APPLICATION OF PCA AS AN ACOUSTICAL SIGNAL PROCESSING TOOL FOR OBJECT CLASSIFICATION PURPOSES

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The authors presented a technique for an optimal representation of acoustical signals for further object classification purposes using different statistical and neural methods. It is based on principal component analysis (PCA) which is a transformation of vectors localized in k-dimensional observation (feature) space into lower n-dimensional component space retaining majority of included information. The resulting improvement in classification efficiency by a chosen statistical classifier was verified by a numerical experiment.

INTRODUCTION

Passive classification of objects using their acoustical signature is a relatively new field of study. However, the importance of a perspective object identification system for both military and civilian applications cannot be underestimated. By object we mean most of the time a ship or military vessel (especially a submarine) that may operate and pose a threat to own vessels in close vicinity and finding the identity of such "object" may play an important role in a decision making process in navigation from both tactical and strategic perspective. Different ways were so far proposed for the purpose of classification of objects but the most promising techniques seem to utilize the intrinsic properties of an acoustic field of a moving vessel and the properties of artificial neural networks [5]. These methods were, however, subject to some disadvantages. One of them were associated with the large amount of information that had to be processed by classifiers in order to make a classification or identification decision. Therefore, a solution had to be found in order to make the amount of information smaller and the effectiveness of classifiers more robust. This article proposes one way of making such data processing reduction possible by means of applying a transformation called principal component analysis which seems adequate for such type of problems. The authors presented a scheme and a method of classification which starts with a feature extraction, continue with a PCA transformation of the feature space and finish with an application of a sample classifier (Fisher's method) for the task of verification of the proposed method.

1. FEATURE EXTRACTION

The necessary condition for any classification process is determination of parameters called features that characterize different kinds of objects. The goal of the selection is minimization of the number of feature under condition that they would still optimally represent the described objects. The process, called feature extraction and constituting almost a science field of its own [2], [3], [6], [7] results in the initial diminishing of the observation space into feature space.

$$f : \Re^k \Rightarrow \Re^n,\ n < k \tag{1}$$

where, k – dimension of original space,
n – dimension of feature space

and is usually based on a linear transformation of a signal vector $\vec{u}$ by some transforming matrix $A_{n \times k}$:

$$\vec{x} = A_{n \times k} \cdot \vec{u}\ . \tag{2}$$

Although there are many types of feature extraction methods for acoustical signals the methods most useful for the future classification depend on finding best features in the frequency domain of an acoustic signal. For the purpose of classification the authors believe that a natural, most convenient and probably most efficient method for determining signal features is the Fast Fourier Transform, although the choice of a method is not a main goal of this paper.

2. PCA IN ACOUSTICAL SIGNAL PROCESSING

Principal components analysis (PCA) is a signal processing method which in underwater acoustics may be used for a reduction of the feature space dimensionality. For the first time it was described by Pearson [4] between XIX and XX century. Recently, it has become one of the primary transformations of signal preprocessing for the purpose of classification and is described in monographs from the field of statistics, pattern recognition and linear algebra, for example [3], [7].

The PCA transformation of a random feature vector may be considered as problem of finding of new axis's in multidimensional coordinate system so that the data transformed into that system retains it maximum amount of information in form of variance. The process based on data rotation around the origin of the coordinate system is shown on Figure 1. From the algebraic point of view this transformation is a linear transformation of data from the initial space to an orthogonal space of lower dimension in which the axis's are called components.

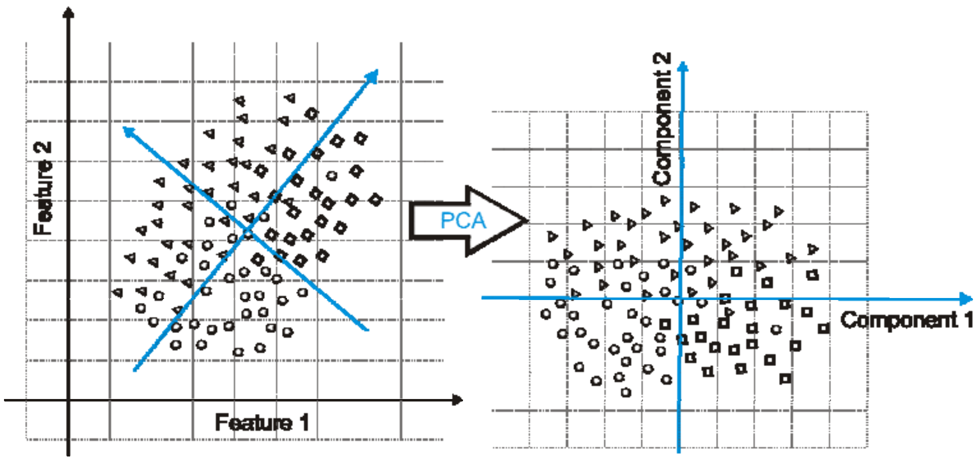

Fig.1 Graphical presentation of PCA transformation in 2-dimensional feature and principal component space

An optimal solution for this problem (in least mean square optimization sense) is based on the application of the eigenvalue and eigenvectore structure of the data covariance matrix and is similar to Karhunen-Loeve transformation used in the problems of image and signal processing [Cich02]. In other words, PCA is an algorithm for finding the eigenvalues and eigenvectirs of the estimated data covariance matrix:

$$\hat{R}_{xx} = \mathrm{E}\left\{\overline{x}\overline{x}^T\right\} = V\Lambda V^T \in \Re^{n\times n} \tag{3}$$

where,

$\Lambda = diag\{\lambda_1, \lambda_\lambda, \cdots, \lambda_n\}$, λ_i - eigenvalues of $\hat{R}_{xx}$ from largest to smallest,

$V = [\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n] \in \Re^{n\times n}$ - orthogonal matrix composed of eigenvectors $\vec{v}$ of $\hat{R}_{xx}$ called principal components corresponding to λ in Λ,

$\overline{x} = [\overline{x}_1, \overline{x}_2, \cdots, \overline{x}_n]^T$ - normalized random observation feature vector.

The transformation to the new principal component space is carried out by means of the following formula:

$$\vec{y}_s = V_S^T \vec{x} \tag{4}$$

where,

$\vec{y}_s = [y_1, y_2, \cdots, y_p]^T$ is an output vector in the principal component space,

$V_S = [\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p]^T \in \Re^{n\times p}$ principal components subspace eigenvectors.

As we can see from the above formulas the application of PCA (only p eigenvectors V_p corresponding to the p eigenvalues have been used) results in the transformation of the signals data localized in the feature space $\Re^n$ to the principal components space $\Re^p$ ($p < n$). The following are the features of PCA [1]:

- Expected values of principal components are equal to zero.
- Principal components are mutually not correlated.
- i-th principal component variance is equal to the i-th eigenvale of the covariance matrix $\hat{R}_{xx}$:

$$\operatorname{var}\{y_i\} = \sigma_{y_i}^2 = \mathrm{E}\{y_i^2\} = \mathrm{E}\left\{\left(\vec{v}_i^T \vec{x}\right)^2\right\} = \vec{v}_i^T R_{xx} \vec{v}_i = \lambda_i . \tag{5}$$

- PCA is optimal in the least mean square error of a random vector $\overline{x}$ recreated using p eigenvectors $\vec{v}$:

$$\hat{\vec{x}} = \sum_{i=1}^{p} y_i \vec{v}_i = \sum_{i=1}^{p} \vec{v}_i \vec{v}_i^T \vec{x}, \; p < n , \tag{6}$$

$$\mathrm{E}\left\{\left\|\vec{x} - \hat{\vec{x}}\right\|^2\right\} = \mathrm{E}\left\{\left\|\sum_{i=p+1}^{n} y_i \vec{v}_i\right\|^2\right\} = \sum_{i=p+1}^{n} \left\{\left|y_i\right|^2\right\} = \sum_{i=p+1}^{n} \lambda_i . \tag{7}$$

Covariance matrix decomposition given by formula (3) is equal to spectral decomposition of data matrix X into singular values [1]:

$$X = U \Sigma V^T \in \Re^{m \times n} \tag{8}$$

where:

$U \in \Re^{m \times m}$, $V \in \Re^{n \times n}$ - orthogonal matrices,

$\Sigma \in \Re^{m \times n}$ - pseudo diagonal matrix, in which n upper rows contain $\Sigma_S = diag\{\sigma_1, \sigma_2, \cdots, \sigma_n\}$ of positive elements with decreasing order and $(m-n)$ lower rows are equal to zero.

The relation between PCA and the above transformation (called Singular Value Decomposition - SVD) is given by the following formulas:

$$XX^T = U \Sigma_1^2 U^T \tag{9}$$

$$X^T X = V \Sigma_2^2 V^T \tag{10}$$

where: $\Sigma_1 = diag\{\sigma_1, \sigma_2, \cdots, \sigma_m\}$, $\Sigma_2 = diag\{\sigma_1, \sigma_2, \cdots, \sigma_n\}$.

This means that singular values of matrix X are the square roots of eigenvalues of matrix XX^T and eigenvectors U of matrix XX^T are the left singular vectors of data matrix X.

3. RESULTS OF EXPERIMENTS

The main goal of this paper was to propose PCA as a tool in acoustical signal processing to improve further signal classification with statistical and neural methods. The best way to verify its suitability is to apply some classification scheme and use both type of input signals: raw signals from feature extraction phase and data processed by the PCA tool. The experiment was conducted in the following way.

Acoustic signals corresponding to 8 different objects was processed by FFT resulting in a set of patterns for all objects contained in matrix X:

$$X^c = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{w1} & x_{w2} & \cdots & x_{wn} \end{bmatrix} \tag{11}$$

where:

n – number of elements in the observation vector (200),
w – number of patterns per class (40),
c – object number.

Each pattern was a vector of 200 elements (features) equal to the acoustical power of a corresponding frequency. Since we used only one frequency band of 1-200 Hz each element was associated with a frequency equal to its index

$$x_j = P_a(j),\ j = 1,..,200 \tag{12}$$

where:

$P_a(j)$ is the acoustical power of acoustical signal in decibels (re 10^{-12} W In 1 Hz band) for frequency j in Hz.

The classification method selected for the purpose of this research was the Fisher's classifier [7], [2] which is a classical example of transformation of data from n-dimensional feature or component space to d-dimensional object space. It is constructed in such a way so that consecutive decision regions were optimally separated. The transformation formula is given by the following equation:

$$\omega = V\vec{x} \tag{13}$$

where:

ω - is a class of an identified object;
V - matrix consisting of d column eigenvectors $\vec{v}_i$ (of n elements) in decreasing order of the associated eigenvalues λ_i obtained from the formula:

$$S_B \vec{v}_i^T = \lambda_i S_W \vec{v}_i, i = 1, 2, \cdots, d \tag{14}$$

where:

$S_W = \sum_{c=1}^{d} \sum_{i=1}^{s} (\vec{x}_i^c - \hat{\vec{x}}_c)(\vec{x}_i^c - \hat{\vec{x}}_c)^T$ - matrix of within class dispersion,
$S_B = \sum_{c=1}^{d} s(\hat{\vec{x}}_c - \hat{\vec{x}})(\hat{\vec{x}}_c - \hat{\vec{x}})^T$ - matrix of between class dispersion,
$\hat{\vec{x}}_c$ - mean data vector for single class c,
$\hat{\vec{x}}$ - mean vector for all data,
s – number of sample vector per class,
d - number of object classes.

The results of classification are presented in Figure 2.

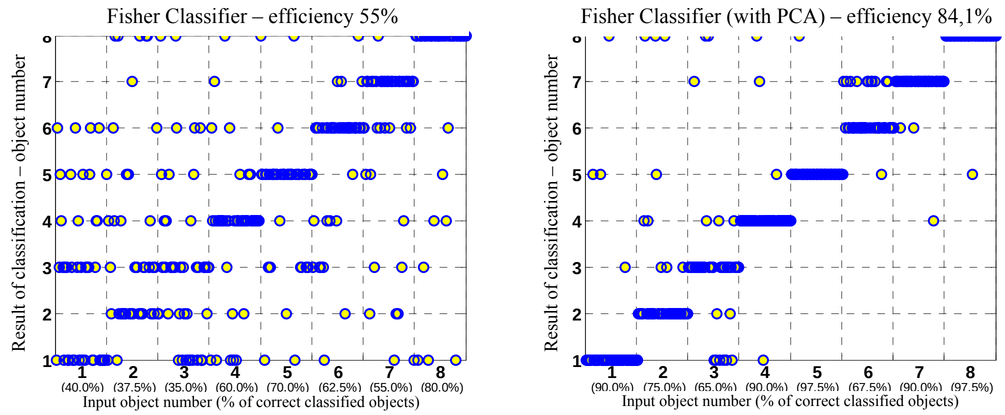

Fig.2 Classification efficiency based on signals processed with PCA and without PCA transformation for 8 classes of objects

4. CONCLUSIONS

The data reduction scheme proposed by the authors in form of the principal component analysis decreased the amount of information that must be processed in the acoustical object identification process and increased the robustness of a classification procedure in terms of more effective classification (Figure 2). Therefore it should be perceived as a candidate for the pre-processing phase of such methods that involve the application of statistical and neural techniques for the above mentioned purpose.

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