INVERSION TECHNIQUES FOR OCEAN ACOUSTIC TOMOGRAPHY AND BOTTOM CLASSIFICATION BASED ON NORMAL MODE THEORY

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The paper presents a short review of the modal representation and inversion techniques applied to problems of ocean acoustic tomography and geoacoustic inversion. The use of the modal inversion techniques is based on the assumption that the modal component of the acoustic field can be recognized in the recorded acoustic signals. The paper is referred to the main techniques that have been applied so far by the group of the Institute of Applied and Computational Mathematics at FORTH and address their benefits and drawbacks.

INTRODUCTION

Acoustical methods for the estimation of critical parameters of the ocean environment including the water column and the sea floor have been extensively used over the last years as complimentary tools to traditional oceanographic and seismic methods.

The methods we are dealing with in this paper are based on measurements of the acoustic field obtained at long distances from a known source. For every application an inverse problem is defined of the form

$$\mathbf{f}(\mathbf{d},\mathbf{m}) = 0 , \qquad (1)$$

where, $\mathbf{d}$ is the vector of the data (taken from the measurements) and $\mathbf{m}$ is a vector of the recoverable parameters. Of course there is no evidence that the equations thus defined for a general inverse problem contain enough information to specify uniquely the model parameters, or that they are consistent. A general inverse problem is known to be ill-posed. Therefore a thorough analysis of the corresponding inverse problem is required.

The recoverable parameters in our case is the sound speed profile in the water column $c(\vec{x})$, the current velocities $v(\vec{x})$, the geometry of the interfaces in the water column and the bottom, the compressional $c_b(z)$ and shear $c_s(z)$ velocities in the bottom and the compressional and shear wave attenuation parameters $a_p(z)$ and $a_s(z)$. Note that in this notation, water parameters are in general considered 3-Dimentional, whereas those of the bottom are functions of the depth only. It should be underlined however that the research for fully 3-D inverse problems is at its very early stages and so far results can be obtained for some specific 3-D cases only.

The vector **d** contains the "observables" of the signal, which are characteristic components that in turn can be associated with the recoverable parameters through an appropriate propagation model. This paper is devoted to observables directly related to the modal character of the acoustic field and we will also emphasize cases where the sound speed profile has to be recovered. The theory of normal-mode propagation in the ocean will be considered known and extended reference to the differential equations governing the forward problem of acoustic propagation will be omitted.

1. METHODS BASED ON MEASUREMENTS OF THE "MODAL PHASE"

This is a rather "old" technique. The modal phase is defined at range *r* by the following notation:

$$\Phi_n(r) = \int_0^r k_n(r')dr' \quad , \tag{2}$$

where $k_n(r')$ is the eigenvalue of the "depth problem" defined at each range *r'*. [1]-[4]. The modal phase can be measured through an additional inverse procedure after measurement of the acoustic field at a vertical array of hydrophones. The procedure is known as "mode filtering"[5] and is based on the normal-mode representation of the time independent acoustic field (pressure) in the form :

$$p(r,z) = \frac{i}{4\rho_0} \sum_n u_n(z_0;0)u_n(z;r)H_0^{(1)}(k_n r) \quad , \tag{3}$$

where, a cylindrical co-ordinate system has been adopted in an axially symmetric environment, u_n is the eigenfunction of the depth problem and $H_0^{(1)}$ is the Hankel function of the first kind and zero order. ρ_0 is the density in the water column where the sound source has been placed. In formula (3), it is assumed that the environment is range-independent and the eigenvalues are defined uniquely for all ranges. However, formula (2) indicates a range-dependent environment. Note that the specific form of the modal phase is derived from equation (3), when the asymptotic form of the Hankel function is considered and an adiabatic approximation is adopted. In order to complete the presentation it should be added that a continuous wave source has been assumed and thus the acoustic wave equation by separation of variables is transformed to the Helmholtz equation. Therefore the following analysis in this chapter is referred to a single circular frequency ω.

As soon as the modal phase is obtained, its variation with respect to a background or reference (known) environment can be associated with the sound speed difference $\delta c(r,z)$ in the water column and the sea-floor, with respect to the same background environment, through an equation of the form :

$$\delta\Phi_n(r) = \int_0^r \int_0^\infty Q_n^0(z;r)\delta c(r,z)dzdr \tag{4}$$

where $Q_n^0(z;r)$ is a known kernel calculated for the reference environment [3]. Equation (4) defines an inverse problem given at integral form :

- Given measurements of the modal phase difference with respect to a background environment for N propagating modes, estimate the corresponding sound speed differences.

The inverse problem thus defined is more easily solved by appropriate discretization of the environment in range and depth. Thus, the sound speed difference is defined at the cells of the discretizatiuon and the inverse problem is now transformed to a discrete inverse problem solved by some appropriate technique, such as the singular valued decomposition method [4]. Note that this technique defines a linear inverse problem and the variations with respect to the reference environment are considered small in order that formulas like (4) are applicable.

Although the method seems to be easily applied when there exists a good knowledge of the background environment, it has several disadvantages. The main disadvantage is that a vertical array of hydrophones is necessary in order to get a mode-filtering. When a range-dependent environment is considered, mode-filtering is not easily performed. Finally, the condition of the kermel matrix of the discrete inverse problem is not always good and therefore the inversion results are not always reliable [4].

In any case it has been shown that the exploitation of the a-priori information on the environment which is expressed in terms of empirical orthogonal functions (EOFs) has many good effects on the performance of the inversion procedure.

2. METHODS BASED ON THE MEASUREMENTS OF THE MODAL TRAVEL TIME

When the experiments are to be performed with few receiving hydrophones, modal filtering is not practical and measurements in the time domain have to be exploited. The usual approach in this case is to get the acoustic field in the time domain by applying inverse Fourier transform to the acoustic field calculated at specific frequencies within the signal bandwidth. Measurements at a single hydrophone are enough for the application of these methods.

It is well known that the acoustic field propagates in a waveguide in the form of energy packets, each one corresponding to a propagation mode under a characteristic velocity which is defined as

$$v_{g,n} = \left.\frac{\partial\omega}{\partial k_n}\right|_{\omega_0} \tag{5}$$

where, ω_0 is the central circular frequency of the signal bandwidth. There are analytical formulas for calculating the group velocity given the environmental parameters [2].

The idea of the inversion procedure in this case is that as soon as the modal field is identified at a single receiver, the arrival times of the propagating modes can be associated with the sound speed in the water column through the normal-mode theory and thus to define an appropriate formula leading to either a continuous or a discrete inverse problem in the same way as in the previous case [6].

Indeed the travel time of a specific mode at range *r* is

$$t_n(r) = \frac{r}{v_{g,n}} \tag{6}$$

and thus the travel time is associated with the eigenvalues at the vicinity of the central frequency and eventually with the modal phase we defined earlier.

Figure 1 presents a simulated acoustic signal in a shallow-water tomographic experiment. The range where the measurements are considered is 10000 m and the central frequeny of the signal is 150 Hz. The modal character of the signal is evident, although an additional process is required to identify the peaks of the signal as modal arrivals (see note at the end of next chapter).

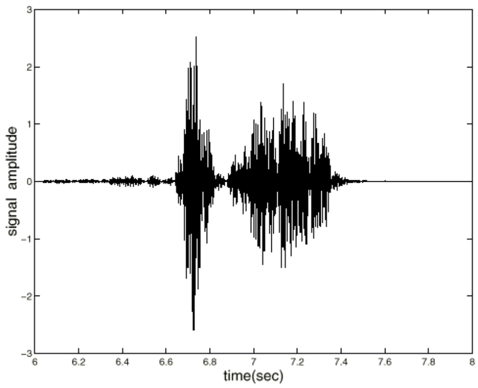

Fig.1 A simulated acoustic signal in a typical shallow water environment

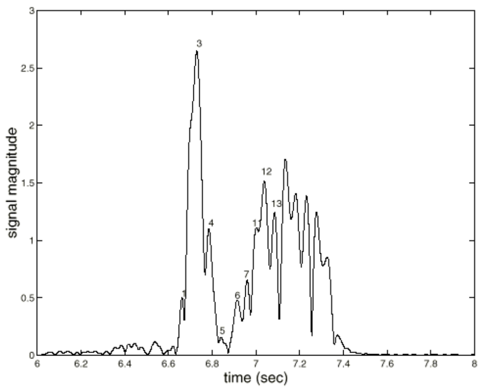

Fig.2 Mode identification for the signal of Fig. 2

2.1 Linear approach

Making the same assumptions as before in what concerns the background environment, and its difference with respect to the actual one, an integral equation of the form

$$\delta t_n(r) = \int_0^r \int_0^\infty \left. \frac{\partial Q_n^0(z;r)}{\partial \omega} \right|_{\omega_0} \delta c(r,z) dz dr \tag{7}$$

is defined, where the kernel is calculated as derivative at the central frequency. In practical applications the derivative is calculated numerically and the integral equation is transformed to a matrix notation through appropriate discretization of the environment [6].

The main advantage of this approach is that it is technologically easier to be applied as there is a need of only one hydrophone for the measurements. From the theoretical point of view it still has drawbacks as the adiabatic approximation used when a range-dependent environment is considered is not always appropriate for every application. Also problems associated with the quality of the kernel matrix remain similar to the previous case.

It should be noted that in order that this technique is applicable in real cases, the identification of the propagating modes should have been performed before the inversion starts. An effective identification process is described in [7] and an example of the mode identification is presented in figure 2, based on the signal of figure 1.

2.2 Non-linear approach

When a reference environment is not known or difficult to be defined, a linear approach is not any more applicable or reliable, as the variations with respect to some reference environment may be large.

In this case it is possible to apply a non-linear optimization technique based on the "matching" of the arrival times of the normal-modes of the actual environment, with the calculated modal arrivals of some candidate environment taken from a set of potential environments.

The optimization technique is built upon an appropriate norm measuring the difference between the actual and calculated travel times of the identified modes.

A very simple norm showing good performance in most realistic cases is

$$P(\delta t_i) = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \delta t_i^2} \tag{8}$$

Where, δt_i are the travel time differences between the actual and predicted arrivals for M identified modes [7].

The main advantage of this approach is that since a non-linear problem is defined, the vector **m** of the recoverable parameters can in principle contain as many parameters as we wish, including water depth, location of the source and the interfaces in the bottom etc. Thus, all the properties of the water column and the sea-floor can be considered simultaneously as the unknowns of this approach. Of course, not all the parameters show the same sensitivity with respect to chosen norm. However, it has been demonstrated that at least the most important ones for practical applications such as the sound speed profile in the water and the bottom can be retrieved with reliability using the suggested approach.

Since the optimization scheme has to be applied using a great number of candidate environments (consider cases where we have a multidimensional search space and many combinations should be studied), suitable techniques minimizing the time required for the realization of the inverse procedure should be applied. Such methods include energy techniques (such as simulated annealing), genetic algorithms, neural networks or equivalent.

In concluding this chapter we should mention that non-linear approaches can be combined with linear ones to improve or fine-tune the results, as the results from the non-linear inversion can be considered as the appropriate background environment upon which the linear approach is based. (See for instance [8] and [9]).

3. METHODS BASED ON THE DISPERSION CHARACTERISTICS OF THE ACOUSTIC CHANNEL

An alternative way to exploit the information contained in an acoustic signal for inversion purposes is to transform the signal into the time-frequency plane. For this purpose the short-time Fourier Transform or the Wavelet Transform can be used, resulting in a modal representation of the acoustic field, for the full signal bandwidth (see Figure 3). Remember that in the previous chapter we considered representations associated with the central frequency only, although the signal is of broad-band character. Although both Transforms result to the same type of representation (spectrogram or scalogram), the Wavelet Transform has been shown to provide a better analysis of the dispersion characteristics in comparison with short-time Fourier Transform especially in cases of relatively broad-band signals. An even better performance is obtained through the reassigned wavelet transform [10], [11].

For inversion purposes, the scalogram can be used by appropriate discertization in frequency and time through an optimization procedure to match the measured with estimated dispersion characteristics from a set of candidate environments in almost the same way as in the case of the travel time measurements described in the previous chapter. Thus, a non-linear inverse problem is again defined [12],[13].

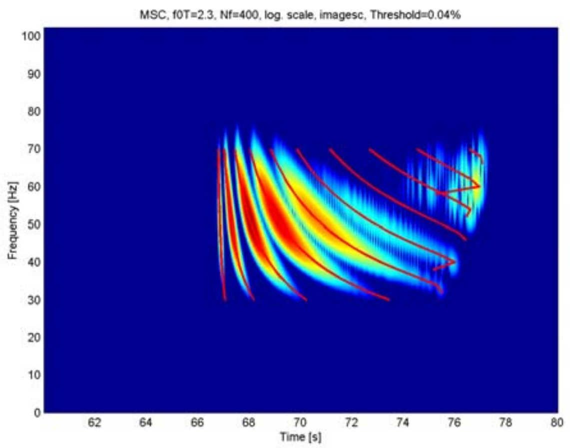

Fig.3 Matching the scalogram and the dispersion curves at a typical shallow water environment based on measurements obtained at 100 km from the source

The main advantage of this approach with respect to the previously mentioned ones is that the inversion procedure exploits more information contained in the signal as the modal character in a continuous spectrum of frequencies is represented. Moreover, it can be applied even in cases where limited possibilities of mode identification in the time domain only exist. Note that again a single hydrophone is enough to get the information required for the inversions.

4. DISCUSSION

The paper outlined a few inversion methods applied in problems of ocean acoustic tomography and bottom classification, where the modal character of the acoustic field is exploited. The exploitation is done both in the definition of the observables, which consist the input data to the inverse problem and in the model which associates the parameters to be retrieved with the observables.

The methods presented are classified as linear or non-linear. The linear methods are associated with a reference environment, the knowledge of which is assumed necessary, whereas the non-linear methods are associated with some optimization approach performed over a relatively wide search space.

It should be noted that modal inversion techniques are more appropriate to be used in shallow water environments and low frequency sources, when the number of propagating modes is relatively small. Moreover, they have to be applied at relatively long ranges where mode identification is ensured.

In deep water environments and high frequency sources, the possibility of exploiting the modal character of the acoustic field for inversion purposes is very low and thus other type of techniques based on observables easily identified in these cases should be applied. However the reference to these methods is beyond the scope of this paper.

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