NUMERICAL SOLVER OF ACOUSTIC FIELD IN SIMULATION OF ARTERY WALL THICKNESS EXAMINATIONS

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The aim of the study was the elaboration of a mathematical model to describe the process of acoustic wave propagation, generated by an ultrasonic probe in a inhomogenous loosing medium. Numerical calculations make it possible to define waveforms for electric signals that are generated when ultrasonic waves, being reflected and backscattered by an artery model, are then received by the ultrasonic probe. It is the signal that pretty well corresponds with the actual RF signal that is obtained during measurements at the output of the ultrasonic apparatus. The developed solver of acoustic field was used for simulation of the artery wall thickness examination. The theoretical model of the artery for the creating the simulated ultrasonic reflected echoes was used. The internal radius of the artery model was 3 mm for the diastolic pressure and 3.3 mm for the systolic pressure. The intima-media thickness (IMT) of the artery wall was changed from 0.48 mm to 0,44 mm respectively. The solver based on zero-crossing method was used for detecting changes of the IMT.

INTRODUCTION

Considerable interest is observed in a contemporary ultrasound medical diagnosis in examining artery walls by means of invasive and non-invasive methods. The basis of assessment of structural changes taking place in the artery wall is the measurement of its thickness and stiffness [2,3]. In case of non-invasive ultrasonic measurement, reproducibility of the obtained results is an extremely important parameter, since it is used to define sensitivity of the diagnostic tool [4]. The major objective of the thesis was to develop a numerical solver that would be capable of describing spatial and time-dependent distribution of an ultrasonic beam that is emitted by a piezoelectric ring transducer and then backscattered on cylindrical surfaces of the walls in artery models. The developed solver was tested for results of experiments when an elastic pipe was immersed in water. The investigations were carried out using the VED equipment, designed and constructed in the Ultrasonic Department of the Institute of Fundamental Technological Research of the Polish Academy of Sciences, purposefully dedicated for elasticity examination of arterial walls in human body [2,3] .

1. PHYSICAL MODEL

With use of non-dimensional variables, the equation that defines the propagation of sonic waves in a homogenous (with undisturbed parameters of the material) non-linear and absorbing medium, can be expressed by the following equation [6]:

$$\Delta P - \partial_{tt} P - 2\partial_t AP + q\beta\partial_{tt}(P)^2 = 0 \qquad (1)$$

where $AP \equiv A(t) \otimes P(x,t)$, $A(t) = F^{-1}[a(n)]$, $P(x,t)$, is the pressure in the 3D coordinate system x at the moment of time *t;* A is a convolution-type operator that defines absorption; *q* is the Mach number (in our case the Mach number is calculated for velocities on the surface of the disturbance); $\beta \equiv (\gamma+1)/2$; $\gamma \equiv (B/A)+1$ or γ - adiabatic exponent, $n \equiv f/f_0$ - non-dimensional frequency; f, f_0 - respectively: frequency and characteristic frequency; a(n) -the small signal coefficient of absorption, $A = F^{-1}[a(n)]$, $F[\cdot]$ - Fourier transform.

For the medium with disturbed materials parameters the equation of the scattered field P^{sc} can be developed from the formula 1 (more details see [7]) end takes the form:

$$\Delta P^{sc} - \partial_{tt} P^{sc} - 2\partial_t AP^{sc} = -\Pi\partial_{tt}(P^{sc} + P^{in}) \qquad (2)$$

where P^{in} - incident field which fulfils the equation (1), $\Pi(x) \equiv 1 - 1/c_r^2$ - scattering potential, c_r^2 - disturbed dimensionless sound velocity.

2. SOLVER

Construction of a solver for backscattered fields is the fundamental issue for setting up a numerical model of an experiment that is aimed to reflect real situations that occur in ultrasonography practice. The solver that we constructed is composed of three parts:

1) Solver for the incident field. It is the solver that bases on codes JWNUT2D and JWNUT3D [8], which we have been using for many years. The first code solves the equation in the axially symmetrical cases, the second one is applicable to whichever one-sided boundary conditions.
2) Solver for the backscattered field. It is the tool that is able to calculate parameters of backs-scattered fields and their pressures on the detector surface, whereas the tool uses numerically determined incident field and information on geometrical and material parameters of the target as the basis for calculations.
3) Simulator of the electronic receiver channel that is used for calculation of pulse responds $h(t)$ of this unit. Distribution of pressure on the surface of the probe is averaged over the entire probe surface (the theory of piezoelectric phenomena says that electric signals at probe output are proportional to the aforementioned average value).

$$P_E(t) = \frac{1}{S} \int_{S(x)} P^{sc}(S(x),t) Ap(S(x)) dS \qquad (3)$$

where $S(x)$ denotes a point on the transducer surface, S stands for the transducer surface area and $Ap(S(x))$ is the apodization function for the transducer surface. In this study $P_E(t)$ is referred to as the echo. The RF signal $P_{RF}(t)$ represents a single line of scanning and is calculated as follows:

$$P_{RF}(t) = h(t) \otimes P_E(t), \; h = F^{-1}[H(n)] \tag{4}$$

where $H(n)$ is the system transmittance.

3. SOLVER EXAMINATIONS

Experimental setup is shown on Fig. 1 [9]. The research was carried out for a pipe made of latex, with internal diameter of 5 mm and wall thickness of 1.25 mm. Both wall thickness and pipe diameter were similar to corresponding dimensions of the common carotid artery for people at the age of 20. The velocity of ultrasonic wave in the pipe wall was found by ultrasonic measurements and amounted to 1.333 of the velocity in water. The investigations employed the VED ultrasonic apparatus. The frequency of the transmitted ultrasound was 6.75 MHz. The pulses were measured by means of the hydrophone of the type: Sonic Technologies Model 800 Bilaminar Hydrophone. During the performed research the front surface of the pipe wall was positioned in the focus of the ultrasonic probe.

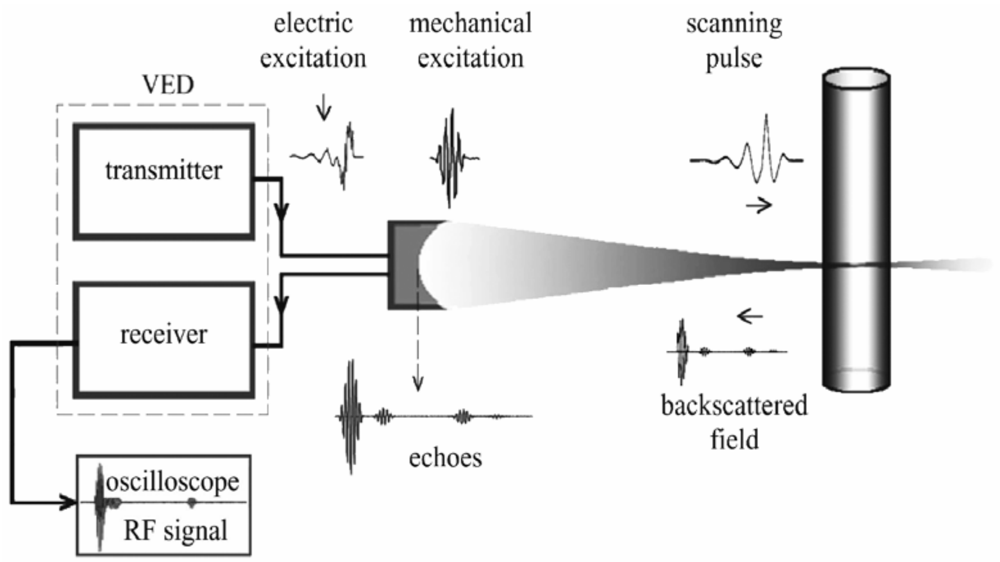


Fig.1 Experimental setup

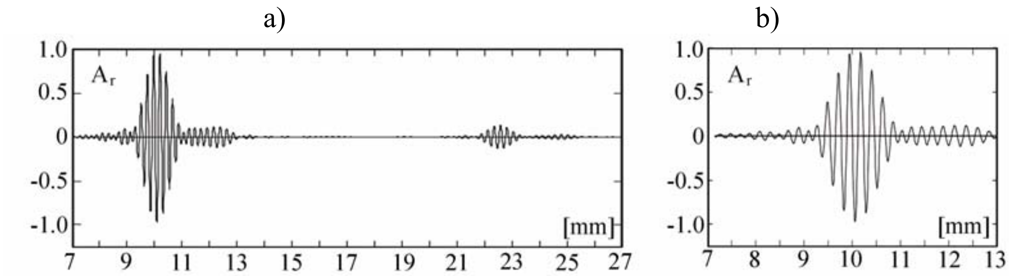


Fig.2 a) The RF signal $P_{RF}(t)$ calculated from the numerical model by means of the formula 2; b) the expanded RF signal, from the first pipe wall. The point 16.5 on the scale above corresponds to location of the pipe centre at the distance of 26.5 mm from the ultrasonic probe, Ar – the relative amplitude (with respect to the maximum value of the RF signal amplitude)

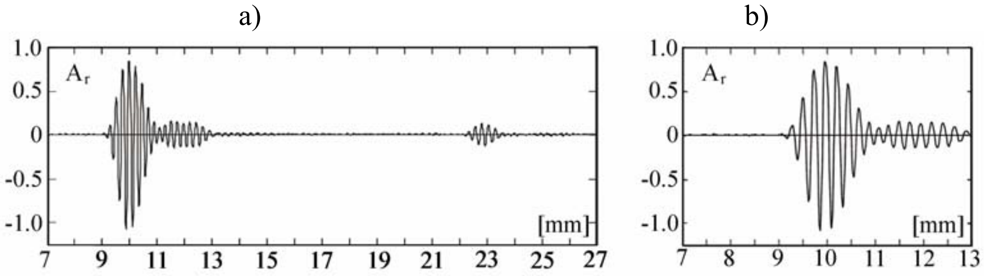


Fig.3 a) The RF signal $P_{RF}(t)$ measured by means of the VED apparatus; b) the expanded RF signal from the first pipe wall, Ar – the relative amplitude (with respect to the maximum value of the RF signal amplitude)

The RF electric signal, $P_{RF}(t)$, corresponding to echoes reflected by the pipe walls, was recorded at the output of the RF receiver by means of the digital oscilloscope AGILENT 54641D. To highlight relationships between target (pipe) dimensions and wavelength of the echoed signals (both acoustic waves and corresponding electric waveforms) the scales were converted all the time into 3D ones and expressed in millimetres. Signal amplitudes were presented as relative values. Results for calculations and measurements are presented in Fig. 2 and 3.

4. SIMULATION OF ARTERY WALL THICKNESS EXAMINATIONS

The artery wall is composed of three layers: the adventitia, the media and the intima. The basic difficulty in ultrasonic examination of wall thickness is limited longitudinal resolution of ultrasonic systems. For applied transmission frequency between 5-10 MHz the longitudinal resolution is from 0.4 to 0.2 mm. Generally it is not enough to measure the intima thickness which value is less then 0.2 mm. In this situation, the intima-media thickness (IMT) was calculated on the basis of the distance between two successive echoes which correspond to reflection from intima and adventitia layers respectively. Moreover, the wall thickness changed under the blood pressure change during the cardiac cycle.

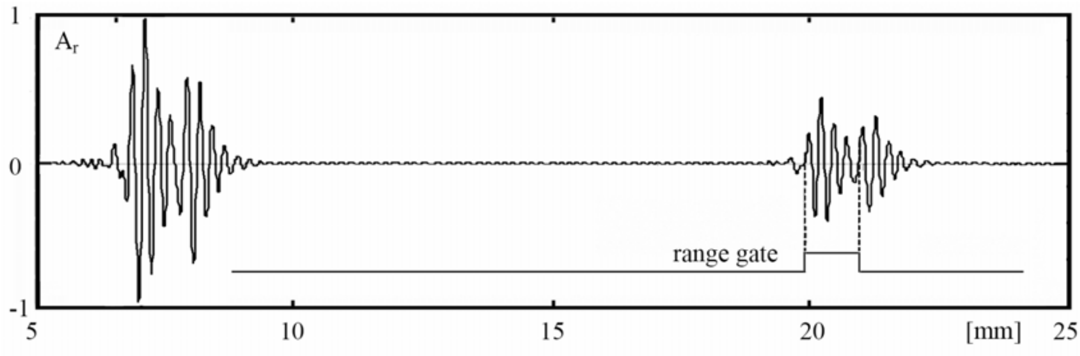
The examination of the change in the artery wall thickness was carried out on a theoretical model. The wall thickness and the artery model diameter were similar to corresponding dimensions of the common carotid artery for people at the age of 30 [5]. The internal radius of the artery model was 3 mm for the diastolic pressure and 3.3 mm for the systolic pressure. For diastolic pressure, the thickness of the intima, the media and the adventitia layer was equal to 0.12 mm, 0.36 mm and 0.12 mm respectively. Taking into account the incompressibility of the material, from which the artery wall was made, the IMT was changed from 0.48 mm to 0.44 mm respectively.

The RF signal was calculated for the following conductions:

- the frequency of the transmitted ultrasound was equal to 6.75 MHz,
- the front surface of the artery wall was positioned in the focus of the ultrasonic probe,
- the density of the wall layer was as follows: 1.05 kg/m^3 for the adventitia and the intima layers, 1.1 kg/m^3 for the media layer.
- the attenuation of the ultrasonic wave in tissue was as follows: 9.2 Np/mMHz in the wall artery [1], 2.1 Np/mMHz in the blood [1], 5 Np/mMHz in the tissue outline of the artery.

The RF signals calculated for artery model are presented in Fig 4. During the calculation of the IMT, the range gate (Fig. 4) was placed in the area of a group of echoes coming from the back surface of the artery wall between the ultrasound echoes coming from the initima layer and the echoes from the adventitia layer. The solver based on zero-crossing method was used for detecting of the IMT value. The precision of determination of the position of the tracked echo in the simulation process was the same as in the VED apparatus and was equal $7*10^{-6}$ m. Figure 5 depicts changes in the IMT.

a)



b)

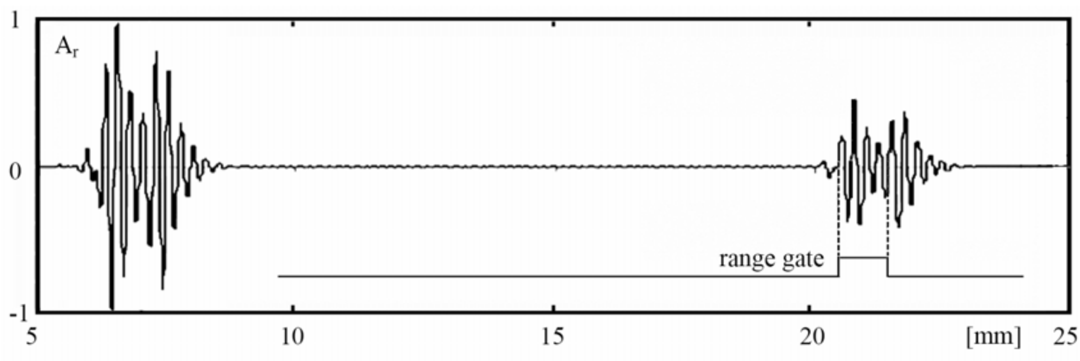


Fig.4 The RF signal $P_{RF}(t)$ from the artery model calculated by means of the developed solver for diastolic pressure (a) and for systolic pressure (b). The point 13.6 on the scale above corresponds to location of the artery centre at the distance of 25 mm from the ultrasonic probe, Ar – the relative amplitude (with respect to the maximum value of the RF signal amplitude)

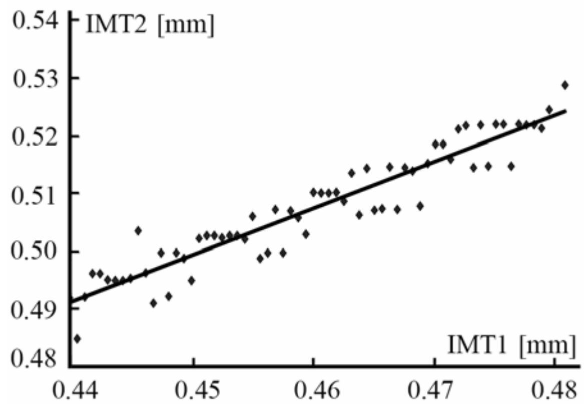


Fig.5 The comparison between the real (IMT1) and calculated from the RF signal (IMT2) values of the imtima-media thickness

5. CONCLUSIONS

Comparison between the results that were obtained from numerical calculations and from measurements (Fig. 2 – 3) serves as a proof that the numerical model that was developed by our own enables simulation of the experiments with a good coherence, which was the actual objective of the study.

The simulation of the artery wall thickness examinations on the basic of the RF signal showed that the numerical solver of the acoustic field can be useful for determination of the IMT and its changes. The value of the IMT determined on basis of the RF signal was equal 0.53 mm for diastolic pressure and 0.49 mm for the systolic pressure. The real value of the IMT assumed in the artery model was equal 0.48 mm and 0.44 mm respectively.

ACKNOWLEDGEMENT

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