NONREDUNDANT 2-D AND 3-D SONAR SYSTEMS BASED ON GOLD RING BUNDLES

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The paper presents new method for configure systems of non-redundant two-dimensional and three-dimensional aperture antenna. The method is based on the Perfect Combinatorial Sequencing theory, namely the concept of Gold Ring Bundles (GRB)s for finding of the optimal placement of array antenna elements for acoustic or underwater acoustic systems. It is shown that the method provides many opportunities of the concept for novel design of nonuniform array with non-redundant aperture of array systems, including acoustics and hydroacoustics.

INTRODUCTION

Problem of structural optimization of sonar systems relates to finding the best placement of structural elements in spatially or temporally distributed systems. The problem to be of great important for improving the quality indices of the system, including active sonar, and it is closely connected with application of fundamental research in an applied finite-field theory [1]. Research into underlying mathematical area involves investigation of novel techniques based on combinatorial models, such as multidimensional Gold Ring Bundles [2].

The finite-field theory and appropriate technique, based on wide-range radar or sonar interferometric synthesis has been suggested in [3]. Some regular methods for constructing non-redundant two-dimensional n-elements masks over $n \times n$ grids, based on special combinatorial structures known as difference sets are described in the publication [4]. It is shown that the construction of a non-redundant two-dimensional mask can be reduced to a similar one-dimensional problem, as well as it is demonstrated that arrangements of masks over an interval can be constructed from difference sets obtainable using methods from finite-field theory. A systematic technique for constructing non-redundant masks is described in the paper [4]. It was shown that such a system of masks would cover nearly all of the required range of spatial frequencies. However, the classical theory of combinatorial configurations based on finite-field theory can hardly be expected effective for solving 2-D and 3-D problems using methods, based on the theory.

Hence, both an advanced theory and regular method for finding optimal solution the problem are needed. Research into techniques based on combinatorial models, namely the concept of the Gold Ring Bundles relates to development both the advanced combinatorial theory and regular method for high performance nonuniform array design, based on application of remarkable combinatorial properties of two- and three-dimensional Gold Ring Bundles, and generalization of these methods and results to improvement of quality indices of sonar systems (e.g. active sonar) with respect to resolving ability and operating range.

1. NUMERICAL BUNDLES

Let us calculate all S_n sums of the terms in the numerical n-stage chain sequence of distinct positive integers $K_n = \{k_1, k_2,, ... ,k_i, ... k_n\}$, where we require all terms in each sum to be consecutive elements of the sequence. Clearly the maximum such sum is the sum S_{max} of all n elements:

$$S_{max} = k_1 + k_2 + ... + k_i + ...+ k_n . \tag{1}$$

A sum of consecutive terms in the chain sequence can have any of the n terms as its starting point p_j, and finishing point q_j, and can be of any length (number of terms) from 1 to n. So, each of j numerical pair (p_j,q_j), $p_j,q_j \in \{1, 2, n\}$ corresponds to sum $S_j = S(p_j,q_j)$, is equal:

$$S_j = S(p_j,q_j) = \sum_{i=p_j}^{q_j} k_i , \; p_j \le q_j. \tag{2}$$

An ordered numerical pair (p_j,q_j) determines sum $S(p_j,q_j)$ in the numerical n-stage chain sequence, and it is a numerical code of the sum. All sums of consecutive terms, calculated by (2), can be illustrated graphically by the Table 1.

To see this, we observe that the maximum number of distinct sums is

$$S'_{max} = 1+2+...+n = n(n+1)/2. \tag{3}$$

The ideal ordered -chain sequence (ideal bundle) is such numerical n-stage of distinct positive integers $k_1, k_2, ... k_n$, which exhausts the natural row of numbers wrote down into cells of the table 1. Table of sums of consecutive terms in ordered -chain sequence could be used for research of numerical sequences in order to speed up finding ideal or optimal solutions.

For example, ordered -chain sequence {1,3,2} is the ideal bundle or Ideal Golomb ruler.

Tab.1 Sums of consecutive terms in ordered -chain sequence

p_j	q_j				
	1	2	...	n-1	n
1	k_1	$\sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n-1} k_i$	$\sum_{i=1}^{n} k_i$
2		k_2	...	$\sum_{i=2}^{n-1} k_i$	$\sum_{i=2}^{n} k_i$
...			...		...
n-1				k_{n-1}	$\sum_{i=n-1}^{n} k_i$
n					k_n

Unfortunately, there are not exist ideal bundle with more of three intersections. The problem, is known, to be of great important in development of regular method based on the idea of "perfect vyazanka" for finding optimal placement of structural elements in 3-D spatially distributed systems (e.g. an acoustic system or sonar) with respect to improving the resolving ability and tuning range.

2. GOLD RING BUNDLES

If we regard the chain sequence K_n as being cyclic, so that element k_n is followed by k_1 , we call this a ring sequence.

Table of sums of consecutive terms in ordered -ring sequence K_n= {k_1, k_2,, ... ,k_i, ... k_n is demonstrated below (Tabl.2). A sum of consecutive terms in the ring sequence can have any of the *n* terms as its starting point p_j, and finishing point q_j, and can be of any length (number of terms) from 1 to *n*-1.

Tab.2 Sums of consecutive terms in ordered -ring sequence

p_j	q_j				
	1	2	...	n-1	n
1	k_1	$\sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n-1} k_i$	$\sum_{i=1}^{n} k_i$
2	$\sum_{i=1}^{n} k_i$	k_2	...	$\sum_{i=2}^{n-1} k_i$	$\sum_{i=2}^{n} k_i$
...			...		...
n-1	$\sum_{i=n-1}^{n} k_i + k_1$	$\sum_{i=n-1}^{n} k_i + \sum_{i=1}^{2} k_i$	...	k_{n-1}	$\sum_{i=n-1}^{n} k_i$
n	k_n+k_1	$k_n + \sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n} k_i$	k_n

So, each numerical pair (p_j,q_j), $p_j,q_j \in \{1, 2,\ldots, n\}$, corresponds to sum $S_j = S(p_j,q_j)$, and can be calculated in case, when $p_j \le q_j$, by equation:

$$S_j = S(p_j,q_j) = \sum_{i=p_j}^{q_j} k_i \tag{4}$$

In case $p_j > q_j$ a ring sum can be calculated by

$$S_j = S(p_j,q_j) = \sum_{i=1}^{q_j} k_i + \sum_{i=p_j}^{n} k_i\ , \tag{5}$$

Easy to see from the table 1, that the maximum number of distinct sums S_n of consecutive terms of the ring sequence is

$$S_n = n(n-1) + 1 \tag{6}$$

Comparing the equations (3) and (6), we can see that the number of sums S_n for consecutive terms of the ring topology is nearly double the number of sums in daisy-chain topology, for the same sequence of n terms.

An n-stage ring sequence $K_n = \{k_1, k_2, \ldots ,k_i, \ldots k_n\}$ of natural numbers for which the set of all S_n circular sums consists of the numbers from 1 to $S_n = n(n-1) + 1$, that is each number occurring exactly once is called an "Gold Ring Bundles" (GRB) with $R = 1$.

Here is an example of a numerical ring sequence with $n = 4$ and $S_n = n(n-1) + 1 = 13$, namely {1, 2, 6, 4}, where $k_1=1$, $k_2=2$, $k_3=6$, $k_4=4$.

Table of circular sums for the sequence is given below (Tabl.3).

Tab.3 Table of circular sums for numerical ring sequence {1, 2, 6, 4}

	q_j			
p_j	1	2	3	4
1	1	3	9	13
2	13	2	8	12
3	11	13	6	10
4	5	7	13	4

Table 2 is calculated in similar way than above, using equation (2) and (3). To see this table 2 contains the set of all $S_n = n(n-1) + 1 = 13$ sums to be consecutive elements of the 4-stage ring sequence {1,2,6,4}, and each sum from 1 to n-1 occurs exactly once. So, the ring sequence is an one-dimensional Gold Ring Bundle (1D-GRB) with $n = 4$.

Here is a graphical representation of one-dimensional Gold Ring Bundle (1-D GRB) containing four (n=4) elements {1,2,6,4}.

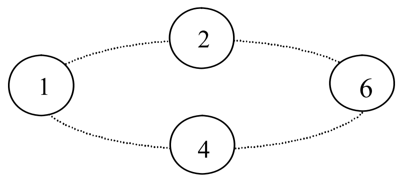

Fig.1 A graph of one-dimensional Gold Ring Bundle (1D-GRB) containing four (n=4) elements {1,2,6,4}

It is known, a number of consecutive elements in the GRB can be of a considerable length and the more length the more number of the GRB [2].

3. TWO-DIMENSIONAL GOLD RING BUNDLE

The idea of "perfect" numerical bundles provides development and design of other "gold" combinatorial constructions, such as one- and two-dimensional Gold Ring Bundles.

Let us regard the n-stage ring sequence $K_{2D}=\{(k_{11},k_{12}), (k_{21}, ,k_{22}),...(k_{i1},k_{i2})...,(k_{n1},k_{n2})\}$, where we require all terms in each circular vector-sum to be consecutive 2-stage sequences as elements of the sequence. A circular vector-sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length from 1 to n-1. An n-stage ring sequence $K_{2D} =\{(k_{11},k_{12}), (k_{21}, ,k_{22}), ...(k_{i1},k_{i2}), ...(k_{n1},k_{n2})\}$, for which the set of all

$$S_{2D} = n(n-1) \tag{7}$$

circular vector-sum forms two-dimensional grid, where each node of the grid occurs exactly R-times, is named a two-dimensional Gold Ring Bundle (2D-GRB).

Next, we consider two-dimensional n-stage ring sequence GRB with four (n= 4) terms in the ring topology, where k_1= (1,1), k_2= (1,2), k3=(1,4), k_4=(1,3) which graph is depicted below (Fig.2).

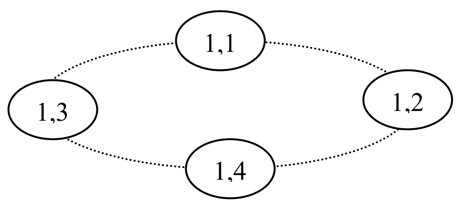

Fig.2 A graph of two-dimensional Gold Ring Bundle (2D-GRB) containing four (n=4) elements {(1,1), (1,2), (1,4), (1,3)}

We can calculate easy the all circular two-dimensional vector-sums, taking modulo m_1 =4 for the first component of the vector-sum and modulo m_2 =5 for the second its component:

$$
\begin{array}{ll}
(2,1) \equiv (1,2)+(1,4) & (3,1) \equiv (1,3)+(1,1)+(1,2) \\
(2,2) \equiv (1,4)+(1,3) & (3,2) \equiv (1,1)+(1,2)+(1,4) \\
(2,3) \equiv (1,1)+(1,2) & (3,3) \equiv (1,4)+(1,3)+(1,1) \\
(2,4) \equiv (1,3)+(1,1) & (3,4) \equiv (1,2)+(1,4)+(1,3)
\end{array}
$$

So long as the elements of the ring sequence themselves are circular vector-sums too, the circular vector-sums set configure the 3×4 matrix as follows:

$$
\begin{array}{cccc}
(1,1) & (1,2) & (1,3) & (1,4) \\
(2,1) & (2,2) & (2,3) & (2,4) \\
(3,1) & (3,2) & (3,3) & (3,4)
\end{array}
$$

The result of the calculation forms the 3×4 grid which exhausts the circular 2D vector-sums and each of its meets exactly once (R=1). So, the ring sequence of the 2D vectors {(1,1), (1,2), (1,4), (1,3)} is two-dimensional Gold Ring Bundle (2D-GRB) with n=4, R=1, m_1=4, m_2=5.

4. NONREDUNDANT 2-D APERTURE MASK SYSTEMS

The present method relates to constructing thinned planar phased antenna array configurations, which have the antenna or sensor elements positioned in a manner as prescribed by the GRB, using appropriate variant of two-dimensional Gold Ring Bundle for the constructing. The method involves technique for minimizing sizes of antenna arrays prescribed by parameter S of appropriate GRB. The search algorithm allows finding optimal solution in the simplest way based on regarding selected matrix of circular two-dimensional vector-sums on the GRB as well as crossing out operations. These procedures make it possible to configure mask system with the smallest possible number of grids.

Here is example of constructing the planar antenna array configuration based on the two-dimensional Gold Ring Bundle with parameters n=13, R=4, and m_1=5, m_2=8 (Fig.3).

We search needed solution after construction of 2-D matrix of all circular two-dimensional vector-sums on the GRB and regarding each of its with respect to search minimum of the sum using crossing out. The method described is illustrated in Fig. 3.

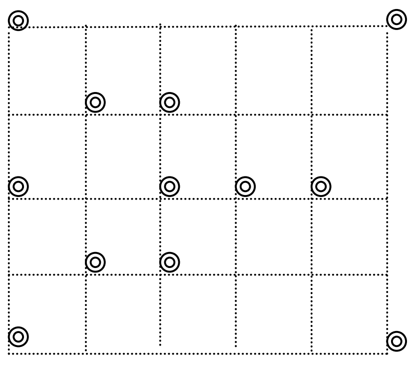

Fig.3 An antenna array over 5 × 6 grids reconstructed from the array over 5 × 8 grids based on the underlying 2-D GRB

The example shows that the grid order based on the GRB can be reduced further without loss of the possibility to construct an antenna array.

5. NONREDUNDANT 3-D APERTURE MASK SYSTEMS

The three-dimensional (t=3) GRB of order n can be represented as n-stage ring-like sequence $\{(k_{11}, k_{21}, k_{31}), (k_{12}, k_{22}, k_{32}),\ldots,(k_{1n}, k_{2n}, k_{3n})\}$ which give us a set of circular 3-vector-sums on the sequence as $M_1 \times M_2 \times M_3$ -matrix exactly R times.

Let the first of six (n = 6) mask elements is the (0,0,0) cell of $2 \times 3 \times 5$ – matrix cycling. Now, we can obtain coordinates of the remaining five elements accordingly the underlying 3-D perfect distribution cycling modulus $m_1 = 2$, $m_2 = 3$, $m_3=5$:
(1,1,1), (0,2,3), (1,2,1), (1,1,3), (1,2,2).

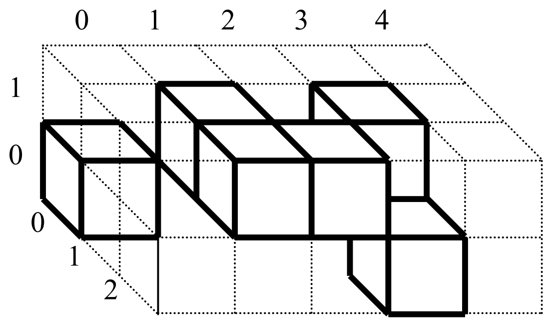

Fig.4 A non-redundant $2 \times 3 \times 5$ – matrix, based on the 3-D Gold Ring Bundle with parameters n= 6, R=1, and $m_1 = 2$, $m_2 = 3$, $m_3=5$

Now, to obtain configuration with smaller grids, we can exclude all right-hand columns (Fig.4), and one can be reconstructed on smaller matrix $2 \times 3 \times 4$.

It is exists a priori an infinite set of GRBs, and its parameters can be of any large number. Underlying technique can be used both for configure sonar systems with high quality indices due to all spacing vectors between their elements are different in order to avoid of interference of components of the same spatial frequency, and for development methods of non-redundant 3-D mask construction.

6. CONCLUSION

Two- and three-dimensional Gold Ring Bundles (GRB)s are perfect combinatorial models of non-redundant planar or 3-D space-tapered arrays of sonar. These models provide the optimal its structure from the point of the convenience to reproduce the maximum number of combinatorial varieties in the system with the limited number of elements. Moreover, the structural perfection has been embedded in the underlying models. Underlying property can be used for finding of the optimal placement of structural elements in planar or three-dimensional arrays of hydroacoustic systems. Proposed technique provides generalization of the GRBs theory and results to the improvement and optimization of the systems, including positioning of elements in array of sonar (e.g. an active sonar) with respect to resolving ability, and the other operating characteristics of the system. Combinatorial multidimensional sequencing theory and methods based on the idea of the combinatorial models provide many opportunities to apply the novel design approach to numerous problems in hydro-acoustics, including synthesis of wide-aperture equipped with non-redundant sets of opening for space and underwater acoustics.

The underlying two- and three-dimensional Gold Ring Bundles provide, essentially, new models for constructing wide-range non-redundant arrays of sonar, based on the idea of "perfect" ordered ring sequences. Method allows finding optimal solution in the simplest way using selected matrix of circular two- or three-dimensional vector-sums on the GRB as well as crossing out operations. These procedures make it possible to configure high performance arrays of sonar.

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