MODIFIED MUSIC ALGORITHM FOR DOA ESTIMATION USING THE A-PRIORI DEFINED UNITARY MATRIX

TADEUSZ JANOWSKI, EDYTA POROSIŃSKA, ZAJĄC RYSZARD

Ośrodek Badawczo-Rozwojowy Centrum Techniki Morskiej S.A.
Poland, 81-109 Gdynia, ul Dickmana 62
Tadeusz.Janowski@ctm.gdynia.pl, Edyta.Porosinska@ctm.gdynia.pl, Ryszard.Zajac@ctm.gdynia.pl

The determination of the number of the simultaneously observed objects is an important problem of the MUSIC algorithm. As the result of the covariance matrix eigendecomposition the eingevector and unitary matrix are obtained. The unitary matrix defines the transformation of the measurement vector. Consequently the obtained vector is orthogonal to the noise subspace. The solution of the eigendecomposition problem can be avoided assuming a-priori knowledge of the unitary matrix. Practically, the passive sonar detects the first appeared noisy object. This enable to beforehand define the set of the unitary matrixes exactly matched to the previously determined directions of the signal arrival. The article presents the form of the matrix and features of the measurement vector transformation. The results of the DOA algorithm computer simulation for passive sonar composed of four hydrophones developed in the OBR CTM S.A. are also presented.

INTRODUCTION

In the year 2008 the international research project entitled “Autonomous Maritime Surveillance System” was initiated in scope of the 7 Frame Program – topic “Surveillance in wide maritime areas through active and passive means”. The aim of the project is to develop the monitoring system safeguarding the sea borders of the European Union from illegal emigration. To perform it the set of the stand-alone surveillance buoys will be manufactured and spaced around the protected seashore. The buoys will be equipped with optical sensors to observe the sea surface and passive array of the hydrophones to control the underwater acoustic background. The hydrouacoustic array is developed in OBR CTM S.A. The requirements regarding to detection of the small ships equipped with inboard or outboard engines are high. Therefore the OBR CTM S.A. undertook the work to develop the very effective algorithm of DOA estimation.

The great deal of sound sources and their random nature create the broadband underwater acoustic noise. These acoustic signals are received by the array composed of four digital hydrophones. When no prevailing signal occurs, the array produces the vector of uncorrelated signals. The situation changes whereas the dominant sound source emerges. This signal is mostly produced by the ship's rotating gears and shafts or generating sets. In this case the particular signals from different hydrophones are shifted in time domain. The time delays directly correspond to the direction of the signal arrival. When the signal is narrowband the time delays can be transformed to the phase shifts of the complex vectors. For this reason the preliminary signal processing relays on extraction of the narrowband signal from the wideband noise spectrum. The narrowband signal contains the information on the amplitude and phase of the received signal. To obtain the maximum angular resolution the size of the array should be matched to the processed signal wavelength. As the result of the preliminary signal processing the series of the complex measuring vectors are obtained. These series is further processed to estimate DOA. Typically the covariance matrix of this vector is similar to the diagonal matrix. The main diagonal of the matrix consists of values corresponding to the noise dispersion in the sea water medium. The remaining elements, such as the uncorrelated noises have significantly less values. Intruder appearance on the desired direction conducts to increase of the mutual correlations among the elements of the measuring vector. The matrix analysis using the MUSIC (Multiple Source Classification) method enables the valuation of the number of the signal sources and independent DOA estimation for each of the source. This method is the theoretical base of the presented algorithm.

1. MODIFIED MEASURING VECTOR

The DOA estimation algorithms are based on the statistical methods. The ***x*** measuring vector is the algorithm input value. Definition of the "received vector" is not correct due to initial parameterization algorithm of the signals obtained from the array of the hydrophones. The size of this vector equals to the number of the sensors:

$$Rank(x)=M \tag{1}$$

Theoretically the vector is considered as the random variable. Practically the series of the cyclically received complex snapshots are obtained instead of random variable. Therefore the ***E(*)*** operator defining the statistical average for random vector or very narrow low-pass filter for series of the complex snapshots can be introduced. The definition of the ***a*** steering vector is the essence of the all parametric methods. The ***a*** steering vector has ***M*** size. The vector elements are the complex numbers having unitary amplitudes and DOA dependant phases. When there is no noise the vector meets equation:

$$x = S \cdot a \tag{2}$$

The ***S*** is the complex amplitude of the signal received by all hydrophones. This equation assumes the same reception characteristics of all hydrophones and operation in the far field. In case of the non-balanced hydrophones the existing differences can be software equalized. The ***y*** modified measuring vector is defined as:

$$y = V \cdot x \tag{3}$$

Where the ***V*** is the unitary matrix defined as:

$$V = \frac{1}{\sqrt{M}} DFT \cdot diag(a^*) \tag{4}$$

The DFT means the M-points Discrete Fourier Transform. Its coefficients are determined as follows:

$$DFT_{l,m} = e^{-2\pi j\frac{l\cdot m}{M}} \qquad (5)$$

The ***()**** operator returns the conjugate value to the ***a*** steering vector.
The ***diag(*)*** operator creates the diagonal matrix from the vector.
The ***V*** matrix is unitary, it results from following equations:

$$DFT^{-1} = \frac{1}{M} DFT^{H} \qquad (6)$$

$$diag(a^*)^H = diag(a) \qquad (7)$$

The ***(*)**$^{\boldsymbol{H}}$ notation represents the Hermitian Conjugation.
Substitution of these dependences to the product of the ***V*** matrix by the $\boldsymbol{V^H}$ matrix conducts to the ***I*** unit matrix.

$$V \cdot V^H = I \qquad (8)$$

The physical interpretation of the measuring vector modification is shown on the *Fig.1*. In the event of the lack of noise the y_0 element represents the entire signal.

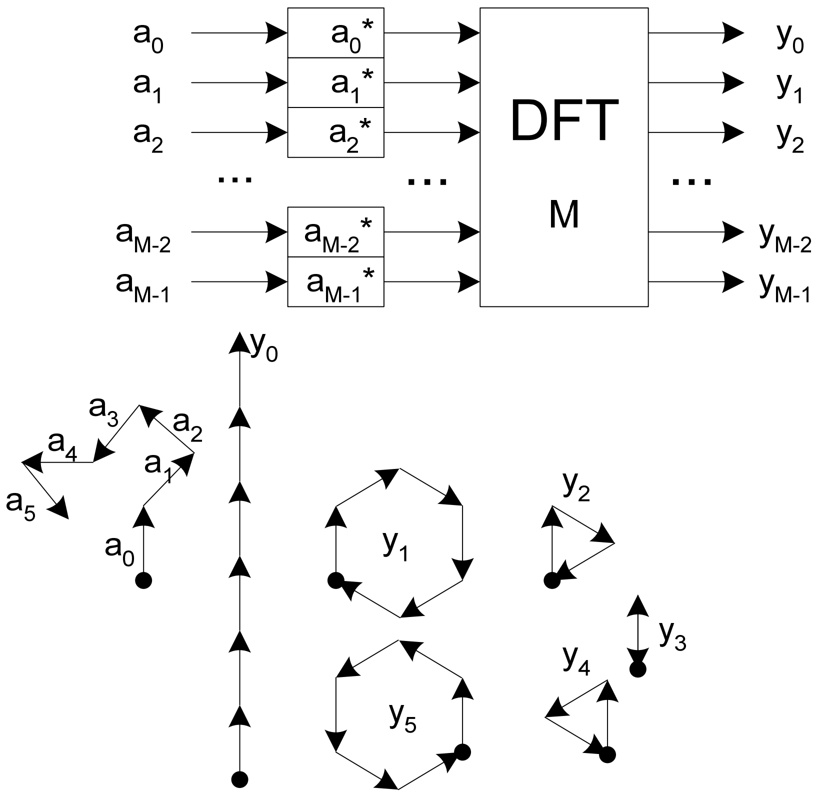

Fig.1. The unitary matrix on the base of the DFT transformation, general scheme, geometrical interpretation

This element represents the signal subspace, the remaining elements are nullified.

2. THE COVARIANCE MATRIX OF THE MODIFIED VECTOR

The principle of the MUSIC method is eigendecomposition of the $\boldsymbol{M \times M}$ covariance matrix of the ***x*** vector.

$$E(x \cdot x^H) = U^H diag(\lambda) U \qquad (9)$$

The $\boldsymbol{\lambda}$ vector includes the eigenvalues, whereas the $\boldsymbol{U}$ unitary matrix contains the eigenvectors. The covariance matrix is the Hermitian one, so all eigenvalues are real. The covariance matrix of the modified measuring vector equals:

$$E(y \cdot y^H) = E(V \cdot x \cdot x^H \cdot V^H) \tag{10}$$

or

$$E(y \cdot y^H) = V \cdot E(x \cdot x^H) \cdot V^H \tag{11}$$

Substitution of the (9) to (11) conducts to equation:

$$E(y \cdot y^H) = V \cdot U^H diag(\lambda) \cdot U \cdot V^H \tag{12}$$

That means, that the covariance matrix of the $\boldsymbol{y}$ vector has the same eigenvalues as the covariance matrix of the $\boldsymbol{x}$ vector. In case of single directional signal source the eigenvalues are following:

$$\lambda_0 = |S|^2 \cdot M + \sigma^2, \lambda_1 = \lambda_2 = \cdots = \lambda_{M-1} = \sigma^2 \tag{13}$$

The $\boldsymbol{\sigma^2}$ value represents the environmental noise.

When the $\boldsymbol{y}$ vector is DOA-matched (*Fig.1*) equalities are met:

$$V = U \tag{14}$$

$$E(y \cdot y^H) = diag(\lambda) \tag{15}$$

The *Fig.2* presents the essential difference between the MUSIC method and a-priori defined unitary matrix method.

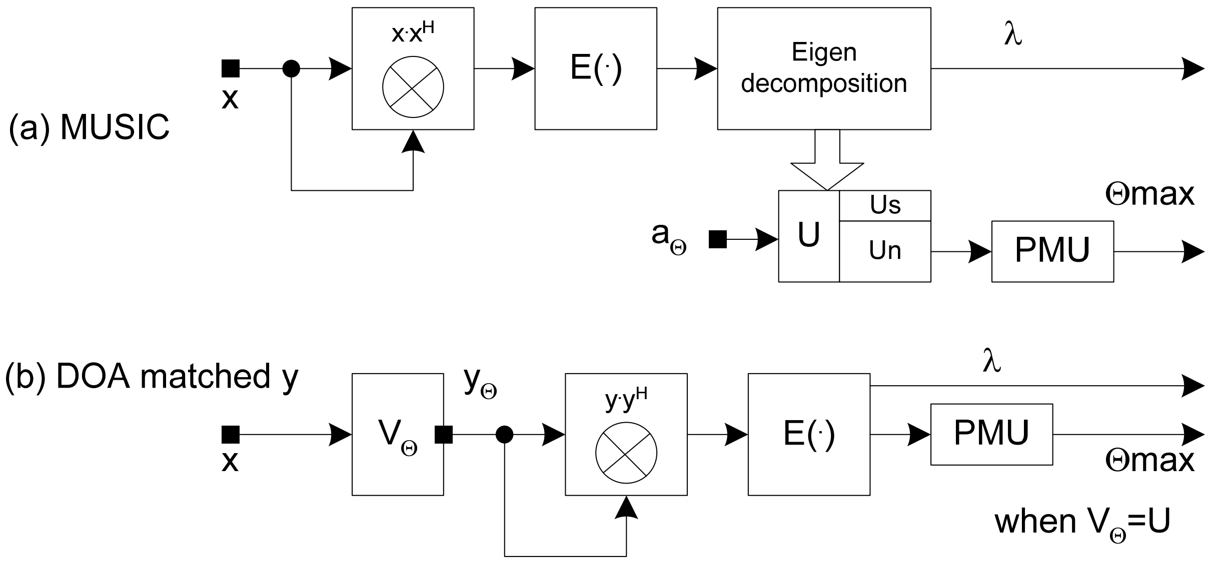

Fig.2. Methods comparison

In both cases the determination of the signal maximum is carried out by tuning of the steering vector as the DOA function $\boldsymbol{a}: \boldsymbol{\Theta} \rightarrow \boldsymbol{a}(\boldsymbol{\Theta})$. In the event of MUSIC method the tuning is realized on the calculated sub-matrix of the defined noise subspace. For discussed method the $\boldsymbol{V}$ matrix is a-priori known (the (4) dependence) before the covariance matrix calculation. The tuning relays on $\boldsymbol{DOA} = \boldsymbol{\Theta_{max}}$ finding for which the covariance matrix is closest to the diagonal one. This criterion achieves the maximum when the absolute value of cross-correlation vector reaches minimum.

3. COMPUTER SIMULATION

The computer simulation of the algorithm was carried out for UCA (Uniform Cylindrical Array) composed of four hydrophones. The target parameters such as distance, signal level and environmental underwater noise were simulated. The reception channel for defined hydrophones sensitivity and I/Q circuit was also simulated. As the result of these operations the series of thousands $\boldsymbol{x}$ vectors were obtained. These vectors were multiplied by V unitary matrix being DOA dependant. The arithmetic average operator of all $\boldsymbol{y} \cdot \boldsymbol{y}^{H}$ matrixes was applied to obtain right approximation of the covariance matrix. The figures show the absolute values of the matrix coefficients $(c_{00}, c_{01}=c_{10}, c_{02}=c_{20}, c_{03}=c_{30}, c_{11}, c_{12}=c_{21}, c_{13}=c_{31}, c_{22}, c_{23}=c_{32}, c_{33})$. The number of the plot means the number of the row, whereas the axis is the number of the column of the covariance matrix. The c_{00} coefficient corresponds to the Bartletta's beamformer and determines the target strength. The *Fig.3* presents the target detection. Setting of the proper detection threshold enables the target detection.

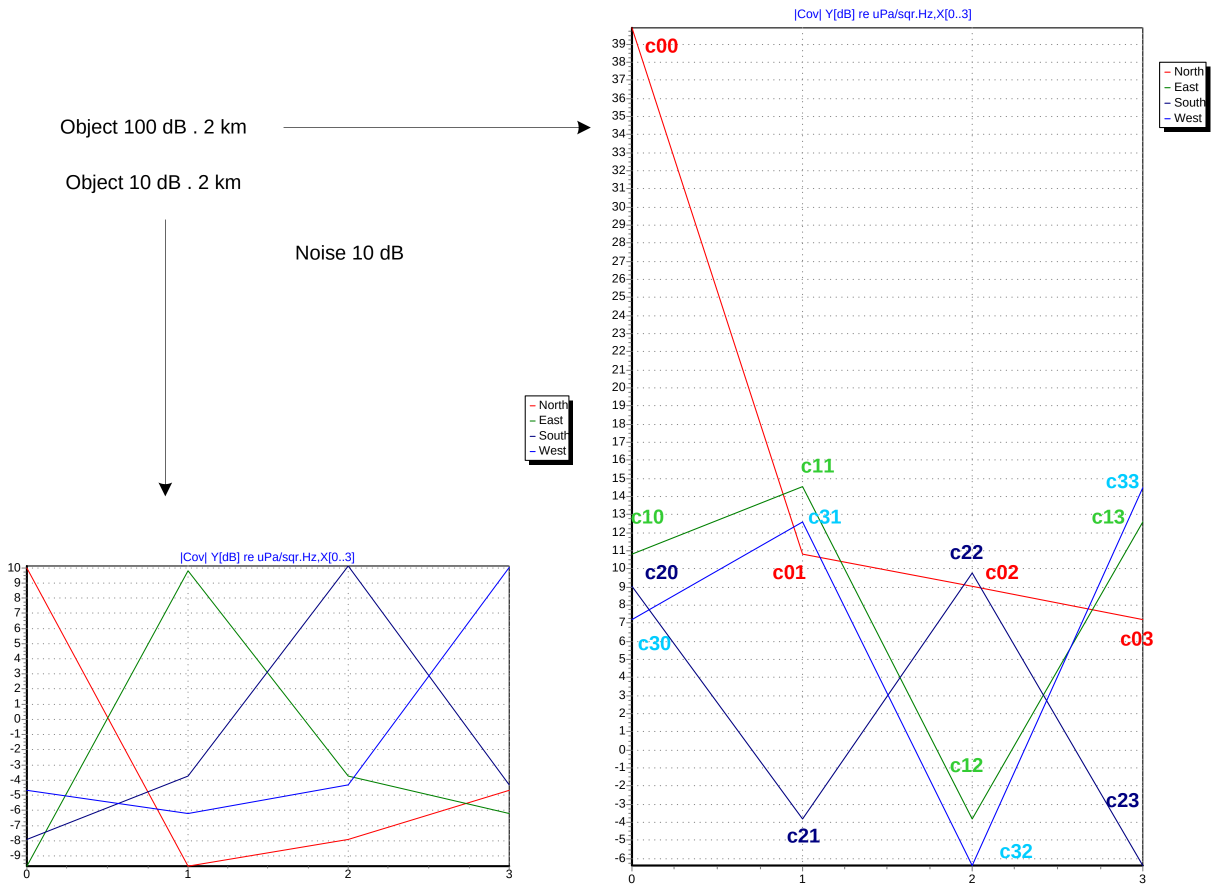

Fig.3. Computer simulation of the target detection algorithms

The *Fig.4* shows the covariance matrix for matched *(210 degrees)* and non-matched *(200 degrees)* direction of the signal arrival.

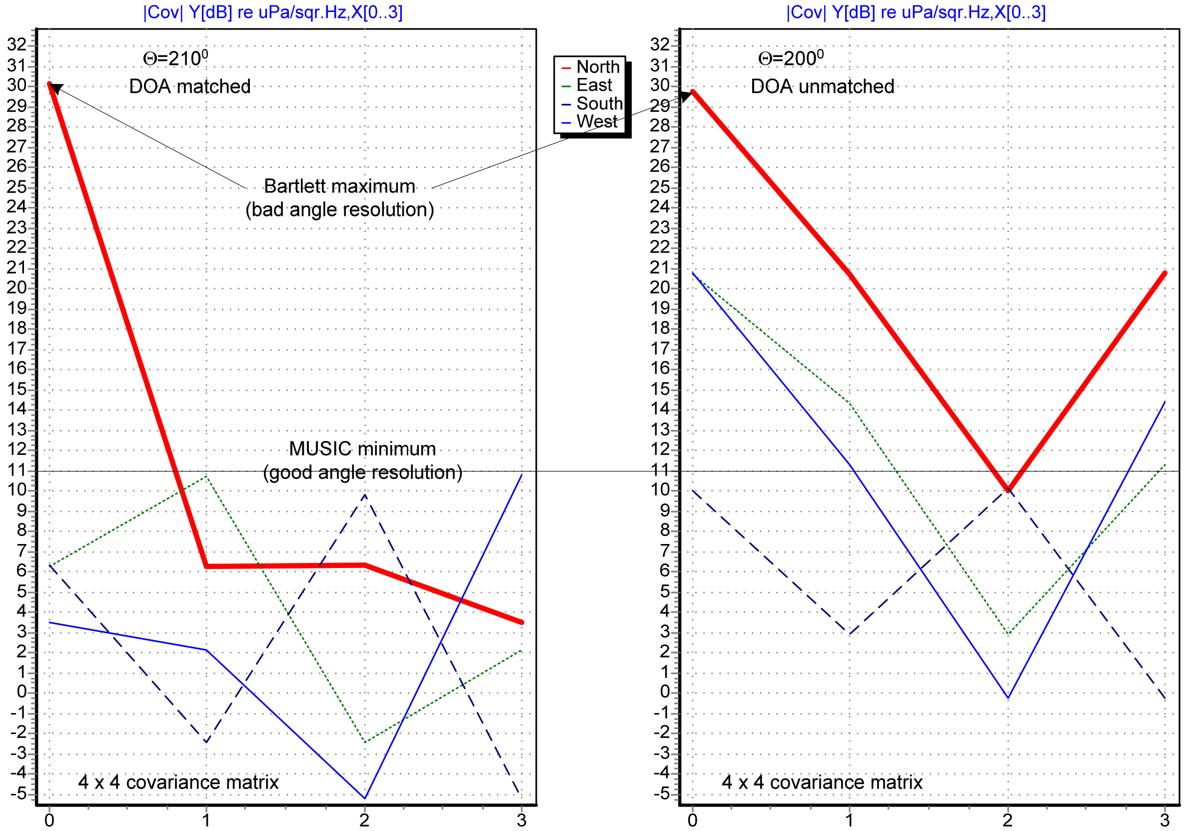

Fig.4. Results of computer simulation of DOA estimation

Bartletta's beamformer is not suitable for DOA estimation in case of the array composed of four hydrophones. Precision calculation of the remaining elements of the covariance matrix enables DOA estimation with higher resolution.

4. CONCLUSIONS

Presented algorithm of DOA estimation is the effective numeric alternative for the commonly used standard methods. Algorithm enables substitution of the eigenvectors by calculation of the maximum function on the set of covariance matrixes for previously determined directions of signal arrival. Such algorithm is especially convenient for hardware implementation e.g. using FPGA technology. This algorithm will be applied in passive array of the hydrophones developed for AMASS system.

REFERENCES

[1] Ralph O. Schmidt, Multiple Emitter Location and Signal parameter Estimation IEEE Transactions on antennas and propagation, March 1986.
[2] F. Belloni, Signal processing for arbitrary sensor array configurations: theory and algorithm, October 2007.
[3] NAXYS Ethernet Hydrophone 02345.
[4] P. Dawkins, Linear Algebra.
[5] A. Pezeshki, Eigenvalue Beamforming using a Multi-rank MVDR Beamformer and Subspace Selection, IEEE transactions on signal processing 2008.