THE DIFFERENCE-FREQUENCY ACOUSTIC SCATTERING FROM NONLINEAR OBJECTS

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The difference-frequency acoustic scattering from nonlinear objects such as bubbles was studied for many years and used for bubble diagnostics. In the present paper we consider theoretically two problems. The first one is the difference-frequency scattering from the nonlinear layer in linearly stratified medium. The second problem is the difference frequency scattering from the system of periodic nonlinear layers. In both problems the resonance phenomena are studied and their influence on characteristics of the scattered fields are analyzed. In the third part of the paper we describe experimental method of acoustic difference-frequency vision of large-scale bubbly objects.

INTRODUCTION

Parametrically generated sound based on interaction of collinear waves of different frequencies is well known in acoustics [1]. It is known also that interaction of noncollinear acoustic waves is possible in a thin (compared to the wavelength) nonlinear layer [2]. In this paper we study more general cases of the parametrically generated sound from nonlinear layers. It is described two problems. The first one is the investigation of the difference frequency sound generation from the nonlinear layer between two acoustically linear half spaces of different densities and sound velocities irradiated with two noncollinear acoustic beams. The second problem is the nonlinear scattering of two acoustic waves by the system of periodic nonlinear layers. Both problems may find applications in nonlinear acoustic diagnostic techniques.

Methods of acoustic imaging have been elaborated for quite a long time. They are mainly based on the use of linearly scattered waves from an object. Some objects, such as gas bubbles in fluids [3-5], are known to have strongly nonlinear acoustic responses. Different nonlinear acoustic methods are used to detect such objects with highly nonlinear responses [5-6]. But it was mainly investigated the detection of small-size nonlinear inhomogeneities compared to the wavelengths. If the size of an object is large compared to the wavelength, methods of nonlinear acoustic imaging can be employed. In the third part of this paper we experimentally demonstrate possibilities of the method of the difference-frequency acoustic vision for such a large-size objects.

1. THE COMBINATION ACOUSTIC SCATTERING FROM THE NONLINEAR LAYER

The formulation of the problem is shown in Fig.1. Suppose that two quasi-plane beams of frequencies f_1 и f_2 incident from the half space (ρ_1,c_1) on plane layer (ρ_2,c_2) having thickness d and the nonlinear parameter ε. Behind the layer is the another half space (ρ_3,c_3).

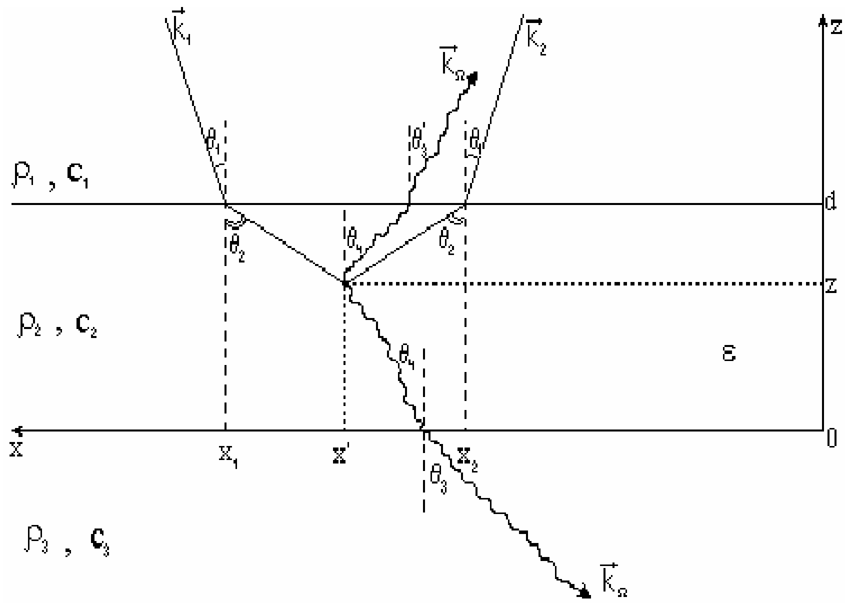

Fig.1 A scheme of the problem

We suppose that two half spaces are acoustically linear media. Two primary beams are supposed to incident on the layer symmetrically relatively to the normal of the layer at the incident angles θ_i. The insonified zone on the layer by two primary beams is considered for simplicity as rectangular $a{\cdot}b$. We will calculate forward and backscattered acoustic fields of the difference frequency $F=f_1-f_2$ in media 1 and 3. The solution of this problem can be found by the integration of the Westerwelt equation accounting the Green function for the incident and the scattered fields in this complex environment. It was analytically obtained expressions for the difference frequency fields in both half spaces, which are unwieldy to be presented here. Those expression were analyzed numerically.

In Fig.2 it is shown an example of the normalized angular dependence of the field in medium 3 at the difference frequency F=10 kHz. The field was calculated for parameters: f_1=100 kHz, f_2=90 kHz, $a=b=d$=1 m, θ_i=0.02 rad, $\rho_{1;2;3}$=1; 1.2; 2 g/cm^3, $c_{1;2;3}$=1.5; 1.7; 2 km/s.

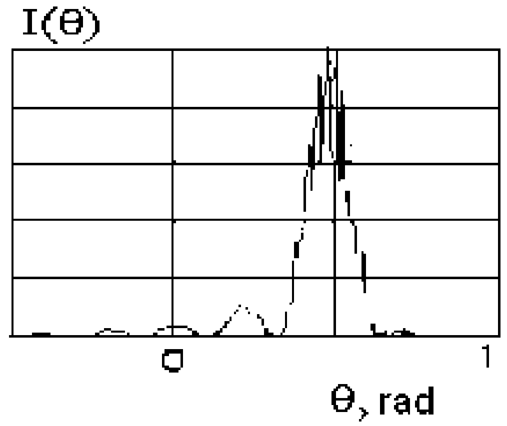

Fig.2 Directivity pattern of the difference frequency field in the medium 3

The main difference of the obtained results from the case of the difference frequency scattering from the nonlinear layer in homogeneous by linear parameters medium is the appearance of resonance peaks in the directivity pattern of the difference frequency field. These peaks are due to the resonances of the layer which is placed between two half spaces. The resonance properties of the layer exists if the layer thickness is comparable with the wavelengths $\lambda_{1,2}$ or Λ. There is also some decrease in the amplitude of scattered field and additional angle shift of the directivity pattern compared to the case of scattering from the nonlinear layer in linearly homogeneous medium.

2. THE COMBINATION ACOUSTIC SCATTERING FROM THE SYSTEM OF PERIODIC NONLINEAR LAYERS

Consider the another problem shown in Fig. 3. Let N nonlinear layers are put in the medium having the same linear parameters (ρ_0, c_0) as the linear parameters of layers. Two noncollinear plane waves insonify this system as shown in Fig. 3. The difference frequency acoustic field scattered from the system in back direction is to be calculated.

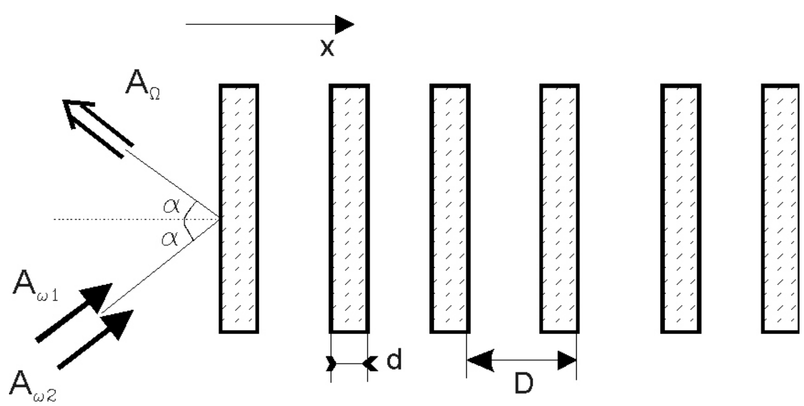

Fig.3 A scheme of the problem of the difference frequency scattering from the periodic nonlinear layers

Two incident plane waves of frequencies ω_1 and ω_2 are:

$$\widehat{A}_{\omega_1} = A_{\omega_1} e^{i(\omega_1 t - k_1 x)}, \qquad \widehat{A}_{\omega_2} = A_{\omega_2} e^{i(w_2 t - k_2 x)}.$$

At each of layers these waves interact to generate the difference frequency wave ($\Omega=\omega_1-\omega_2$).

In order to find the total scattered field one need to sum all the fields generated by N layers. Analytical calculation will be done for the following assumptions:

1. Each of layers are thin compared to the wavelengths ($d<<\lambda_1, \lambda_2, \lambda_\Omega$);

2. Incident waves loss some part of energy passing through the each of layers (we denote δ as the transmission coefficient for one layer. ω_1

3. The generation of the difference frequency wave at each of layers can be formulated as follows $A_\Omega=Q\, A_{\omega 1}^{*} A_{\omega 2}$, where Q is the coefficient of the nonlinear transformation.

4. Since frequencies ω_1 and ω_2 are closed to each other ($\Omega<<\omega_1$, ω_2) one may suppose that these waves have the same value of the transmission coefficient δ, while for the difference frequency scattered field there is no attenuation of sound passed through layers. For normal incident of primary waves at the layer system the scattered field generated from the n-th layer is

$$\widehat{A}_{\Omega}^{(n)} = \frac{1}{2} Q A^{*}_{\omega_1} A_{\omega_2} e^{i[(\omega_2-\omega_1)t-(k_2-k_1)x]-iD(n-1)[k-k_1-k_2]} \delta^{2(n-1)}.$$

Summing coherently all the field from each of layers one may obtain the scattered field from all the N layers:

$$\widehat{A}_{\Omega}^{(N)} = \widehat{B} \delta^{2(N-1)} e^{i2(N-1)kD},$$

where B is the complex amplitude of the difference frequency field in back direction, the expression for which is unwieldy and therefore omitted.

From this expression it can be easily find a condition of the maximum amplitude of the backscattered field: $D=m\lambda/2$, where $m=1,2, \ldots$. This condition is the same as for the Bragg resonance for linear scattered waves in periodic systems. Figures 4 and 5 presents the dependence of the backscattered difference frequency wave from the system of periodic nonlinear layers as function of D/λ ratio and of number N of layers, respectively (for two figures shown in Fig.4 δ=0.98 and 0.99).

For the case of $\alpha\neq0$ one can also obtain expressions for the scattered field which is the same as for the normally incident primary waves. The differences between these two cases are the another form for the expression for B, and besides it is necessary to change D by $D_1=D/\cos\alpha$. Consequently, the condition for the resonance scattering will be as follows: $D_1=D/\cos\alpha = m\,\lambda/2$, where $m=1,2,\ldots$.

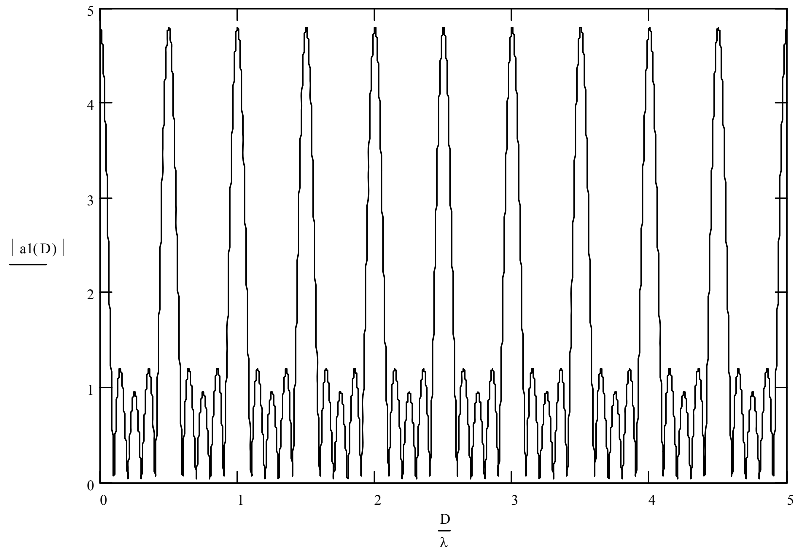

Fig.4 The dependence of amplitude of the backscattered difference-frequency acoustic wave on the ratio *D/λ*

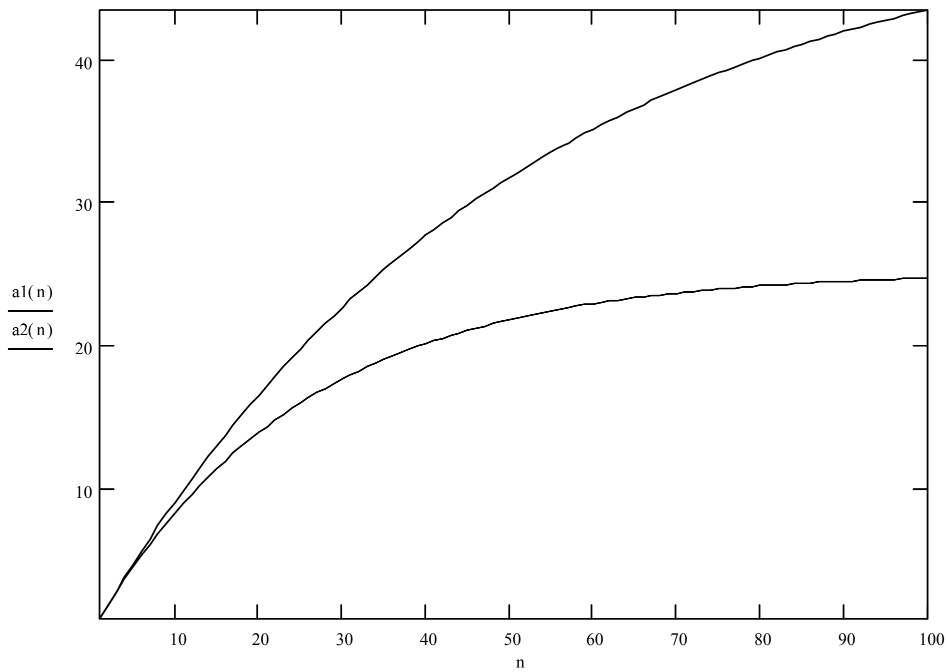

Fig.5 The dependence of amplitude of the backscattered difference-frequency acoustic wave on number of layers *N*. For upper curve δ=0.99, and for the lower curve δ=0.98

3. DIFFERENCE-FREQUENCY ACOUSTIC VISION OF BUBBLY OBJECTS

The experiments on the difference-frequency acoustic vision of bubbly objects were carried out in a water tank. Steel cylinders were chosen to be the observed objects. They were vertically arranged and have diameter 40 mm and length 300 mm spaced 20 cm apart. Acoustic images were constructed by line-angular scanning of sonar with a spherical mirror. The cylinders were sounded by two noncollinear CW ultrasonic beams at 195 kHz and 130 kHz. The cylinder surfaces were covered by gas electrolysis microbubbles to produce highly nonlinear scattering from the surfaces at higher harmonics and the combination frequencies. In the experiment we registered the difference-frequency scattering. A spherical receiving mirror 35 cm in diameter with focus distance of 27.5 cm was made of foam plastic. A synchro-drive provided angular scanning. Signals focused by the mirror on detection were registered by an ADC, fed to a computer, and the image was displayed on the monitor.

The geometry of the experiment is shown in Fig.6, where 1 and 2 are cylinders, 3 and 4 are emitters (f_1=195 kHz, f_2=130 kHz), 5 is a receiving hydrophone, 6 is a spherical mirror, and 7 is a unit of electron-computer processing. The distance between the mirror and the cylinders was 2 m.

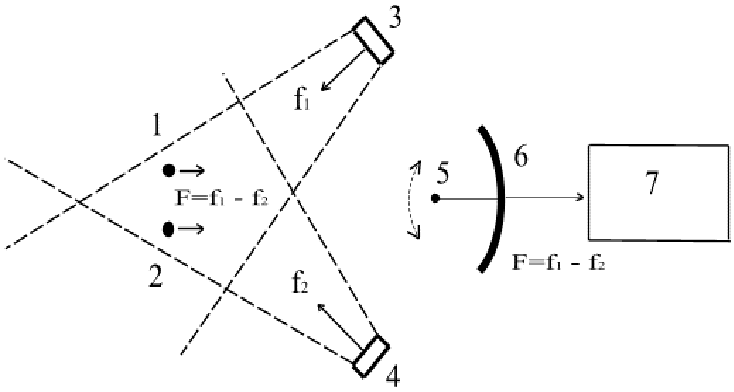

Fig.6 A scheme of the experiment

An obtained image of two cylinders at the difference frequency of 65 kHz is shown in Fig. 7.

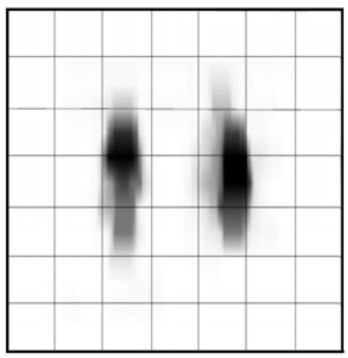

Fig.7 An image of two metal cylinders covered with electrolysis bubbles obtained at the difference frequency 65 kHz

The images of cylinders were also obtained at the primary frequencies due to linear scattering of primary high frequency waves by the objects. A section of the image along one horizontal line obtained in the linear regime at a high primary frequency is shown in Fig. 8. Fig. 9 presents a section along the same horizontal line of the image shown in figure 2 at the difference frequency of 65 kHz. For comparison values of amplitudes given along the vertical axes in figures 8 and 9 were normalized to unity.

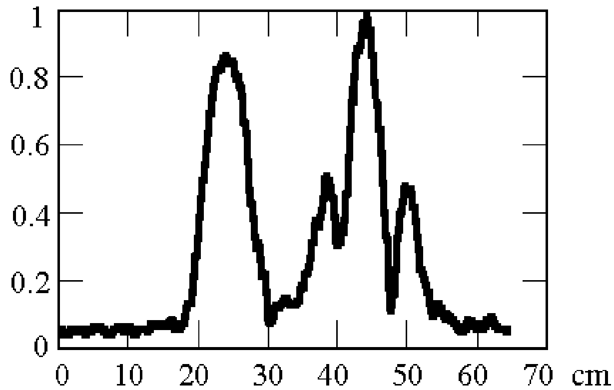

Fig.8 A section of the image of two metal cylinders covered with electrolysis bubbles along one horizontal line obtained in the linear regime at a high primary frequency 195 kHz. Amplitudes along the vertical axis are normalized to unity

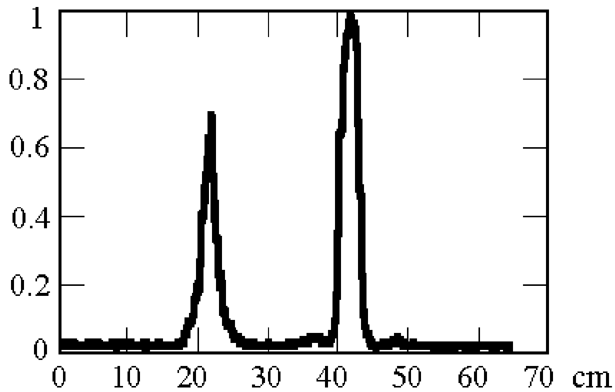

Fig.9 A section of the image of two metal cylinders covered with electrolysis bubbles along one horizontal line obtained in the nonlinear regime at the difference frequency 65 kHz. Amplitudes along the vertical axis are normalized to unity

One can see from figures 8 and 9, that the image in the nonlinear regime is more contrast compared to the linear one. The nonlinear regime provides also better spatial resolution.

4. CONCLUSION

In this paper two problems of the difference frequency scattering of noncollinear primary waves from nonlinear layers were considered. It was obtained and analyzed the analytical expression for the difference frequency field scattered from the nonlinear layer between two linearly different half spaces. The main feature is the appearance of peaks in the directivity pattern of the difference frequency field due to the layer resonances. For the problem of the difference frequency scattering from the system of periodic nonlinear layers in linearly homogeneous medium it was revealed the resonance amplification of the backscattered field is possible. The resonance phenomenon is similar to Bragg resonances in linearly backscattered field from periodic systems.

In this work it was demonstrated experimentally potentialities of obtaining images of nonlinear scattering objects in acoustic fields at the difference frequency. Acoustic imaging at the difference-frequency can be an effective technique for investigating various objects. The spatial resolution and sharpness of the image obtained at difference-frequency waves are higher than in the case of a linearly scattered signal. This method may be useful for example for acoustical observation different underwater object that are frequently covered by microorganisms and gas microbubbles producing high acoustic nonlinearity at the surface of the object.

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REFERENCES

[1] L. Zarembo, D. Krasilnikov, *Introduction into the Nonlinear Acoustics*, Nauka, Moscow 1966.

[2] S. Zamolin, D. Donskoy, L. Kustov, V. Nazarov, A. Sutin, Nonlinear backscattering of acoustic waves in a bubble layer, Acoustics Letters, Vol. 6, 15-17, 1982.

[3] E. Zabolotskaya, S. Soluyan, Emission of harmonic and combination frequency waves by air bubbles, Sov. Phys. Acoust., Vol. 18, 396-398. 1972.

[4] A. Eller, H. Flynn, Generation of subharmonics of order one-half by bubbles in a sound field, J. Acoust. Soc. Am., Vol. 46, 722-727, 1969.

[5] V. Zverev, Yu. Kobelev, D. Selivanovsky, A. Sokolov, On a method of detection of gas bubbles in a liquid, Sov. Tech. Phys., Vol. 50, 1544-1545, 1980.

[6] I. Didenkulov, A. Martyanov, D. Selivanovsky, N. Pronchatov-Rubtsov, Bubble diagnostics with the nonlinear acoustic scattering method. Hydroacoustics, Vol. 4, 43-52, 2001.