APPROXIMATION OF UNDERWATER CHANNEL IMPULSE RESPONSE BY NON-ORTHOGONAL FUNCTIONS

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This paper presents the idea and methods of approximation of underwater channel impulse response by non-orthogonal functions. The family of functions that correctly estimate response of underwater channel is chosen. For these functions, method of calculation of parameters and coefficients is introduced. Besides that, two examples are presented. The first one is the approximation of response in deep reservoir, the second one is approximation of response in shallow reservoir, where appear problems of reflections from surface of water and bottom of reservoir.

INTRODUCTION

In the process of signal transmission, it is important to find suitable representation of the impulse response of transmission channel, which allows to describe very easily the process of transmission. In paper [5] response to a special kind of functions was presented. Unfortunately the result was very complicated. Impulse response can be obtained as a result of measurements and can be representated in a form of samples. The next step is to gain mathemathical interpretation. The problem is to choose appropriate approximation function. Commonly used cosinus and sinus functions or Chebyshev polynomials fulfill their role very well. Unfortunately to achieve satisfying approximation a notable number of approximating functions need to adopted. This leads to complication of mathematical representation and handicaps following physical realisation as well as further process of mathemathical analysis. In publication [1] approximation of underwater channel impulse response in acoustic transmission in deep waters with usage of Chebyshev and Laguerre polynomials was suggested. Correct estimation of channel response was obtained by the application of several approximating functions. It is too many, that transmission system could perform properly with expected data processing rate and low level of structure complicity. It is important to find such a functions, which allow to achieve desired solution (estimation of underwater channel impulse response with a low number of approximating functions). Unfortunately these functions very often do not fulfill basis of orthogonality, which leads to complication of the process of approximation. The goal is to find method, that would allow with usage of computing machine to compute coefficients of any approximation function in unrestricted interval (a,b). The function mustn't have complicated Laplace transform. In other way, finding spectrum would be very difficult.

1. BASIS OF METHOD OF APPOXIMATION BY NON-ORTHOGONAL FUNCTIONS USED TO CALCULATE CHANNEL RESPONSE

Every function can be represented by [2]:

$$f(t) \cong \sum_{n=1}^{k} C_n \cdot g_n(t) \tag{1}$$

where:
$f(t)$ – approximated function; C_n – coefficient;
$g_n(t)$ – approximating functions; k – natural number

The evaluative criteria is the mean squared error, which can be described by the equation below:

$$\varepsilon = \frac{1}{b-a}\int_a^b \left[f(t) - \sum_{n=1}^{k} C_k \cdot g_n(t) \right]^2 dt \tag{2}$$

The mean squared error must be the smallest. To fulfill this, the following formulas must be performed:

$$\frac{\partial \varepsilon}{\partial C_1} = 0; \quad \frac{\partial \varepsilon}{\partial C_2} = 0; \quad \dots \quad \frac{\partial \varepsilon}{\partial C_k} = 0 \tag{3}$$

After calculations, system of k equations with k unknown variables is obtained:

$$\begin{aligned}
&\int_a^b f(t)\cdot g_1(t)dt - C_1\cdot\int_a^b g_1(t)\cdot g_1(t)dt - C_2\cdot\int_a^b g_2(t)\cdot g_1(t)dt - C_k\cdot\int_a^b g_k(t)\cdot g_1(t)dt = 0\\
&\int_a^b f(t)\cdot g_2(t)dt - C_1\cdot\int_a^b g_1(t)\cdot g_2(t)dt - C_2\cdot\int_a^b g_2(t)\cdot g_2(t)dt - C_k\cdot\int_a^b g_k(t)\cdot g_2(t)dt\\
&\dots\\
&\int_a^b f(t)\cdot g_k(t)dt - C_1\cdot\int_a^b g_1(t)\cdot g_k(t)dt - C_2\cdot\int_a^b g_2(t)\cdot g_k(t)dt - C_k\cdot\int_a^b g_k(t)\cdot g_k(t)dt = 0
\end{aligned} \tag{4}$$

If approximating function was orthogonal, every integral $\int_a^b g_l(t) \cdot g_k(t)dt$ for l different than m would equal 0 and formulas for sequent coefficients would be represented by:

$$C_k = \frac{\int_a^b f(t) \cdot g_k(t)dt}{\int_a^b g_k^2(t)dt} \tag{5}$$

In the case, all the functions $g_k(t)$ are not orthogonal, values of this integrals (4) may not be equal 0. So the problem of approximation by any function resolves to solution of k linear equations with k unknown variables. Unfortunately, for a large number of coefficients C_k,, finding solution would be practically impossible without computer. For a number of 3 values of coefficients calculations become very combplicated.

Integrals, that are in formulas (4) have to be computed before. When the approximated function is presented in the form of samples and Simpson critera is used to calculate intergrals, formulas will be reprsesentated by:

$$\int_a^b f(t) \cdot g_k(t)dt = \sum_{n=1}^{m-2} \frac{1}{3} \cdot (f_n + 4 \cdot f_{n+1} + f_{n+2}) \cdot (g_{k,n} + 4 \cdot g_{k,n+1} + g_{k,n+2}) \cdot \Delta f \tag{6}$$

where:
Δf - distance between samples
f_n, f_{n+1}, f_{n+2} – samples of approximated function
$g_{k,n}, g_{k,n+1}, g_{k,n+2}$ – values of k approximating function
m – number of samples of channel impulse response

Integrals by the aproximation coefficients are to be solved the same way:

$$\int_a^b g_j(t) \cdot g_k(t)dt = \sum_{n=1}^{m-2} \frac{1}{3} \cdot (g_{j,n} + 4 \cdot g_{j,n+1} + g_{j,n+2}) \cdot \cdot (g_{k,n} + 4 \cdot g_{k,n+1} + g_{k,n+2}) \cdot \Delta f \tag{7}$$

After calculation of these values, expected equation system can be defined.

2. OBTAINING IMPULSE RESPONSE IN DEEP RESERVOIR

Let's now consider a deep reservoir. Signal, that is spreading in water reaches receiver only in one path. During transmission, there is no reflections from water surface nor the bottom of reservoir. The input signal is explosion, which is adequate to Dirac impulse in underwater transmission.

For a better realization of the impulse response in underwater channel function must fulfill the following conditions:

$$\lim_{t \to \infty} f(t) = 0 \tag{8}$$

and

$$f(0) = 0 \, . \tag{9}$$

Besides that, function in the whole interval from 0 to infinity should have only positive values and only one maximum. This is expected because underwater channel responses have similar character.

The best family of functions which can be used to approximate underwater channel impulse response is:

$$g(t) = t^n \cdot e^{-\frac{1}{A} \cdot t^m} \tag{10}$$

where:

m, n – natural numbers

A – any number > 0

In our example we will use the folowing function:

$$g(t) = t^3 \cdot e^{-\frac{1}{A} \cdot t} \tag{11}$$

This function fulfills all assumptions and doesn't have complicated Laplace transformation, which allows to easily calculate spectrum in the later phase.

In this function parameter A is very important. This parameter describes the velocity of increase and decrease of the impulse response. That's why, it is important to find this parameter, with which approximation function would estimate the response of the channel as good as it's possible. To gain this, for the sequent parameters A changing from 1 to 20 process of approximation was executed. After that for every possibility, mean squerre error was computed. In this way, the optimal approximation of the impulse response was obtained..

After approximation function is obtained:

$$f(t) = -0{,}0099 \cdot t^3 \cdot e^{-\frac{1}{2} \cdot t} + 0{,}6633 \cdot t^3 \cdot e^{-t} \tag{12}$$

Fig. 1 presents channel impulse response, which was obtained as a result of measurements and its mathematical representation obtained from approximation by non-orthogonal functions.

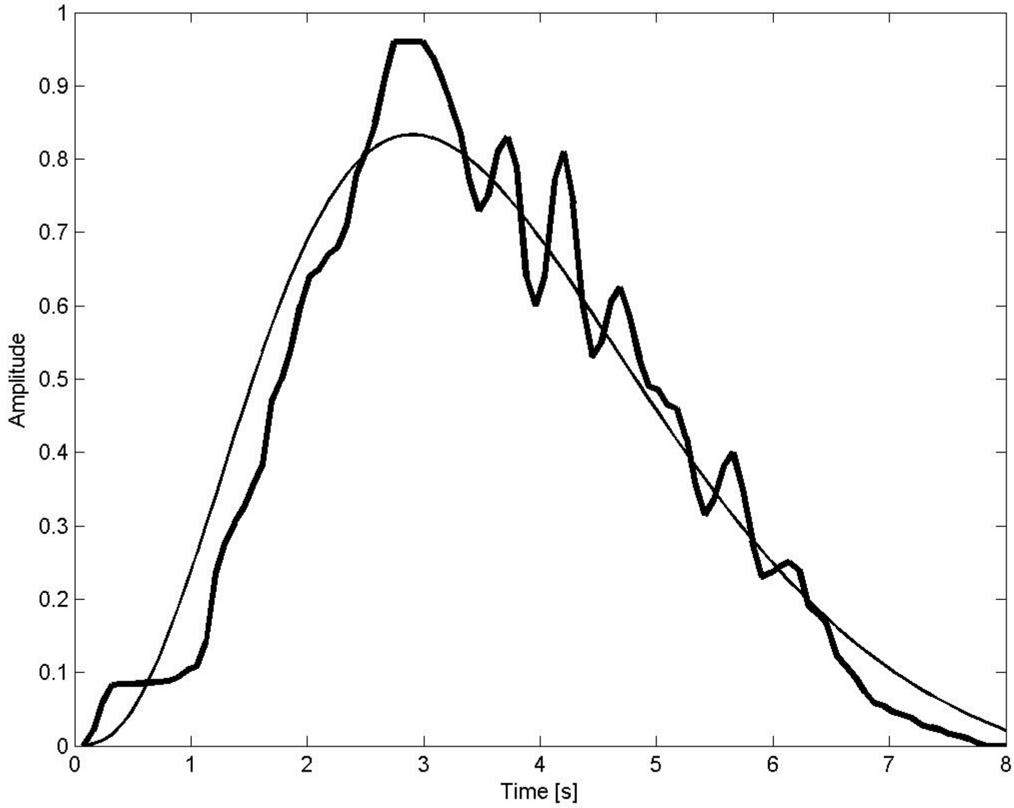

Fig.1 Impulse response and its approximated function

Figure shows, that obtained function correctly estimates response of channel. All deformations caused by aleatory phenomena are obliterated.

Afterwards, Laplace transform of the channel response is calculated. This allows to nominate spectrum function. Laplace transformation is computed, because all the values for t < 0 in channel function are eliminated.

After Laplace Transform:

$$F(s) = -\frac{0{,}9504}{(2\cdot s+1)^4} + \frac{3{,}9798}{(s+1)^4} \tag{13}$$

Compound variable s is replaced by jω to gain spectrum of channel response.

After conversion:

$$F(j\omega) = -\frac{0{,}9504}{(2\cdot j\omega+1)^4} + \frac{3{,}9798}{(j\omega+1)^4} \tag{14}$$

Spectrum function is representated by equation:

$$F(j\omega)=\frac{-0{,}9504\cdot\left(16\cdot\omega^{4}-24\cdot\omega^{2}+1\right)}{256\cdot\omega^{8}+256\cdot\omega^{6}+96\cdot\omega^{4}+16\cdot\omega^{2}+1}+\frac{3{,}9798\cdot\left(\omega^{4}-6\cdot\omega^{2}+1\right)}{\omega^{8}+4\cdot\omega^{6}+6\cdot\omega^{4}+4\cdot\omega^{2}+1}+$$
$$+j\cdot\left(-\frac{-0{,}9504\cdot\left(32\cdot\omega^{3}-8\cdot\omega\right)}{256\cdot\omega^{8}+256\cdot\omega^{6}+96\cdot\omega^{4}+16\cdot\omega^{2}+1}\right)+\frac{3{,}9798\cdot\left(4\cdot\omega^{3}-4\cdot\omega\right)}{\omega^{8}+4\cdot\omega^{6}+6\cdot\omega^{4}+4\cdot\omega^{2}+1} \quad (15)$$

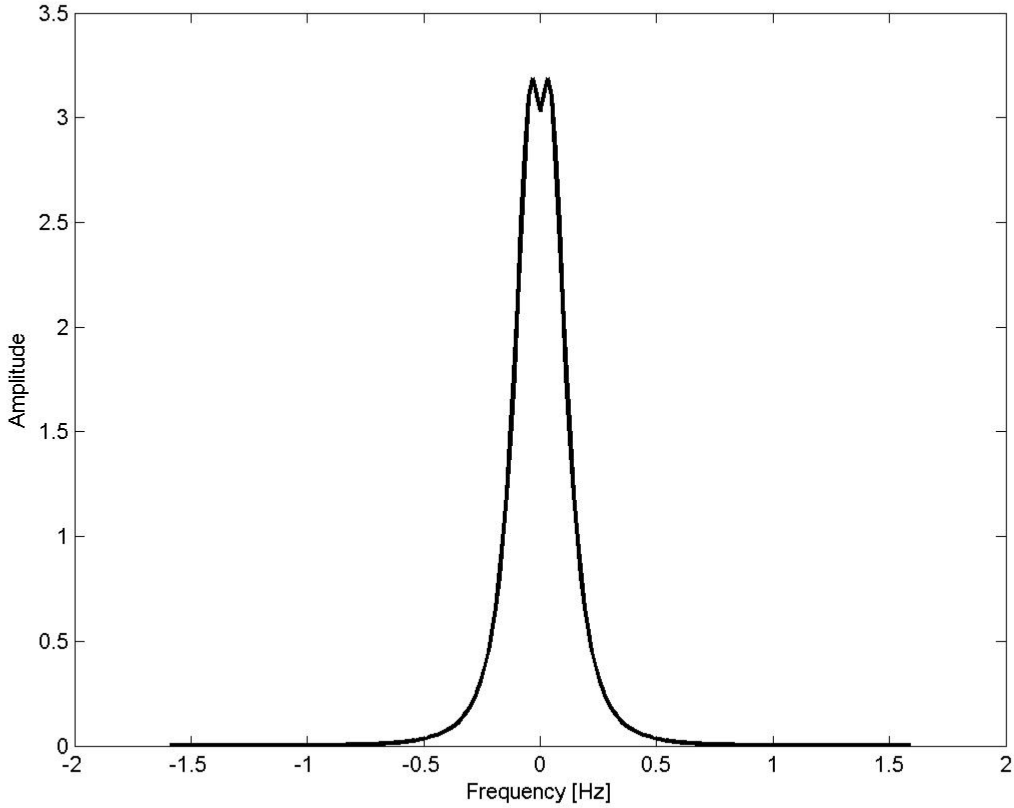

Fig.2 Amplitue spectrum of approximated channel response

3. OBTAINIG IMPULSE RESPONSE IN SHALLOW RESERVOIR

This example shows the process of approximation of channel response to explosion in shallow reservoir. Figure 3 presents the channel response taken from [4]. This picture shows the main impulse reaching the receiver directly from the place of explosion. The following impulses, but with smaller amplitude, reach receiver after reflections from the surface of water and the bottom of the reservoir. Explosion took place in reservoir at depth of 350 meters. Depth of the reservoir was 1000 meters. It is important to know this, because in that kind of reservoirs, reflections from the surface of water as well as reflections from the bottom of reservoir have significant influence on the shape of the response.

For the better representation of channel response, function, which equation is shown below was used:

$$g(t) = (t - T)^3 \cdot e^{-\frac{1}{A}(t-T)} \cdot 1(t - T) \qquad (16)$$

where:
$1(t-T)$ - step function shifted by time T

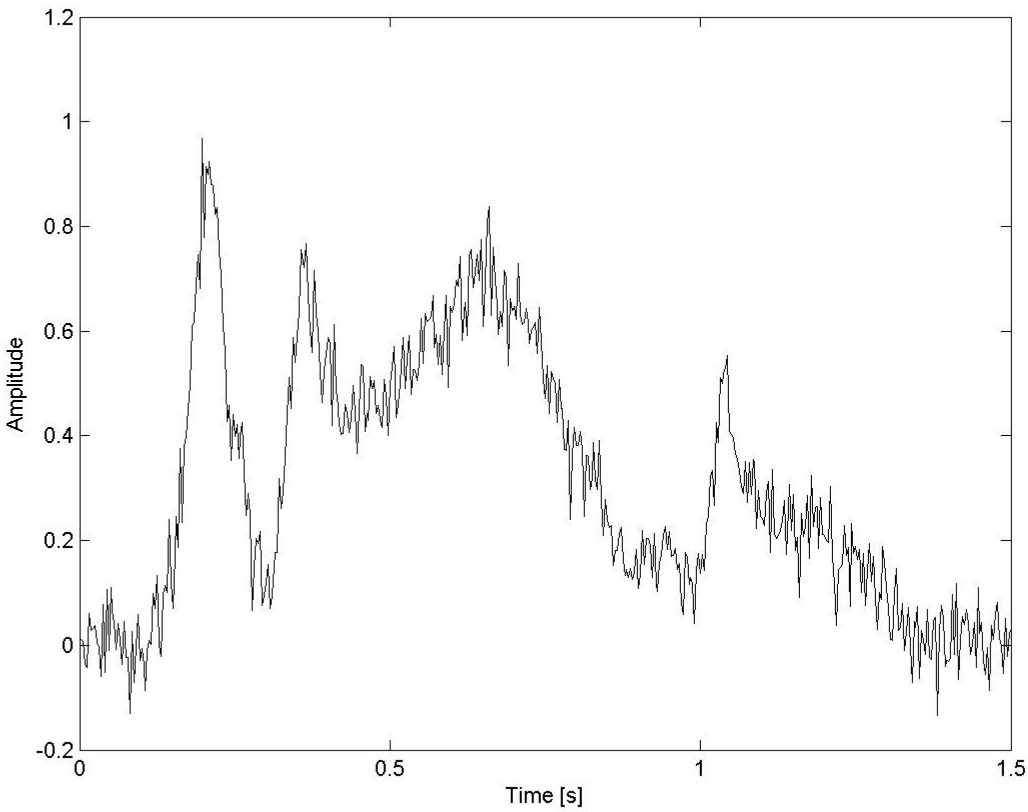

Fig.3 Channel response to explosion

After filtering noise out, the process of approximation was proceeded. Similarly to the previous chapter very important role plays coeficient A, which describes increase and decrease of function. Another very important parameter is parameter T, which describes delay of appearance of peek. Just like in first case, number of calculations were executed, that allowed to approximate function for different parameters A and T. Afterwards all results were surveyed and best approximationg function was chosen. The evaluative criteria, just like in the first example, was mean squared error.

After computing the following function was obtained:.

$$f(t) = 95000 \cdot (t-0{,}15)^3 \cdot e^{-50\cdot(t-0{,}15)} \cdot 1(t-0{,}15) + 75000 \cdot (t-0{,}3)^3 \cdot e^{-50\cdot(t-0{,}3)} \cdot 1(t-0{,}3) + 1200 \cdot (t-0{,}4)^3 \cdot e^{-13\cdot(t-0{,}4)} \cdot 1(t-0{,}4) + 6000 \cdot (t-0{,}95)^3 \cdot e^{-29\cdot(t-0{,}95)} \cdot 1(t-0{,}95) \tag{17}$$

On Figure 4 are shown: channel response after filtering noise out and obtained approximating function.

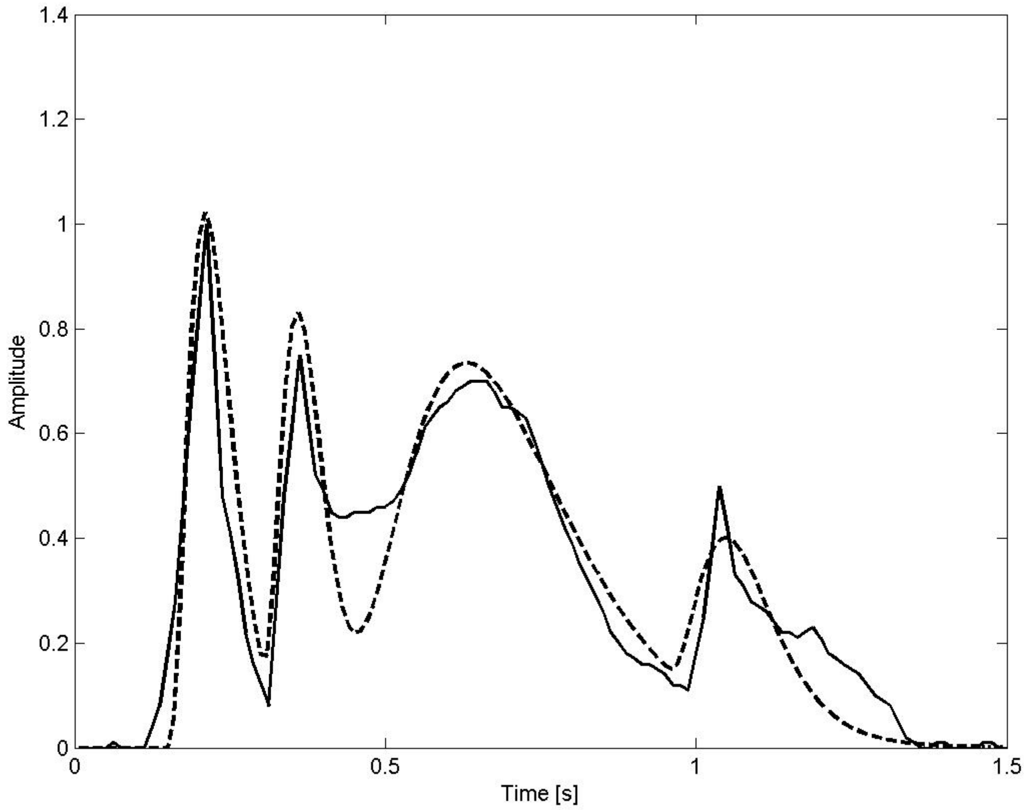

Fig.4 Impulse response and its approximated function (dashed line)

After Laplace transform:

$$F(s) = \frac{6 \cdot 95000}{(s+50)^4} \cdot e^{-0{,}15\cdot s} + \frac{6 \cdot 75000}{(s+50)^4} \cdot e^{-0{,}3\cdot s} + \frac{6 \cdot 1200}{(s+13)^4} \cdot e^{-0{,}4 s} + \frac{6 \cdot 6000}{(s+29)^4} \cdot e^{-0{,}95\cdot s} \tag{18}$$

After conversion:

$$F(j\omega) = \frac{6 \cdot 95000}{(j\omega + 50)^4} \cdot e^{-0,15 \cdot j\omega} + \frac{6 \cdot 75000}{(j\omega + 50)^4} \cdot e^{-0,3 \cdot j\omega} + \frac{6 \cdot 1200}{(j\omega + 13)^4} \cdot e^{-0,4 j\omega} + \frac{6 \cdot 6000}{(j\omega + 29)^4} \cdot e^{-0,95 \cdot j\omega} \quad (19)$$

Amplitude spectrum is shown on the figure 5.

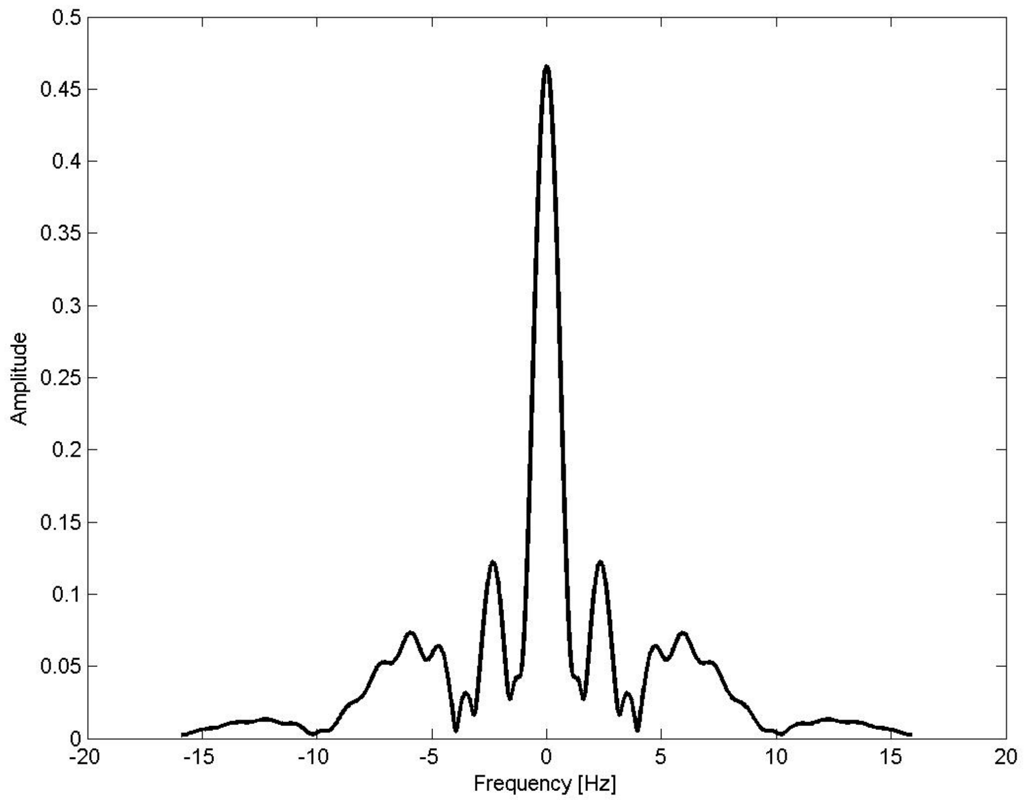

Fig.5 Amplitude spectrum of approximated channel response

4. CONCLUSION

Non-orthogonal functions were used to approximate underwater channel impulse response. The results of this process were satisfying. The level of complication of obtained functions was very low. This allows to construct transmission system based on popular microprocessors, because process of computing doesn't need complicated calculations. It can be assumed, that method presented in this paper is correct and can be applied and developed in subsequent research. It is important to elaborate and analize methods of verification of gained functions. These methods should allow uncomplicated implementation in adaptation receiver systems working in underwater environment, which is the subject of further work.

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