THE ANALISYS OF THE SYSTEM TOWING ROPE WITH LONG UNDERWATER OBJECT

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The paper describes solution of the boundary problem for the system long rope with object density insignificantly greater than the density of the water and small rigidity. There is only known a point of hooking the rope on the ship and depth on which the end of the towed object is sunk. Article presents the analysis of the system described above with calculation of his forms and forces of weights, buoyancy force and lift with changeable longitudinal rope's tension. The results of calculations show capabilities and manners steering form. But what is more important it shows how to steer with the sink of the object on the water of different depths and allows safe maneuvering of the object. Besides, employment of described method and/or similar, (see [1]), for calculating the geometry of the system towed by a maneuvering ship will give better accuracy of the executed measurement by the towed object.

INTRODUCTION

In the article contained is the analysis of the system towing rope and the long object. The formulated boundary value problem is solved iterative with finite element method. Talking about boundary conditions there is known only first point of the rope attached to the ship and the depth on which the end of the towed object is sunk.

Solution of the problem brought us to the equations system with wrongly identified conditions that require labour intensive methods to solve them. Therefore Author is going to use his another method (see [1]) to solve temporary conditions (transitional states) before reaching equilibrium state by the system.

Presented results haven't been confirmed experimentally yet, but it is going to happen soon.

1. PROBLEM FORMULATION

Below you can see formulated equations that serve to determine the shape and internal forces in the towing rope that connects a certain ship afloat with the long and not much elastic object. The shape of the towed object will be analyzed too. Object can change his shape depending on the direction, if moving up deflection is positive, if moving down deflection is negative. Regarding to the boundary conditions of the system the following is known: $z(A)$ – height of the catching point of the rope on the ship (see Fig. 1), and $z(C)$ – depth of point C that is the end of the system.

Additionally there is a possibility to measure a longitudinal tension inside rope in point A. It is assumed that the system is moving straight and the ends A and C of the rope are found within the plane OZX. The static equilibrium state of the rope is sought, without considering its load imposed by possible movements of the water particles caused by waves. Any elastic strain in the rope is omitted too. The equilibrium state of the rope is a reached by the following types of loads:

- The weight of the rope and the system;
- The buoyancy;
- The hydrodynamic forces.

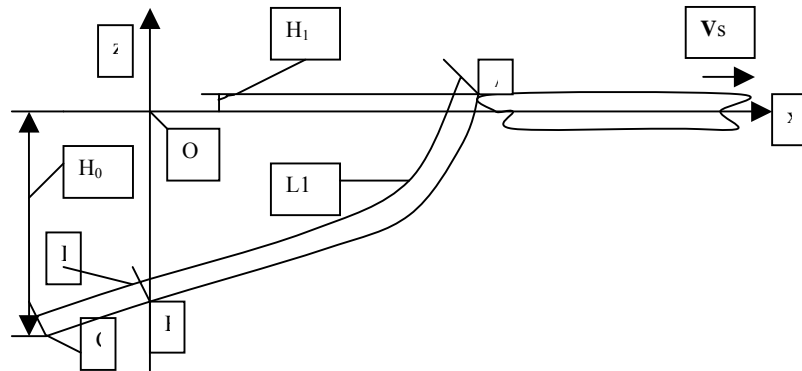 As a consequence the rope axis may be represented by a 2-dimensional curve.

Fig.1 Coordinate system Oxyz

The hydrodynamic forces that have some impact on the rope are defined by the formulas, which follow from Morison's equation [2]:

$$F_n = \frac{1}{2}\rho C_n V_n |V_n| D$$
$$F_t = \frac{1}{2}\pi\rho C_t V_t |V_t| D$$

(1)

where:

 F_n – the continuous load in the rope acting along the axis 'n', directed as the flow speed vector projected on the plane perpendicular to the rope's axis;

 F_t – the continuous load acting tangentially to the rope's axis;

 ρ_W - the water density;

 C_n, C_t – the dimension-less coefficients depending on the cross-section of the rope;

 V_n – the co-ordinate of the flow speed vector projection on the axis normal to the rope;

V_t - the co-ordinate of the flow speed vector projection on the axis tangential to the rope;

d – the characteristic dimension of the cross-section of the rope (the rope diameter for a rope with a round cross-section), (D - for the object).

The equilibrium equations for the rope are formulated based on Fig. 2. Those are the equilibrium conditions for the load and internal longitudinal force in the rope at any point of the rope, in a tangential direction and normal direction to the rope within the plane OZX.
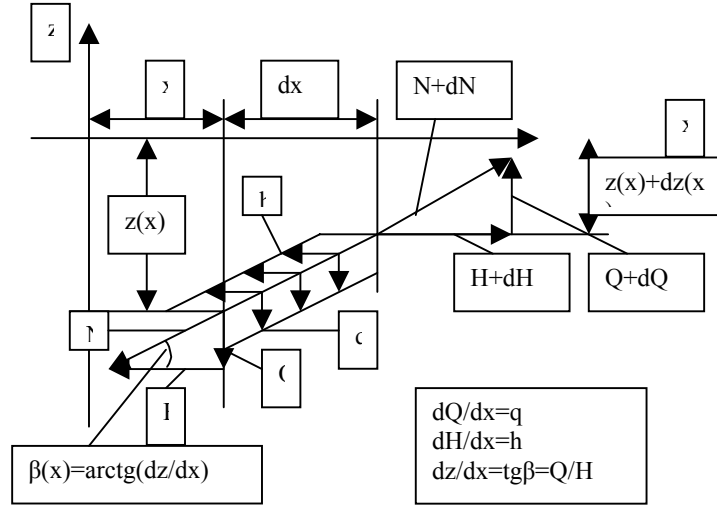

Fig.2 The continuous load and internal longitudinal force impact on the element of rope

The horizontal force h impacting on the element of the rope is the horizontal part of the hydrodynamic force **F** generated by the water flow around the rope. As shown below on Fig. 3 the magnitude of h are defined as follows:

$$h = Fn * \sin \beta + Ft * \cos \beta \qquad (2)$$

The continuous perpendicular load of the rope q is a vector sum of weight and buoyancy of the rope and vertical part of the hydrodynamic force **F** and is as follow:

$$q = (\rho_L - \rho_W) * S_L * g - Fn * \cos \beta + Ft * \sin \beta \qquad (3)$$

where: ρ_L - the density of the material of the rope,

ρ_W - the sea water density,

S_L – the area of rope cross-section,

g – the gravity acceleration.

The equation of a curve of the rope is determined by the following formula derived from Figure 2:

$$\frac{d}{dx}(H \frac{dz}{dx}) = q \qquad (4)$$

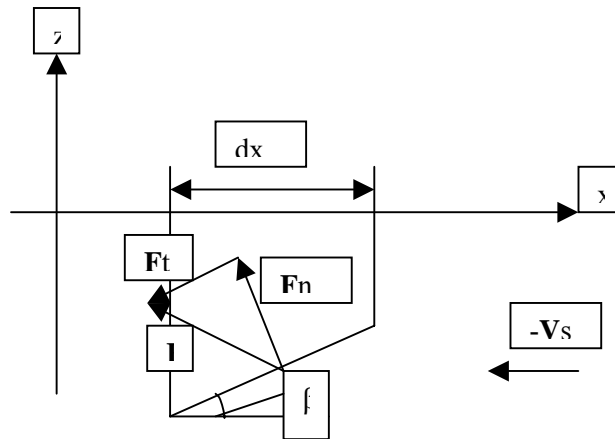where: H – the horizontal component of an internal longitudinal force in the rope.

Fig.3 The normal and tangential parts of the hydrodynamic force impact on the element of the rope

After differentiation equation (4) takes a form:

$$H * \frac{d^2 z}{dx^2} + A_{Fn} * (\frac{dz}{dx})^4 + A_{Fn} * (\frac{dz}{dx})^2 - q_o = 0 \qquad (5)$$

where: $F_n = A_{Fn}*\sin^2\beta$,
$F_t = A_{Ft}*\cos^2\beta$,
$A_{Fn} = \frac{1}{2}*\rho_W*V_s^2*d*C_n$,
$A_{Ft} = \frac{1}{2}*\rho_W*V_s^2*d*\pi*C_t$,
$q_0 = (\rho_L - \rho_W)*S_L*g$.

The equilibrium equations of the object have the same form as for the rope. But boundary condition and the bending stiffness are different from these for the rope. Besides the tangential part of hydrodynamic force is greater than the one for the rope because the object have collars that combine 10m long parts of the object. The diameter of the collar is equal 100mm approximately. For measurements of the object bending stiffness we can use a simple experiment with the object submerged in the water with small pontoons (see Fig. 4). The maximum deflection of the object axis f under static load q_{o0} (see equation (6)) has the following relationship:

$$f = \frac{q_{o0} * l^4}{384 * EI} \qquad (6)$$

where: EI – the object bending stiffness,
f – deflection of the object axis,
l – pontoon distance,
$q_{o0} = (\rho_O - \rho_W)*g* \pi *D^2/4$,
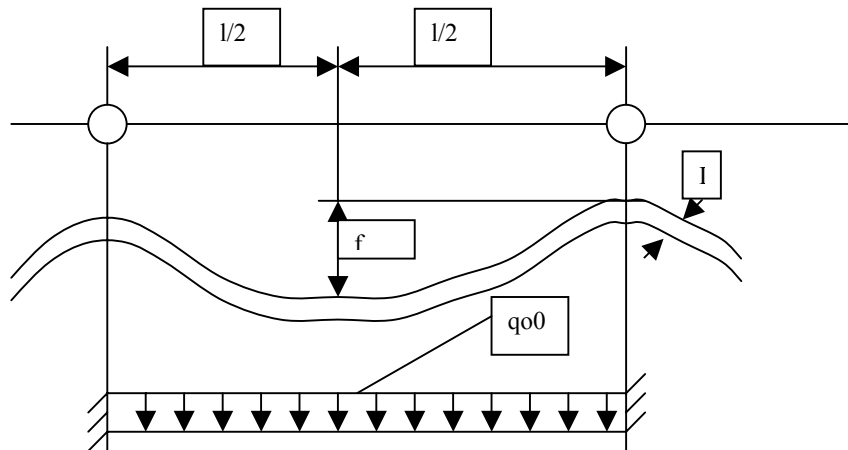ρ_O – the density of the object material.

Fig.4 The maximum deflection of the object axis f under static load q_{o0} (q_{o0} - is a vector sum of weight and buoyancy forces of the object)
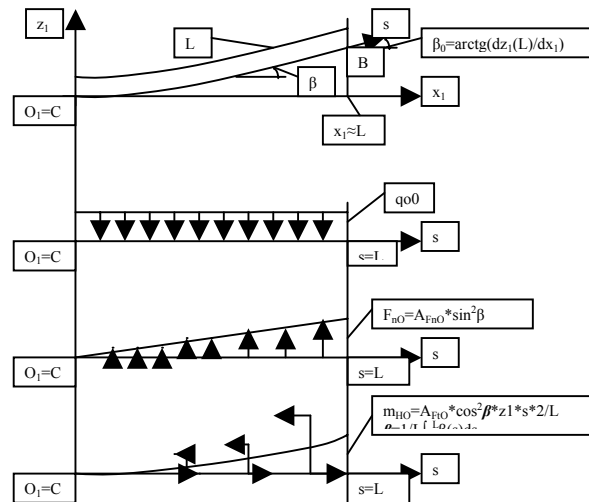

Fig.5 The continuous load impact on the object

The forces and moments impact on object are illustrated on Fig. 5. The boundary conditions are formulated as follows:

- $\dfrac{d^2 z_1(x_1 = 0)}{dx_1^2} = 0$, the end of the object is straight, (point C),

- the curve of the object deflection is quadratic parabola as a first approximation.

With above assumptions and loads as in Fig. 5 the equation of the object deflection gets a form as shown below:

$$z_1(x_1) * EI = -\frac{1}{24} q_{o0} * x_1^{4} + \frac{1}{360} A_{FnO} * \sin^2 \beta_0 * x_1^{6} * L^{-2} +$$

$$\frac{1}{140 * K * L^K} * A_{FtO} * \cos^2 (\beta_0 / 2) * \beta_0 * x_1^{7} \qquad (7)$$

where: $A_{FnO} = \frac{1}{2}*\rho_W*V_s^2*D*C_{nO}$,

$\qquad A_{FtO} = \frac{1}{2}*\rho_W*V_s^2*D*\pi*C_{tO}$.

Whereas the angle β_0 of the object deflection in point B would be computed from the equation as below:

$$\frac{dz_1}{dx_1}(x_1 = L) = \tan(\beta_0) \tag{8}$$

2. SOLUTION OF THE PROBLEM

The calculations of the system shape were carried out in Matlab system with finite element method. For the problem solution were used own procedures for boundary problem and also some of them were taken from Matlab system. First, program calculates the shape of the object and based on the object deflection, calculates the resisting force of the object. This force must be balanced by the rope.

Boundary conditions for the rope equilibrium are as follows:
- internal longitudinal force in the rope in point B is equal to the resisting force of the object,
- in point B the angle inclination of the rope β is equal to the angle of the object deflection β_0,
- initial point of the rope A has known coordinate z(A) equal H_0,
- there is a possibility to measure internal longitudinal force in point A,
- x(A) coordinate is a function of the length L1 and the shape of the rope.

3. FINAL CONCLUSIONS AND RESULTS OF CALCULATION

The shape of the rope and values of the longitudinal force are calculated for the rope with a given length and values of the parameters as below:

$H_1 = 2$m, (Fig. 1),

H_0 – object end depth is calculated from equilibrium equations, (Fig. 1),

$Vs = 12$kn, ship velocity,

$\rho_W = 1025$kg/m^3, the sea water density,

$g = 9.81$m/ s^2 , the gravity acceleration,

$d = 10$e-3m, the nominal diameter of the towing steel rope,

$\rho_L = 7$e3kg/m^3, the density of the material of the rope,

$L1 = 1500$m, the length of the rope, (Fig.1),

$C_n = 1.2$, the dimension-less coefficient depending on the cross-section of the rope, (see (1)),

$C_t = 2$e-2, the dimension-less coefficient depending on the cross-section of the rope, (see (1)),

$C_{nO} = 1.2$, the dimension-less coefficient depending on the cross-section of the object, (see (1)),

$C_{tO} = 2.02671$e-2, the dimension-less coefficient depending on the cross-section of the object,(see (1)),

$D = 90$e-3m, the diameter of the object,

$\rho_O = 1.030$e3kg/m^3,the density of the object material,

$L = 300$m, the length of the object.

The computations were made dividing the rope into 30 equal length elements. At the first iteration the shape of the rope was represented with a cubical parabola between the points A and B (Fig. 1). The depth of the end of the object H_0 was derived in iterations from equilibrium equations. The system obtained equilibrium of the forces when the end of the object flooded on depth of $H_0(C)$ = 151m and value of $\beta_0(B)$=0.0384rad. The geometric values of the system shape as above and values of internal longitudinal force in the rope N(A) and the horizontal component H(B) are presented in Table 1 for different lengths of the rope.

Tab.1

Lp.	L1	$H_0(C)$	$H_0(B)$	$B_0(B)$	H(B)	N(A)
[-]	[m]	[m]	[m]	[rad]	[kN]	[kN]
1	1500	151	145.2	0.0384	33.54	52.19
2	1250	118	112.2	0.0384	33.54	48.92
3	1000	88	82.25	0.0384	33.54	45.725
4	750	60	54.24	0.0384	33.54	42.58
5	500	36	30.25	0.0384	33.54	39.514

Presented in Fig. 6, 7, 8: are: the shape of the system, internal forces and the angle inclination β of the rope calculated for above parameters.
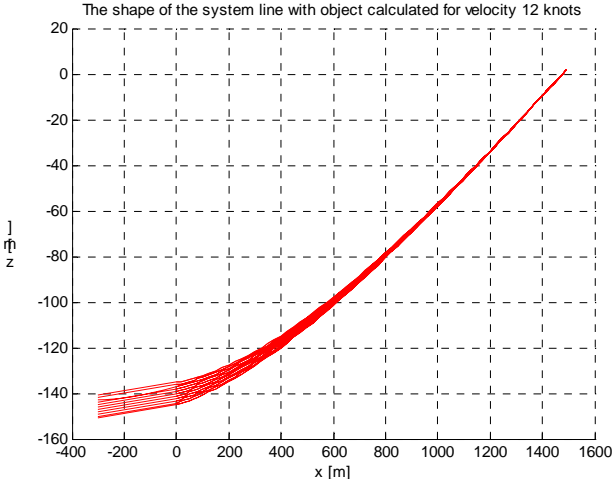

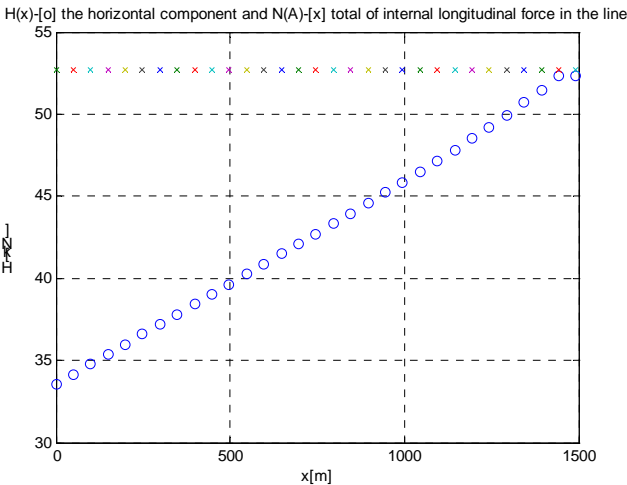Fig.6 The curve of the system for ship velocity equal 12 knots

Fig.7 The horizontal component of the internal longitudinal force in the rope - H(x),[kN] and total longitudinal force in the rope in point A - N(A),[kN]
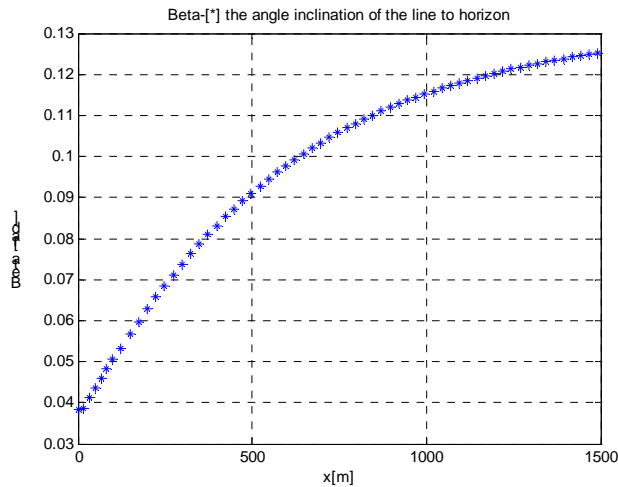
Fig.8 Angle β inclination of the rope

4. FINAL CONCLUSIONS

- presented method allows to steer with the form but also with the sink of the system;
  - change of the above-mentioned geometric parameters can be performed :
- during project design phase by changing rope diameter and average densities of the material of ropes
- during exploitation phase by changing the towing speed or rope length;
- calculation of the object bendiness for transitional states of rope balances during ship maneuvering in horizontal surface will give better accuracy of the executed measurement by the towed object.

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