THE UTILITY OF SYNTHETIC APERTURE SONAR IN SEAFLOOR IMAGING

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Synthetic Aperture Sonar (SAS) is a high-resolution acoustic imaging technique which is an extension of normal sidescan sonar imaging. The cross-range (azimuthal) resolution in real aperture sonar is dependent on distance to the illuminated scene and signal frequency. Moreover, required azimuth resolution usually needs a beamwidth narrower (enlarged real aperture) than what can be achieved in practice by sized physical antenna. Synthetic aperture sonar overcomes all these constraints, which are discussed in this paper. The specific mode of SAS explained and analyzed here is known as stripmapping. In this mode, platform which contains a transmitter and a receiver, emits consecutively pulses to the examined area in range direction, which is perpendicular to the direction of travel. An appropriate coherent combination of received echos leads to formation of synthetically enlarged aperture. Synthetic aperture processing allows to obtain a high-resolution image of the examined seafloor. This paper covers basic SAS processing algorithm and shows some results of numerical simulation.

INTRODUCTION

Conventional side-scan (side looking) sonar systems send successive sound pulses perpendicular to the direction of travel (figure 1a). The corresponding received echo signals are recorded graphically as consecutive lines, resulting in a two-dimensional reflectivity map of acoustic backscatter strength. The along-track resolution in the final image corresponds to the beam width at illuminated area – the narrower beam the better resolution. Narrowing the beam is possible by either increasing the length of the real aperture or the frequency of the acoustic pulses. The first manner is often very impractical because of structural constraints. The disadventage of using high frequency is that attenuation of the signal is greater at higher frequencies. Therefore, this way of increasing along-track resolution is limited and possible only at short range. Moreover, the azimuth resolution of a conventional side-scan sonar system decreases with increasing distance to the scatterer on seafloor because of increasing beam width at illuminated area.

Synthetic **A**perture **S**onar (SAS) systems overcome these drawbacks and allow to obtain a high-resolution image of the examined seafloor independently of both range and signal frequency and that is determined only by the width of the real aperture. In contrast to conventional side looking sonar, a shorter antenna produces higher resolution in SAS. In figure 1b, SAS system during the data acquisition in shown. The positions of the platform, where pulses are transmitted, are evenly spaced. This is because both a constant **P**ulse **R**epetition **F**requency (PRF) and platform speed v are assumed. The platform position and system parameters determine the size and shape of the aperture footprint on the imaged surface. This footprint is swept along-track as the platform moves along ping by ping insonifying the swath, so that the response of a scatterer on the seafloor is contained in more than single sonar echo. An appropriate coherent combining of the signal returns leads to the formation of syntheticaly enlarged antenna, so-called synthetic aperture.

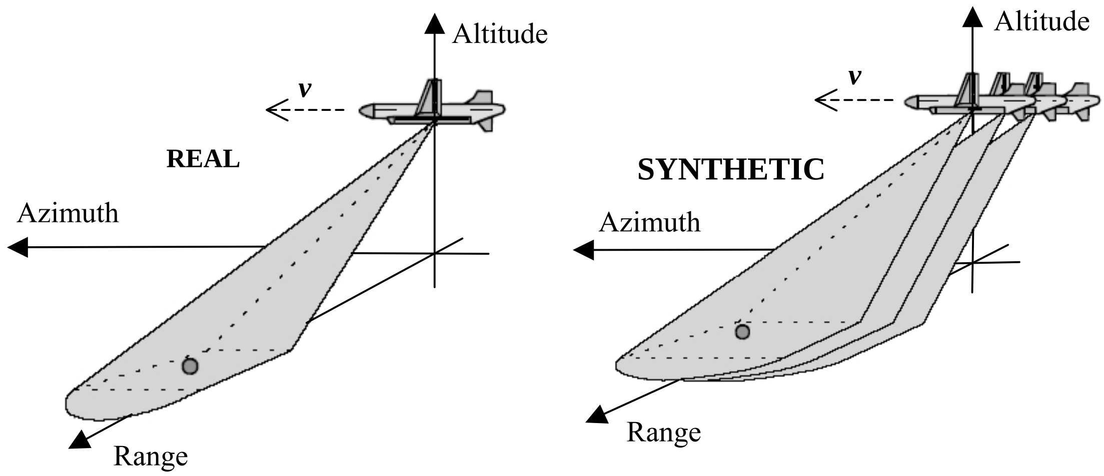


Fig.1 Real vs. Synthetic Aperture Imaging

Generally, there are two main types of synthetic aperture imaging, stripmap and spotlight modes. In stripmap mode, the sonar beam always points in the same direction (typically perpendicular to the direction of travel) during imaging. In the the other one, the beam is continuously steered onto the centre of illuminated scene. The subject of this paper is only stripmap mode of SAS.

1. SLANT RANGE RESOLUTION

The resolution of a system is defined as the minimum separation of two target responses that can be distinguished as separate by the system. The look direction of the antenna is normally called slant range or simply range. The range resolution of a sidescan SAS system is the same as for any pulse-echo ranging system. Assuming the interrupted continuous wave of amplitude a and pulse length τ is given by

$$p(t) = \begin{cases} a & 0 < \mathrm{t} < \tau \\ 0 & \mathrm{t} < 0;\ \ \mathrm{t} > \tau \end{cases}, \quad (1)$$

we can define resolution in the following way. If a transmitted pulse reflects off two targets located at different ranges, they can be resolved by the sonar if the trailing edge of closer target's echo arrives to the receiver before the forward edge of the further target's echo. Thus, the achievable resolution δ_{sr} in slant range depends on the transmitted pulse length τ, or alternatively, on the bandwidth B of the pulse.

$$\delta_{sr} = \frac{\tau \cdot c}{2} = \frac{c}{2B}. \quad (2)$$

To achieve high resolution in range, very short pulse duration is necessary. Unfortunately, in order to maintain received Signal to Noise Ratio (SNR) at appropriate level for the system, shortening the pulse needs increasing energy density. It is difficult to handle in practice and limited by cavitation phenomenon. Therefore, high bandwidth is reached by transmitting a longer pulse with Linear Frequency Modulation (LMF) instead. The energy of this pulse is distributed over longer duration, but it can be compressed after receiving by matched filtering operation. The LMF signal can be defined as

$$p(t) = rect\left(\frac{t}{\tau}\right) \cdot \exp(i\omega_o t + j\pi K t^2), \quad (3)$$

where $K=B/\tau$ is the chirp rate and ω_o is carrier frequency. One of the most important feature of this waveform is its correlation property, that is, when LMF signal is correlated with itself, a *sinc* function is the result. The correlated waveform contains the same energy as the original modulated signal and this energy is concentrated around the time bin corresponding to the time delay. The ideal matched filter is represented by correlation of transmitted pulse p(t) with the received echo s(t). The result of the matched filtering is then

$$V(t) = \int_{-\infty}^{\infty} s(\xi) \cdot p^*(t+\xi) d\xi = s(t) * p(t) = s(t) \odot p^*(-t) = IFFT[FFT[s(t)] \cdot FFT[p^*(-t)]], \quad (4)$$

where * and $\odot$ denote correlation and convolution respectively. The last part of this equation gives possibility to accelerate calculations by utilizing the convolution theorem. Matched filtering operation can be performed in frequency domain where convolution operation (called fast convolution) becomes simple multiplication.

2. SIDESCAN AZIMUTH RESOLUTION

In the along-track or azimuth direction, the resolution of a conventional side-scan sonar corresponds to the size of the footprint on seafloor in this direction. This distinction criteria can be the mainlobe null-to-null width, the 3 dB width, or any suitable criteria specified by the system designer. We will define resolution to be equivalent to the 3 dB width of the main lobe of the target impulse response. The angular resolution of an antenna of length L_{ra} in the azimuth direction can be approximated for a wavelength λ as

$$\theta_{3dB} \approx \frac{\lambda}{L_{ra}}. \quad (5)$$

The spatial resolution at a given range r then results as

$$\delta_{az} = 2r \cdot \tan\left(\frac{\theta_{3dB}}{2}\right) \overset{\text{for small } \theta_{3dB}}{\approx} 2r \cdot \frac{\theta_{3dB}}{2} = r \cdot \theta_{3dB} = r \cdot \frac{\lambda}{L_{ra}}. \quad (6)$$

It follows that the azimuth resolution of a conventional side-scan sonar system decreases with increasing both distance to the scatterer on seafloor and wavelength and is inversely proportional to the size of the real aperture. Therefore, for given distance to the scaterrer we can improve resolution by using large antenna, what is very inconvenient and impractical, or increasing the frequency of transmitted pulses. However, the attenuation of the signal is greater at higher frequencies. The tradeoff between these parameters makes conventional side-scan sonar an inappropriate tool for high-resolution acoustic imaging technique.

3. AZIMUTH RESOLUTION OF A SYNTHETIC APERTURE SONAR

A synthetic aperture sonar overcomes mentioned above problems and is designed to achieve high resolution with small antennas over long distances. A SAS system takes advantage of the fact that response of a scatterer on the seafloor is contained in more than a single sonar echo. An appropriate coherent combination of received pulses leads to the formation of a synthetically enlarged an antenna, the so-called synthetic aperture. This formation is very similar to the control of an antenna array, with the diffrence that only one antenna is used and diffrent antenna positions are generated sequentially in time by movement of the platform. Since in a synthetic aperture system the elements radiate individually at each location the relative phase shifts seen from the target are doubled when compared to an equivalent real aperture system. This double phase sensitivity causes that the resolution of synthetic aperture system is defined as

$$\delta_{az} = r \cdot \frac{\lambda}{2L_{sa}}, \quad (7)$$

where L_{sa} is the length of synthetic aperture. The maximum length for the synthetic aperture is length of the track which a scaterrer is illuminated as depicted in Fig. 2b. This is equal to the size of the antenna footprint on the seafloor at distance *r* where the scaterrer is located at.

$$L_{sa\,\max} = r \cdot \frac{\lambda}{L_{ra}} \quad (8)$$

The maximum along-track resolution, which is possible to obtain by synthetic aperture system in stripmap mode, is given by

$$\delta_{az} = r \cdot \frac{\lambda}{2L_{sa\,\max}} = \frac{L_{ra}}{2} \quad (9)$$

Thus, a synthetic aperture system offers along-track resolution that is independent of range and signal frequency, and is determined solely by the real aperture width, where smaller L_{ra} equates to better resolution.

4. PHASE HISTORY OF A POINT TARGET

The manner of data acquisition by a SAS system is depicted in figure 2a. The antenna is moved along x-axis (azimuth) end emits, perpendicular to track direction, the sonar pulses to the seafloor. The distance between the platform at position *x* and the scatterer on the bottom at x=0 can be expressed as $r = \sqrt{x^2 + r_o^2}$, where r_o denotes minimum distance. The Taylor series expansion for r is

$$r = r_o\sqrt{1+\frac{x^2}{r_o^2}} = r_o\left(1+\frac{x^2}{2r_o^2}+\frac{x^4}{8r_o^4}+...\right) \approx r_o + \frac{x^2}{2r_o}. \quad (10)$$

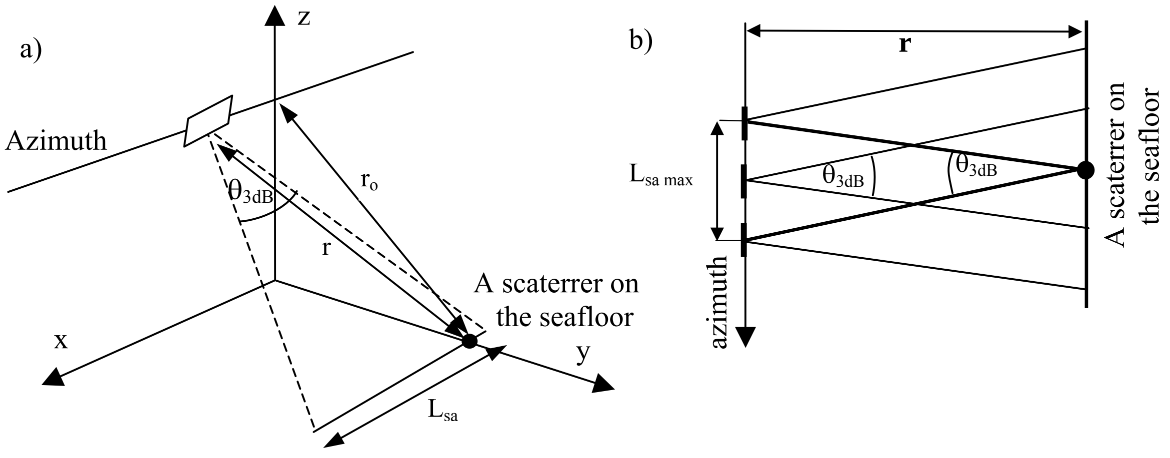


Fig.2 (a) Synthetic aperture imaging geometry. (b) The SAS geometry viewed from the top- the representation of the maximum possible synthetic aperture length $L_{sa\ max}$

Because the extension of the sonar footprint is usually much smaller than the object distance, $x << r_o$, we can neglect the third and high terms of this series. Then the phases of the received echos, resulting from the two-way distance r are defined as

$$\varphi(x) = 2 \cdot \frac{2\pi}{\lambda}(r_o + \frac{x^2}{2r_o}) = \frac{2\pi \cdot x^2}{\lambda \cdot r_o} + const\,. \quad (11)$$

Assuming a constant platform velocity v and the abbreviation $K_{az}=2v^2/\lambda r_o$, neglecting the constant phase part, which has no time dependency, we obtain a quadratic phase behaviour in time

$$\varphi(t) = \pi \cdot K_{az} \cdot t^2\,. \quad (12)$$

This corresponds to the Doppler effect, a linear change in the received azimuth frequency because

$$f(t) = \frac{1}{2\pi}\frac{\partial\varphi(t)}{\partial t} = K_{az} \cdot t\,. \quad (13)$$

We can assume the linear frequency behaviour so long as x is really small in comparison to r_o, otherwise high order components has to be taken into account. It can happen for SAS systems with very long apertures and for those operating not exactly perpendicular to the track direction. The maximum illumination time of a point target is defined by the length of the synthetic aperture and the velocity of the platform.

$$\tau az = \frac{L_{sa}}{v} = \frac{\theta_{3dB} r_o}{v}\,. \quad (14)$$

The factor K_{az} is analogous to chirp rate K in the formula 3, this can be expressed as

$$K_{az} = \frac{B_{az}}{\tau az}\,. \quad (15)$$

Therefore, the bandwidth of the signal in azimuth B_{az} is defined as

$$B_{az} = K_{az}\tau_{az} = \frac{2v\theta_{3dB}}{\lambda}\,. \quad (16)$$

5. PROCESSING IN AZIMUTH

The response of a scatterer on the seafloor is contained in many sonar pulses and appears therefore defocused. The goal of SAS processing (called compression) is to focus all the received energy of a scatterer, distributed over illumination time τ_{az}, on one point . It is achieved by the matched filtering operation using the phase history coming from the data acquisition process, discussed in the previous section. The aim of this processing is to adjust all phase values to the same value and the put signal in azimuth through coherent summation. We try to calculate the analytical result of this operation. The received signal in azimuth direction can be written as

$$S_a(t) = A_o \cdot \exp(i\varphi(t)) = A_o \cdot \exp(i\pi K_{az} t^2), \quad (17)$$

where A_o denotes the backscattering complex amplitude. It is assumed the process of backscattering of a point object is time and angular independent. Then the matched filtering is performed by means of the correlation of S_a with a reference function which is constructed in a way it has in every point exactly the opposite phase of the ideal impulse response

$$R(t) = W(t)\exp(-i\pi K_{az} t^2), \quad (18)$$

where W(t) is box-like weighting function

$$\mathrm{W(t)} = \begin{cases} 1 & \text{at } -\frac{\tau_{az}}{2} < t < \frac{\tau_{az}}{2} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

The result of the correlation is then

$$V(t) = \int_{-\infty}^{\infty} S_a(\xi) \cdot R(t+\xi) d\xi = \int_{-\infty}^{\infty} A_o \cdot \exp(i\pi K_{az}\xi^2) \cdot \exp(-i\pi K_{az}(t+\xi)^2) \cdot W(t+\xi) d\xi =$$
$$= A_o \cdot \exp(-i\pi K_{az} t^2) \int_{-\infty}^{\infty} W(t+\xi) \cdot \exp(-2i\pi K_{az}\xi t) d\xi \quad (20)$$

Because, only small values of t are important, we can substitute $W(t+\xi)$ for W(ξ). Denoting FT[...] as a Fourier transform the result of the correlation can be written as

$$V(t) = A_o \cdot \exp(-i\pi K_{az} t^2) \cdot \sqrt{2\pi} \cdot \mathbf{FT}_{2K_{az}t}[W(\xi)], \text{ where}$$

$$\mathbf{FT}_{2K_{az}t}[W(\xi)] = \int_{-\frac{\tau_{az}}{2}}^{\frac{\tau_{az}}{2}} \exp(-2i\pi K_{az}\xi t) d\xi = \frac{\exp(-i\pi K_{az}\tau_{az} t) - \exp(i\pi K_{az}\tau_{az} t)}{-2i\pi K_{az} t} =$$
$$= \frac{-2i\sin(\pi K_{az}\tau_{az} t)}{-2i\pi K_{az} t} = \tau_{az}\left[\frac{\sin(\pi K_{az}\tau_{az} t)}{\pi K_{az}\tau_{az} t}\right]$$

The final result is then

$$V(t) = A_o \cdot \tau_{az} \cdot \sqrt{2\pi}\exp(-i\pi K_{az} t^2) \cdot \left[\frac{\sin(\pi K_{az}\tau_{az} t)}{\pi K_{az}\tau_{az} t}\right] \quad (21)$$

The example of this compression is shown in Fig. 3. After the matched filtering signal in azimuth it appears well focused at zero point. The maximum amplitude increased from $|A_o|$ to $\sqrt{2\pi} \cdot \tau_{az} \cdot |A_o|$ (unitary values of K_{az} and $|A_o|$ parameters were chosen and so-called object phase which results from the scattering process on the object was neglected).

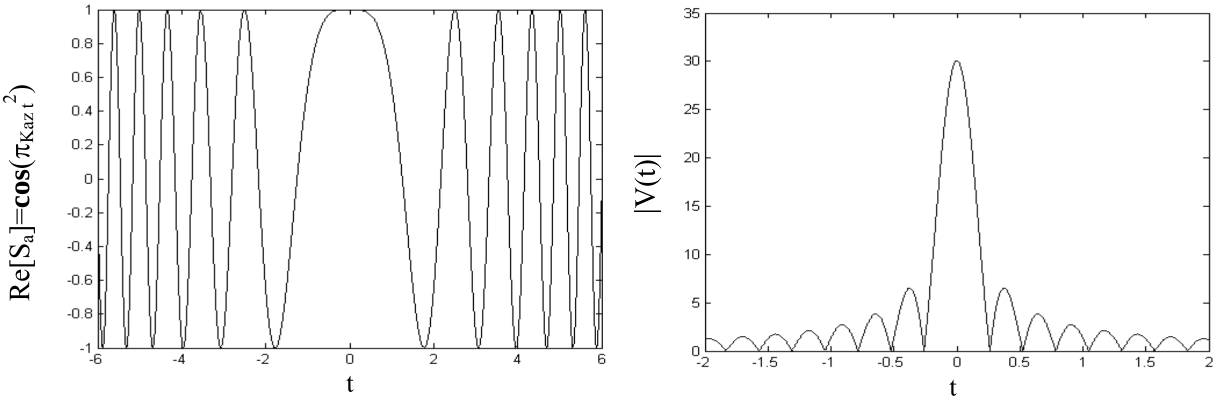


Fig.3 The example of the signal compression in the azimuth direction for an ideal point target

It can be noticed, that longer synthetic aperture (the bigger τ_{az}) causes the narrower main peak, thus the better resolution. The first minima of the main peak appear at $t=\pm 1/K_{az}\tau_{az}$. Therefore, defining the resolution as the half distance between the first minima, we get azimuthal resolution which has already been defined in section 3.

$$\delta_{az} = \frac{v}{K_{az}\tau_{az}} = \frac{v}{B_{az}} = \frac{\lambda}{2\theta_{3dB}} = \frac{L_{ra}}{2}. \quad (22)$$

6. SYSTEM CONSTRAINTS

The unambiguous range of the system is limited by the pulse repetition frequency (PRF). A higher PRF implies that the maximum unambiguous range reduces. The maximum range is given by the distance that the transmitted sound pulse travels out and back before the next pulse is transmitted,

$$r_{\text{unambiguous}} = \frac{c \cdot \tau_{rep}}{2}, \quad (23)$$

where τ_{rep} is the repetition period of the transmiting system. If a larger unambiguous range is desired, either the pulse repetition frequency must be decreased or multiple orthogonal signals must be sent. However, a system employing orthogonal chirp transmission suffers a degradation in signal SNR. The technique is not considered in this paper.

Decreasing the system PRF tends to cause along-track undersampling requiring slower system speed. The bandwidth B_{az} in azimuth, described by equation 16, sets the lower limit of PRF with which pulses are emited to the seafloor. According to the Nyquist-criterion, a sampling frequency of two times the maximum frequency is necessary for unambiguous recording of the data. The PRF corresponds to sampling frequency in azimuth direction.

The constraint of the closest range is dictated by *The Near/Far field limit* (NF) and is approximated to L_{ra}^2/λ. All conventional sidescan sonar systems should operate in their far-field, in conditions where the acoustic wavefront arriving at antenna aperture can be considered a plane wave.

7. ALGORITHM OF SAS PROCESSING AND NUMERICAL SIMULATION

The raw data of SAS are two-dimensional complex array obtained from a data acquisition process. To achieve a SAS image, usually only the absolute values are used, because the phase has a random distribution. The phase of a pixel in a conventional SAS consists out of two terms. The first one is the geometric phase due to the distance between antenna and object. This is strictly deterministic term. The second one is so-called object phase which results from the scattering process on the object. For distributed targets, this is a random value, because the phase of coherent summation of all indivudual scatterers can result in any value.

The example of simulated SAS raw data for an ideal reflected point target at location *(Range, Azimuth)=(30,0)* is shown in the fig. 4a. This point target is smeared in the range direction because of the range pulse duration and in the azimuth direction due to data acquisition process.

We can also notice the phenomenon called range migration. When the point scatterer is first "seen" by the antenna we have a certain distance to this object. As the antenna flies by, this distance will decrease to a minimum, when the scatterer is broadside and after passing broadside the distance will increase again. The target migrates closer and the farther from the sonar as the antenna approaches and then passes the location of the target. If the distance between the maximum and the minimum range is more than the range resolution, the returns will be in different range bins causing a hyperbola (curvature) in azimuth direction.

Depending on system parameters, range migration may be a problem in azimuth processing. Our considerations in the previous section do not take into account changing range positions to the point scatterer as the antenna flies by. This phenomenon appears in our simulated raw data but there is only slightly curvature. The SAS processing neglecting correction and with ideal correction of this phenomenon was simulated. The parameters of simulation are shown in table 1.

The chosen values allowed to obtain the resolution of 5cm both in azimuth and range direction, using the maximum possible synthetic aperture length.

Tab.1 Assumed system parameters

Velocity of platform	0.8 [m/s]
Speed of sound in water	1500 [m/s]
Chirp bandwidth	15 [kHz]
Chirp duration	8 [ms]
Carrier frequency	100 [kHz]
Pulse repeat frequency	18 [Hz]
Sonar length	0,1 [m]

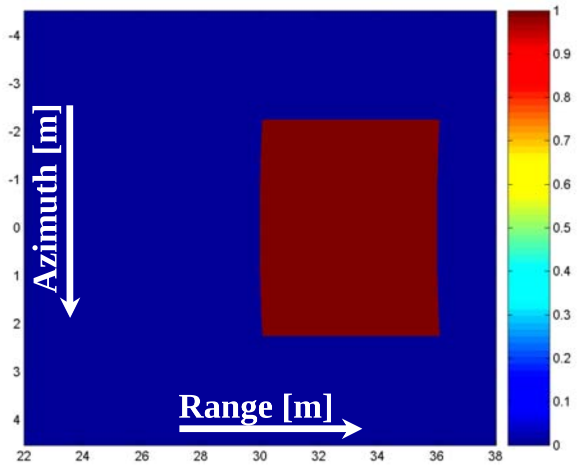
The aim of SAS processing is now to focus all received energy on one point (it is assumed single point scatterer). The principal sequence of SAS processing is following.

a) *Range Compression*

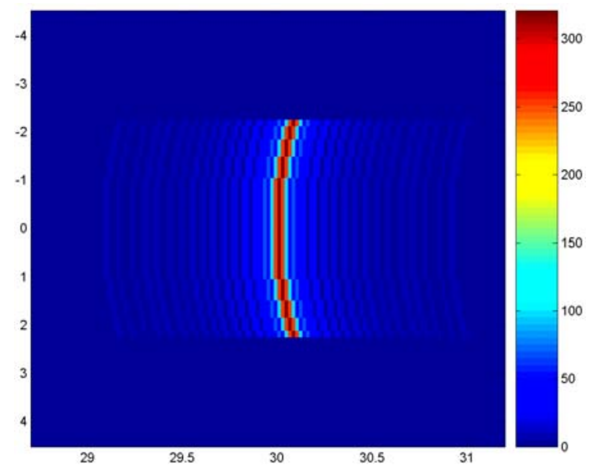
To achieve a high resolution in the range direction, matched filtering of a received LMF signal is performed, using the fast convolution described in section 1. After one-dimensional Fourier transform of the raw SAS data in range direction, each range line is multiplied with the Fourier transform of the reference function being conjugate replica of the transmited LMF signal. After inverse FFT back to time domain, the data are compressed in range. The result of this step is depicted in figure 4b.

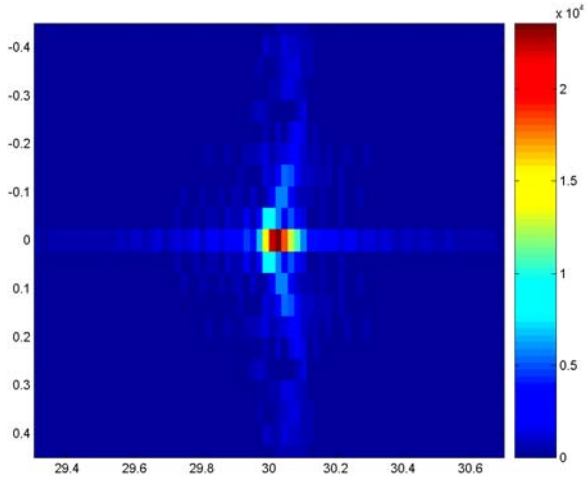
b) *Range Curvature Correction*

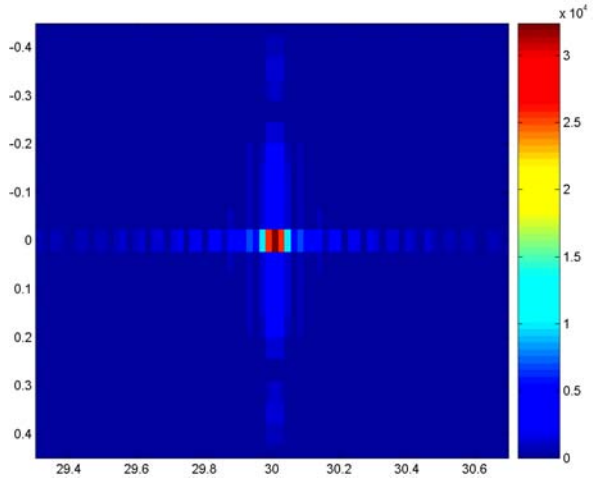
At this point correction of range migration phenomenon can happen if it is needed (compare Fig. 4c with 4d).

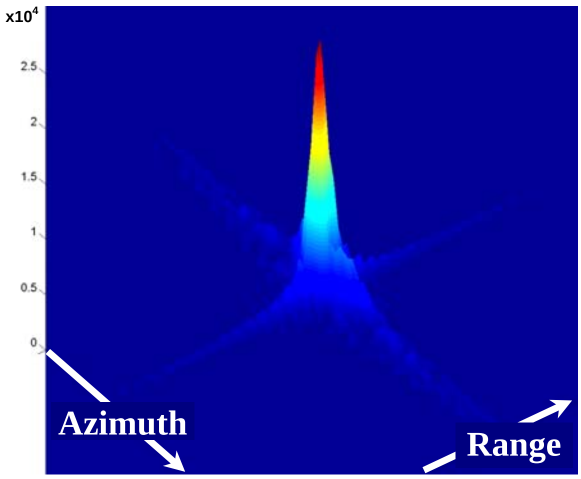


a) Received SAS raw data


b) After compression in range


c) After compression in range and azimuth


d) After compression in range, ideal range curvature correction and compression in azimuth.



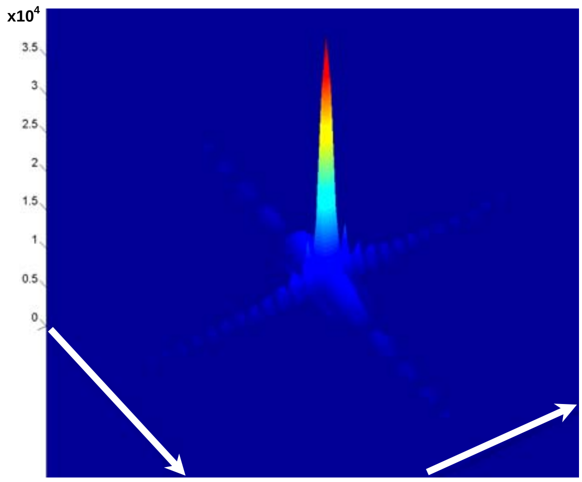


Fig.4 The results of the simulation for an ideal point target at location (30,0)

c) *Azimuth Compression*

The next step is data compression in azimuth direction. This is done with the use of matched filtering operation in frequency domain again. The only difference lies in the fact that the reference function in azimuth has to be adapted to each column. This is because the occuring Doppler efect depends on distance to a scatterer. The following columns represent a growing distance to the antenna, determined by the range sampling rate of the sonar and the speed sound in water in the following way $\Delta r = c/2/f$ sampling. If a correct reference function is prepared a whole azimuth line can be focused in one step, using the fast convolution. The result of SAS processing for single point scatterer is shown in figure 4c or 4d.

8. CONCLUSIONS AND REMARKS

In the derivation of the model of SAS presented above, a number of simplifications were assumed. One of them is that the sonar transmits at particular position and waits for all echoes to return before moving instantaneously to the next transmit position. A real sonar moves continously along the aperture during data collection. The so-called stop-and-hop model, commonly used in Synthetic Aperture Radar (SAR), in SAS may be insufficient because of low speed sound in water. The next is not taking into account motion errors (ideal straight track was assumed) which can appear during the data acquisition process. These problems can result in a Doppler shift of the received data and even unwanted image artifacts.

Synthetic aperture sonar in comparison with SAR systems requires taking into consideration numerous factors that are not likely to be present in SAR operating environments. These factors are multipath, sea state, strong reverberation and many other. These phenomena make execution of SAS systems very difficult in practice.

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