

# INFLUENCE OF CHIRP SIGNAL PARAMETERS ON WINDOWED MATCHING FILTRATION RESOLUTION

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*In this work we studied the proposed effective methods of increasing of a resolution of a matched filtration in time domain of the short broadband signals with the use of linear modulation of frequency (the so-called chirp signals) which product  $BT$  does not exceed 50 and initial frequency is not zero. The choice of matched filter pulse response parameters as smoothing window and matched to it an initial phase and a sampling rate as well as the influence of non linear operations on the matching filtration results have been investigated in order to essentially narrow the output signal main lobe as well as increase the chirp compression. The best results of the recognition resolution and compression improvement occur when at the same time there are used the non linear operations on the convolutions and the pulse response parameters are matched to the chosen window.*

## INTRODUCTION

Chirp signals are characterized by resistance to interferences and therefore they find application in many fields, especially in location systems [1,2,3,4]. So far chirp signals with big values of a product at the signal band and duration  $BT$  ( $BT \gg 100$ ) have been sufficiently examined, especially when a initial frequency equals 0 [6]. Detection of signals with high  $BT$  does not bring difficulties on condition that their compression depends on boundary effects only to a small degree, especially when smoothing windows are used. Behavior of such signals is predictable and with the use of digital matching filtration (including fast convolutions) high compression can be obtained [2,3]. This situation is entirely different when short chirp signals with  $BT \leq 50$  and initial frequency between 0 and  $B$  are considered. For these signals apart from lower compression there occur more boundary effects. However, smoothing windows usage is not always advisable as it decreases the signal power and at same time widens a main lobe that leads to a resolution deterioration.

For short chirp signals with small  $BT$  it is optimal to use digital matching filtration on the convolutions in time domain, and not fast convolutions in the frequency domain because of:

1) higher speed of matching filtration operation in the time domain basing on transversal filters. Such a filtration requires N operations (where N is a number of signal samples) for obtaining all convolutions as opposed to the use of fast convolutions of  $N \log_2 N$  operations;

2) better accuracy of calculation because the filtration in time domain does not need as much transformations as the filtration on the fast convolution in frequency domain [3].

The study results show, the short chirp signal compression in connection with recognition resolution depends much on a initial frequency, phase as well as a sampling rate and window. The available to the authors literature shows that such effects have not been sufficiently examined and therefore the purpose of this work is working out the effective ways of an improvement of the short chirp signals recognition resolution and compression in the time domain on the basis of non linear operations on filtration results and choice of windows, critical initial frequencies, phases as well as sampling rates of these signals.

## 1. MATCHING FILTRATION ALGORITHM

Chirp-signal with constant amplitude and linear frequency modulation gives as follows:

$$x(t) = A \cos[2\pi(at + f_1)t + \varphi_0], \quad (1)$$

where  $a = B/2\tau_i$ ,  $B = f_2 - f_1$  - deviation of the frequency (band),  $f_1$  - initial frequency,  $f_2$  - final frequency,  $\tau_i$  - duration of the chirp signal,  $\varphi_0$  - initial phase.

For realisation of the digital matching filtration the signal (1) was represented in a form of time series  $\{x_n\}$  with a sampling rate  $f_s \geq 2f_2$ , whence the number of samples equals N, where  $N = \text{ENT}(\tau_i f_s)$ , ENT – the whole part of a number.

Discrete form chirp-signal (1) gives as follows:

$$x_r = x(rT_s) = A \cos\left[2\pi\left(\frac{Af}{2N}r + f_1\right)rT_s + \varphi_0\right], \quad (2)$$

where  $\tau_i = NT_s$ ,  $r = \overline{0, N-1}$ .

The pulse response of the matching filter without a smoothing window is a mirror display of an input signal (2):

$$h_n = x_{N-n}, \quad \text{where } n = \overline{1, N}. \quad (3)$$

For reduction of the Gibbs' oscillations which occur at filtration of chirp-signals smoothing windows are applied to the impulse response. In time domain the use of the windows realizes by multiplication of the appropriate weight factors of the pulse response and a window  $\{h_n w_n\}$ . Basic algorithm of the matching filtration basing on convolution in time domain and PCM format is as follows [5]:

$$y_n = \sum_{m=0}^{N-1} x_{n-m} h_m w_m, \quad (4)$$

where  $\{y_n\}$  – filtration results;  $\{x_n\}$  – input signal samples;  $\{h_n\}$  – weight factors of pulse response,  $N$  – number of weight factors and input signal samples.

or in matrix form which is more suitable for specialized processor:

$$y_n = \mathbf{X} \cdot \mathbf{H}_w, \quad (5)$$

where

$$\mathbf{X} = \begin{bmatrix} x_0 \\ \vdots \\ x_{N-1} \end{bmatrix}, \quad \mathbf{H}_w = \begin{bmatrix} h_0 w_0 \\ \vdots \\ h_{N-1} w_{N-1} \end{bmatrix}.$$

Here  $y_n$  –  $n$ -th convolution result;  $\{x_n\}$  – input signal samples;  $\{h_n\}$  – weight factors of impulse response,  $N$  – number of weight factors and input signal samples,  $\{w_n\}$  – smoothing window samples. For rectangular window there is assigned  $\forall w = 1$ .

When a matching filtration result is obtained directly on the basis of algorithm (5), part of convolutions  $\{y_n\}$  has negative values. Usually those values are converted and, afterwards, together with others positive convolutions form the set  $\{|y_n|\}$  which is an envelope basis. At this, some former negative convolutions become a part of the main lobe expanding it. Therefore, we propose rejecting of the negative convolutions in order to increase  $\text{SNR}_{\text{out}}$  as well as decrease a main lobe width to one sample only what is ideal case in relation to resolution. Non linear operations for the elimination of the negative convolutions are realized on the basis of the next algorithm:

$\forall y, \text{sgn } y, \exists y^+ ((\text{sgn } y_n = 1) \rightarrow (y_n^+ = y_n)) \vee (\text{sgn } y_n = -1) \rightarrow (y_n^+ = 0)$  where  $y_n = \text{sgn } y_n \cdot |y_n|$ ,  $\text{sgn } y_n \in \{1, -1\}$ ,  $\rightarrow$  - sequence sign,  $y_n^+$  -  $n$ -st processing result.

## 2. THE WORKED OUT WAYS OF IMPROVING OF THE SHORT CHIRP SIGNALS RECOGNITION AND COMPRESSION

Smoothing window permits to increase a ratio between a main and side lobes, which are a result of filtering, but simultaneously causes an increase of the width of the main lobe. Such broadening worsens a resolution of the chirp recognition. Therefore choice of that window depends on the compression requirements and a signal character.

Our approach relies on matching the pulse response (3) parameters to specific window. In case of using of rectangular window, we propose to match those assigned in time domain parameters to the window in order to obtain maximal rectangularity of pulse response amplitude spectrum in frequency domain.

In the presented work matching filtration of short chirp signals was carried out with the use of rectangular window and classical Hamming's smoothing window to a pulse response of the matching filter respecting or disrespecting negative results of the convolutions. For each of these cases there was examined the influence of the window as well as initial frequency, phase, and sampling rate of pulse response on the chirp signal compression and recognition. Detailed results were presented for chirp signal with duration 2.5 ms and momentary frequency changing linearly in a range from  $f_1=5$  kHz to  $f_2=25$  kHz ( $BT=50$ ). Sampling rate  $f_s$  was changed within 49.80 kHz to 60 kHz and the initial phase of the signal was changed in a range from  $0^\circ$  to  $180^\circ$ . Examinations were performed with use of simulation methods by means of a specially created program. The ratio between values of the main lobe and the highest side lobe ( $\text{SNR}_{\text{out}}$ ) was accepted as criterion and expressed in dB.

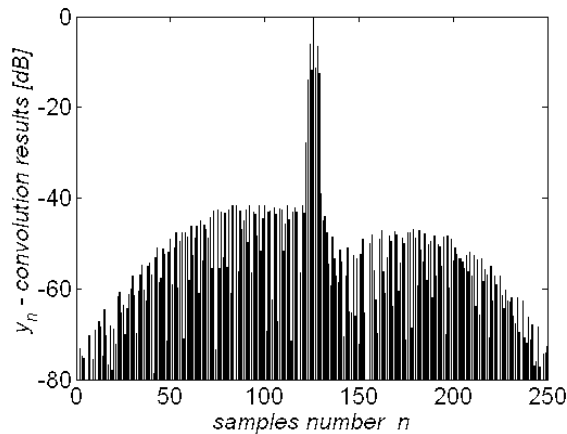


Fig.1 Result of the chirp signal compression with the use of Hamming's window and all convolutions

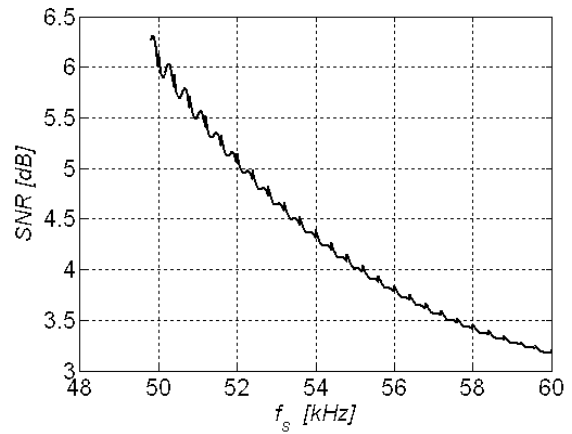


Fig.2 Relationship between  $SNR_{out}$  and  $f_s$  by using Hamming's window,  $\varphi_0 = 100^\circ$  and all convolutions. Main lobe consists of 1 sample only

The result of computer simulations of algorithm (5) with the use of Hamming's window and without the non linear operations is presented in Fig. 1. As a point of reference there was used the results of matching filtration of the chirp signal with the following parameters:  $\tau_l = 2,5$  mS,  $f_1 = 5$  kHz,  $f_2 = 25$  kHz,  $f_s = 50$  kHz,  $\varphi_0 = 100^\circ$ ,  $BT = 50$ ,  $N = 126$ . All values of the convolutions  $\{y_n\}$  have been taken into consideration.

The result of a matching filtration is shown in Fig.1 as a set of different samples. Each sample corresponds an appropriate convolution result. As diagram in Fig.1 shows, maximal  $SNR_{out}$  not exceed 27.8 dB when the main lobe includes 7 samples.

From the obtained simulation results it can be concluded that the signal initial phase as well as the sampling rate have an influence on  $SNR_{out}$ . In case when sampling rate is changed from the Nyquist's frequency  $f_N = 50$  kHz to 60 kHz and by using all convolution results and the width of main lobe equals 1 sample only,  $SNR_{out}$  changes cyclically with decreasing in function of  $f_s$ . It situation by  $\varphi_0 = 100^\circ$  is shown in Fig.2.

Carried out simulations show that usage of non linear operations (removing negative convolutions) in digital matching filtration in the time domain as well as the pulse response sampling rate and initial phase ,appropriately matched to specific window, allows to achieve nonlinear effects which are conducive to the short chirp signal ( $BT \leq 50$ ) compression.

In Fig.3 is shown an example that compression of chirp signal with above mentioned parameters and operations. The sampling rate is  $f_s = f_N = 50$  kHz , and then number of chirp samples is  $N = 126$ . Values of the  $SNR_{out}$  depends on the width of main lobe too. When we are considering the width of the main lobe on a level equalling 7 samples,  $SNR_{out} = 27.8$  dB, whilst on the level of 1 sample,  $SNR_{out} = 12.6$  dB.

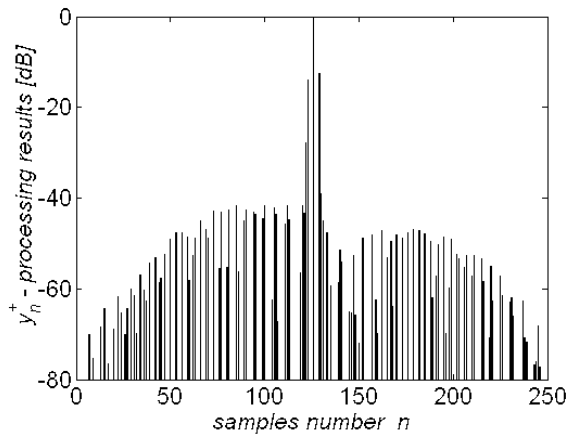


Fig.3 Result of the chirp signal compression with the use of Hamming's window and the positive convolutions only

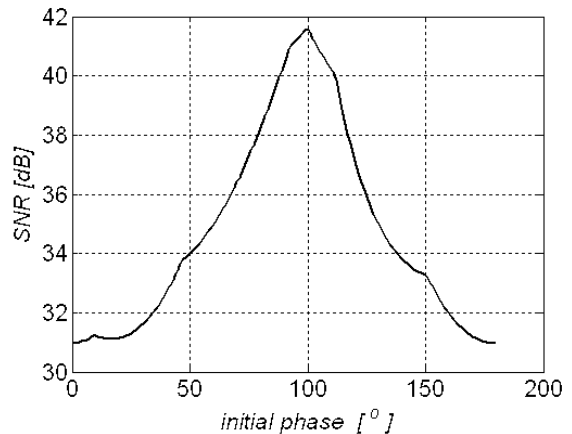


Fig.4 Relationship between  $SNR_{out}$  and  $\varphi_0$  when there are considering positive convolutions only. The main lobe consists of 9 samples

As results of simulation showed, for each initial phase of the pulse response there is possible to match the sampling rate in order to obtain the maximal  $SNR_{out}$ . For example, the  $SNR_{out}$  dependence on the initial phase for positive convolutions only by  $f_s = 50$  kHz is shown in Fig.4. Its maximal value is 41,9 dB.

The result of computer simulations of algorithm (5) with the use of rectangular window and all convolutions  $\{y_n\}$  is presented in Fig. 5. In this case,  $SNR_{out} = 13,4$  dB when main lobe is on a level of 1 sample and  $SNR_{out} = 24$  dB when one is on level of 7 samples. Fig. 6 represents the result of computer simulations of algorithm (5) with the use of the rectangular window and positive convolutions  $\{y_n^+\}$  only. At this  $SNR_{out} = 23,7$  dB when main lobe includes 1 sample. At that there are used the chirp signals with the same parameters as above.

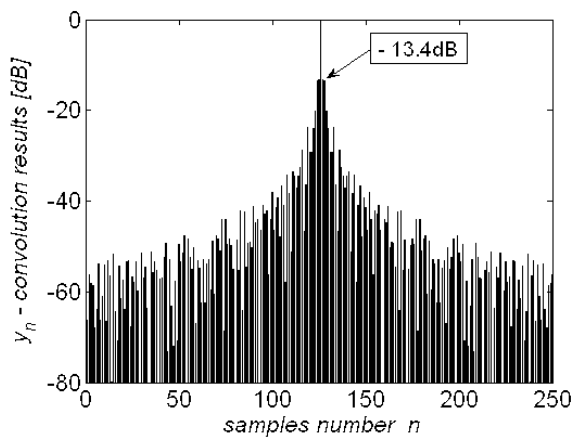


Fig.5 Result of the chirp signal compression with the use of rectangular window and all convolutions. Here are  $\varphi_0=100^\circ$   $f_s=50$  kHz Main lobe includes 1 sample

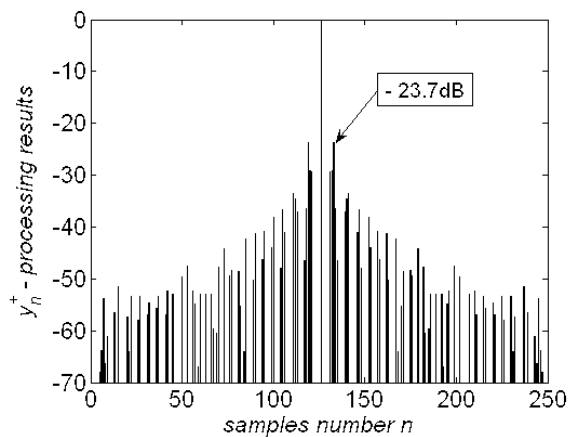


Fig.6 Result of the chirp signal compression with the use of rectangular window and positive convolutions only by matched initial phase and sampling rate. Main lobe includes 1 sample

Examples of studying of the  $SNR_{out}$  dependence on the initial phases and the main lobe width are shown in Fig.7 and Fig.8 by using both all convolutions and positive ones. Here is used the rectangular window and  $f_s=50$  kHz.

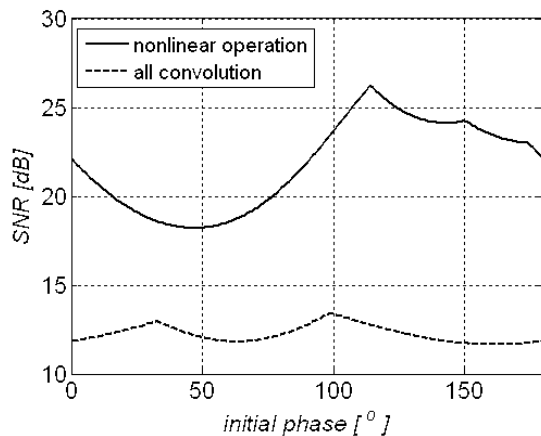


Fig.7 The  $SNR_{out}$  dependence on the initial phase, main lobe includes 1 sample

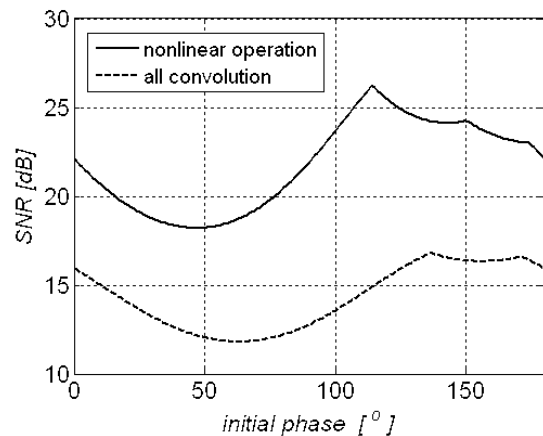


Fig.8 The  $SNR_{out}$  dependence on the initial phase, main lobe includes 3 samples

### 3. CONCLUSION

Obtained results show that usage of Hamming's window and matched parameters of the pulse response of the filtration in time domain of short chirp signals with  $BT \leq 50$  and non zero initial frequencies ( $f_1 \geq 0.035 f_2$ ) leads to obtaining higher  $SNR_{out}$  values than in case of using of the rectangular window if a main lobe includes 7 or more samples. In case of better recognition resolution, when it includes less than 7 samples, the best results take place by using of the rectangular window and positive convolutions only.

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