\{N_{i,p}(h)\}$ are the *p*-th degree B-spline basis functions defined on the nonperiodic (and nonuniform) knot vector

$$H = \{\underbrace{a,\ldots,a}_{p+1}, h_{p+1},\ldots,h_{m-p-1},\underbrace{b,\ldots,b}_{p+1}\}$$

4. RESULTS

Vertical distribution of the sound speed in water have been recorded 5 – 8 of May, 2006 using Valeport Midas sound speed profiler by the hydrographic ship OH 265. Below location of measurements have been presented.

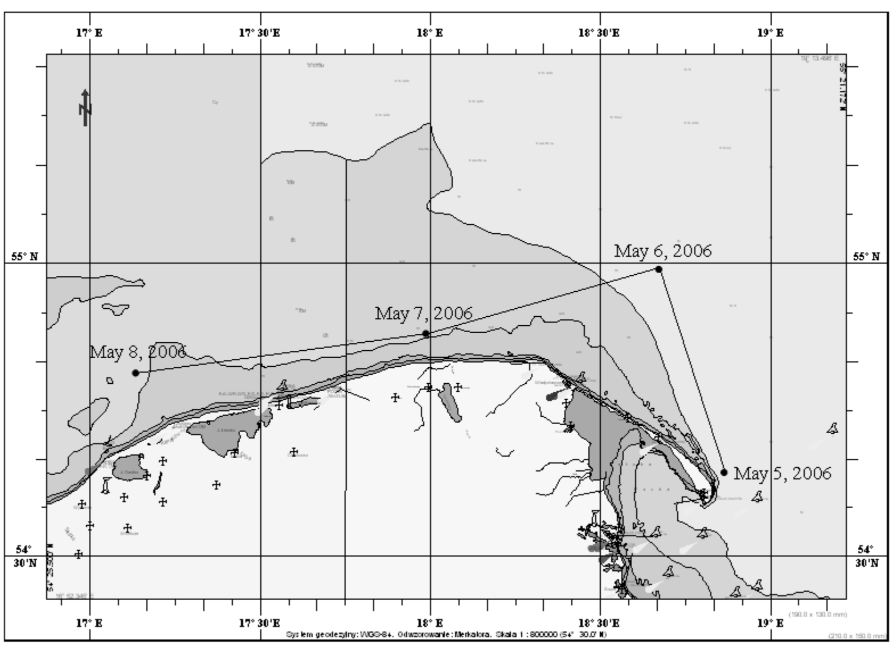

Fig.4. Location of sound speed measurements

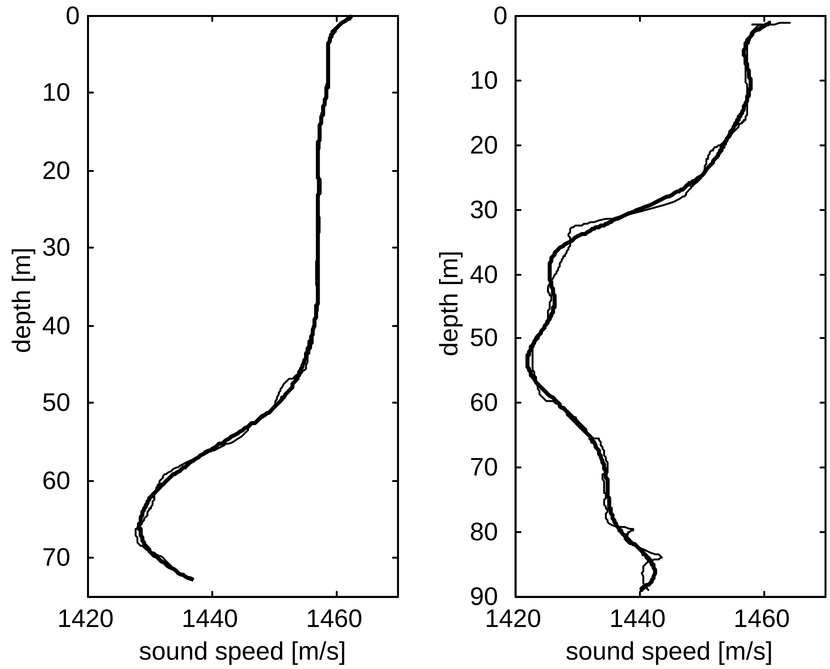

Fig.5. Real and approximated vertical distributions of the sound speed in water – 5-th and 6-th of May, 2006

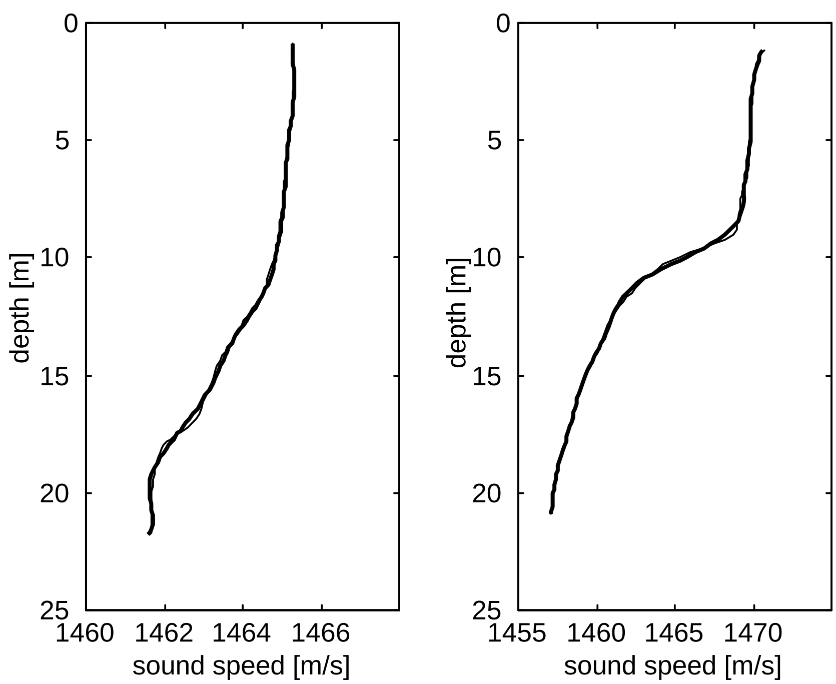

Fig.6. Real and approximated vertical distributions of the sound speed in water – 7-th and 8-th of May, 2006

	correlation coefficient	reminder variation
May 5, 2006	0,999	0,268
May 6, 2006	0,999	1.279
May 7, 2006	0,999	0,003
May 8, 2006	0,999	0,025

5. CONCLUSIONS

Uniform and non-uniform B-Splines are attractive algorithms for description curves (1D functions) and surfaces (2D functions) and they are alternative for polynomial methods, which usage can lead to loss of stability with high coefficient of the polynomial for obtaining the high accuracy of the approximation.

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