ANALYSIS OF NOISY SIGNALS ON BASE OF DIFFERENTIAL CORRELATION ALGORITHMS

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Correlation analysis (CA) in PCM format is often used in signal recognition. Signals representation in PCM format allows for calculate correlation functions (CF) with assign accuracy, however at the same time causes a large number of multi-bit operations and leads to insufficient resolution and fast-acting of CA. Therefore, in such tasks it is expedient to use small-bit differential methods. However, the known approaches to differential CA are not enough studied.

This paper purpose is the resolution and fast-acting increase of noisy signals CA in real time on base of modified differential PCM (DPCM) algorithms.

INTRODUCTION

Periodic determined probe signals are widely used in numerous fields of science and technology especially in hydroacoustics [1]. During propagation the noisiness them takes place. On the basis of correlation analysis (CA) the periods of the noisy periodic determined signals, time or phase shifts etc. can be specified. Therefore, CA is so often used in signal recognition. Signals representation in PCM format allows for calculate correlation functions (CF) with an assigned accuracy, however at the same time causes a large number of multi-bit operations. It limits to the resolution and the fast-acting of CA. Therefore, in such tasks it is expedient to use small-bit differential methods. However, the known approaches to differential CA are not enough studied. We can use the mixed signals representation when one signal is in PCM format whilst another is in the differential format. It allows to obtain an advantages of both above mentioned formats. In works [2,3] there was discussed the use of mixed formats on base of different kinds of Delta Modulation (DM). DM includes such popular kind as DPCM too. However, the use of Modified DPCM (MDPCM) which is based on the quantization step sizes equal to powers of 2 in CA not studied yet.

Therefore this work aim is the resolution and fast-acting increase of the noisy signals CA in real time on base of MDPCM algorithms.

1. DIFFERENTIAL CORRELATION ANALYSIS ALGORITHMS

General formula of the Cross CF estimator in format PCM-PCM for *m* time shifts is:

$$\hat{K}_{xy}(m) = \frac{1}{N-m+1}\sum_{i=0}^{N-m}\dot{y}_i\dot{x}_{i+m} \tag{1}$$

$$\hat{K}_{xy}(-m) = \frac{1}{N-m+1}\sum_{i=m}^{N}\dot{y}_i\dot{x}_{i-m} \tag{2}$$

where $m = \overline{-P+1, P-1}$, P – number of time shifts with the equal signs, $\dot{x}_i, \dot{y}_i$ -centered values of the *i*-ths signal samples, N - signal samples number, $N=ENT(\theta/T_d)$, $T_d^{-1}=f_d$, θ - analysis signal length, f_d - sampling rate.

Writing down these relationships in recurrence form allows to avoid of the execution of the repetitive operations. For this event we take that some discrete variables $\{z_i\}$ we can represent as

$$z_i = z_{i-1} + \nabla z_i = z_0 + \sum_{r=1}^{i}\nabla z_r \tag{3}$$

where

$$\nabla z_i = z_i - z_{i-1} \tag{4}$$

In the book [2] it is proved that on the base of the only calculated estimator of CF for *m*=0 we can calculate the others estimators with use the next relationships.

For the positive shifts:

$$\hat{K}_{xy}(m) = \hat{K}_{xy}(m-1) + \nabla K_{xy}(m) \tag{5}$$

For the negative shifts:

$$\hat{K}_{xy}(-m) = \hat{K}_{xy}(-m+1) - \nabla K_{xy}(-m+1) \tag{6}$$

where

$$\nabla K_{xy}(m) = \frac{1}{N-m}\sum_{k=0}^{N-m-1}\dot{y}_k\dot{x}_{k+m} - \frac{1}{N-m+1}\sum_{k=0}^{N-m}\dot{y}_k\dot{x}_{k+m-1} \tag{7}$$

$$\nabla K_{xy}(-m+1) = \frac{1}{N-m+1}\sum_{k=m-1}^{N-1}\dot{y}_k\dot{x}_{k-m+1} - \frac{1}{N-m}\sum_{k=m}^{N-1}\dot{y}_k\dot{x}_{k-m} \tag{8}$$

When a signal $\{x_i\}$ or $\{y_i\}$ is represented in differential form then finally we obtain on base of [3]:

$$\hat{K}_{xy}(m) = \frac{1}{N-m}\left[(N-m+1)\hat{K}_{xy}(m-1) - \dot{x}_{m-1}\dot{y}_0 - \sum_{k=m}^{N-1}\dot{x}_k \nabla y_{k-m+1}\right] \tag{9}$$

$$\hat{K}_{xy}(-m) = \frac{1}{N-m}\left[(N-m+1)\hat{K}_{xy}(-m+1) - \dot{y}_{m-1}\dot{x}_0 - \sum_{k=m}^{N-1}\dot{y}_k \nabla x_{k-m+1}\right] \tag{10}$$

where

$$\hat{K}_{xy}(0) = \frac{1}{N}\sum_{k=0}^{N-1}\dot{x}_k\dot{y}_k \tag{11}$$

The use of the small-bit kinds of DM to code of the differences $\{\nabla y_i\}$ or $\{\nabla x_i\}$ allows to decries of the length words by the multiplications.

But we can go farther and use the modification of the DPCM encoder in order to exchange of the multiplications on simple shift operations. For this way we need use the quantization step sizes equal to powers of 2 that is 1,2,4,8, ...

2. THE CHARACTERISTICS OF THE ENCODERS

The characteristics of the 3-bits: classical DPCM encoder and two MDPCM encoders is shown in Figure 1. In Figures 1 a),b),c) the characteristic of multi-bit classical DPCM is shown by the thin lines. Irregularity of the MDPCM1 and MDPCM2 encoders characteristics is the advantage in case of fast variable signals such as the noises because MDPCM encoder will be kept up with the signal which has bigger speed of a slope increase than the usual DPCM encoder. In Figure 1c) is shown the characteristic of the MDPCM2 encoder without of 0 that allows to obtain an additional quantization step. As our researches showed, it is especially useful for differential CA when a overload of the encoder plays a main role in comparison with quantization noise.

For signal $\{x_i\}$ coded in the MDPCM format with word length ***c*** we can use the characteristics showed in Figure 1 to write down next equals:

$$\nabla x_i = x_i - x_{i-1} \approx \varepsilon \cdot s_i^{(x)}; \quad s_i^{(x)} = \mathrm{sgn}\nabla x_i \cdot r_i^{(x)} \tag{12}$$

$$r_i^{(x)} = 2^j; \quad j = \{0..c-1\} \tag{13}$$

where ε - modul of minimal quantization step, r_i - quantization coefficient which depends on a form of the characteristic.

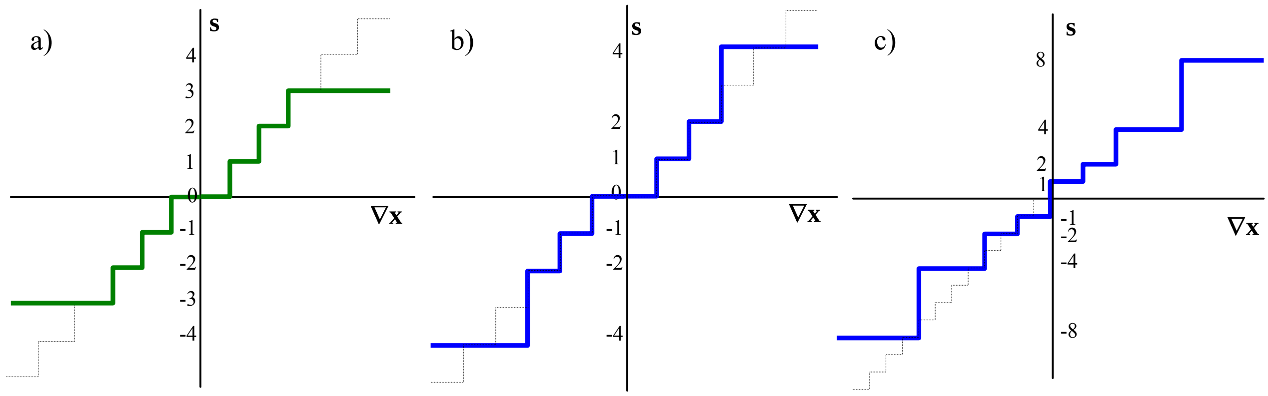

Fig.1 **C**haracteristics of the 3-bits classical DPCM encoder (a), MDPCM1 encoder with the use of the zero step (b), and MDPCM2 encoder without the zero step (c)

As it is shown in [3], substitution of the equal (12) to the relationships (9), (10) allows to obtain:

$$\hat{K}_{xy}(m)=\frac{1}{N-m}\left[(N-m+1)\hat{K}_{xy}(m-1)-\dot{x}_{m-1}\dot{y}_0-\varepsilon^{(y)}\sum_{k=m}^{N-1}\left(\left|\dot{x}_k\right| r_{k-m+1}^{(y)}\right)\cdot \operatorname{sgn}\nabla y_{k-m+1}\cdot \operatorname{sgn} x_k\right] \quad (14)$$

$$\hat{K}_{xy}(-m)=\frac{1}{N-m}\left[(N-m+1)\hat{K}_{xy}(-m+1)-\dot{y}_{m-1}\dot{x}_0-\varepsilon^{(x)}\sum_{k=m}^{N-1}\left(\left|\dot{y}_k\right| r_{k-m+1}^{(x)}\right)\cdot \operatorname{sgn}\nabla x_{k-m+1}\cdot \operatorname{sgn} y_k\right] \quad (15)$$

For the signs multiplication we can use the logical operations and write down:

$$\operatorname{sgn} x\cdot \operatorname{sgn} y=2\left(B^{(x)}\oplus B^{(y)}\right)-1;\quad \operatorname{sgn} x\Rightarrow B^{(x)},\operatorname{sgn} y\Rightarrow B^{(y)},B\in\{0,\,1\} \quad (16)$$

whilst the multiplication some number by a power of 2 realizes by shift operation only.

3. RESEARCH METHOD

We used the simulation for calculating of the Auto CF estimator $\hat{K}_{xx}(m)$ in formats PCM-PCM and PCM-MDPCM for a complex signal which is a sum of the sin signal and the white noise which spectrum is limited to a frequency of sin signal. Sampling rate f_d was chosen with the use of the assigned ratio α of the frequency f_d to the Nyquist's frequency f_N

$$\alpha=\frac{f_d}{f_N} \quad (17)$$

The signal/noise ratio (SNR) was held on a level –18 dB. The results were being averaged on a base of 100 realizations of the signal. The successive realizations of the noise were realized by a change of the stochastic numbers on output of the generator. The results of CA on a base of 8-bit PCM was taken to compare. As the criterion of the comparison of the obtained estimators CFs was used the MSE which was calculating as

$$\sigma_k = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\hat{K}_{xx}^{(PCM)}(i) - \hat{K}_{xx}^{(DM)}(i)}{D_{xx}^{(PCM)}}\right)^2} \tag{18}$$

where N – number of the CF estimators shifts, $\hat{K}_{xx}^{(PCM)}, \hat{K}_{xx}^{(DM)}$ - estimators of CFs suitably in PCM-PCM format and MDPCM-PCM format, $D_{xx}^{(PCM)}$ - variance in PCM format.

Number of the time shifts of the CF was equaled to 20% length of the studied sequence. DPCM minimal quantization step size ε was equaled to a PCM quantization step ε_{PCM} .

4. SIMULATION RESULTS

The first stage was connected with the accuracy analysis on base of the MSE $\boldsymbol{\sigma_x}$ by the comparison between the amplitudes of the signal coded with the use of DPCM and converted after it into the format PCM and the standard signal in PCM format.

The analysis averaging results are shown in Figure 2 where the used kinds of the modulations are marked in the following way: MDK13 is 3-bit MDPCM1 characteristic, MDK43 is 3 bit MDPCM2 characteristic, MDK44 is 4-bit equivalent of the MDPCM2 characteristic, MDK14 is 4-bit equivalent of the MDPCM1 characteristic, DPCM3 is 3-bit standard DPCM characteristic, DPCM4 is 4-bit standard DPCM characteristic.

Results were normalized relatively of a minimal error which DPCM4 obtains.

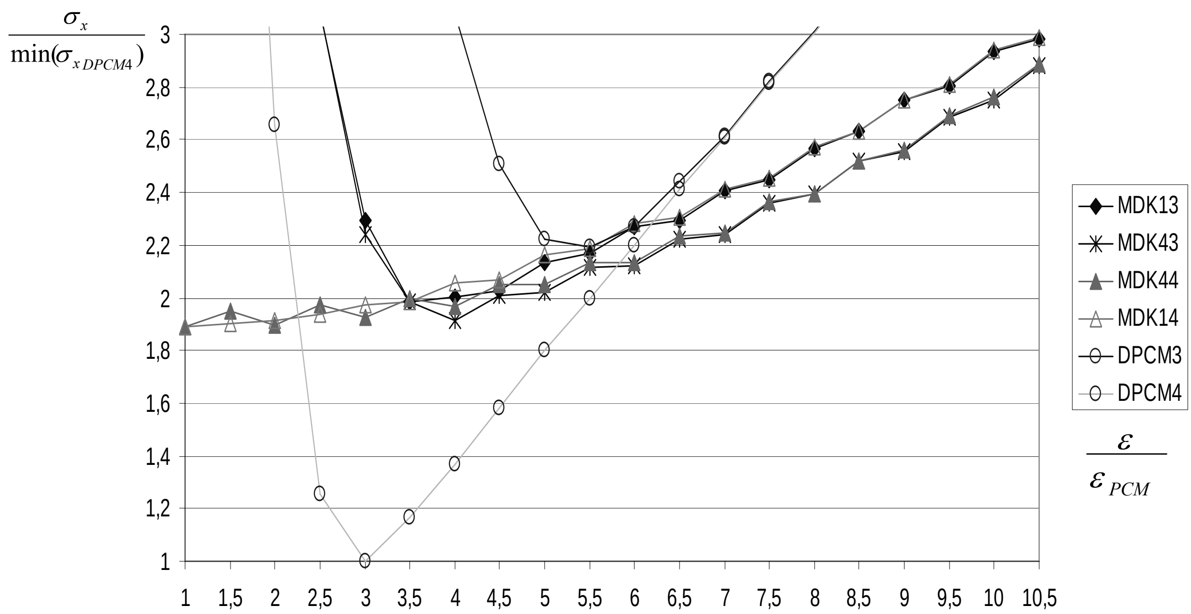

Fig.2 Normalized MSE of the decoded DPCM signal relatively PCM signal, in relation to the quantization step ε by α=2

From above showed characteristics it is followed:

a) 3-bit MDPCM encoder better represented input signal than 3-bit DPCM encoder,
b) MDPCM2 encoder provide better parameters of the signal representation than MDPCM1 with use a whole range of the quantization steps, particularly in case of the big increases of the signal,

c) The use of the MDPCM length words more than 3 bit does not lead to any increase of the CA accuracy.

Farther we study CA with use of MDPCM2 only. In Figure 3 are presented some families of the relationships of the correlation functions MSE in relation to the normalized quantization step $\frac{\varepsilon}{\varepsilon_{PCM}}$ and sequence length N.

These Figures allow to see that for little α values the choice of the optimal quantization step is critical because even little deviations of the step ε relatively the optimal value of this step lead to the big grows of the error.

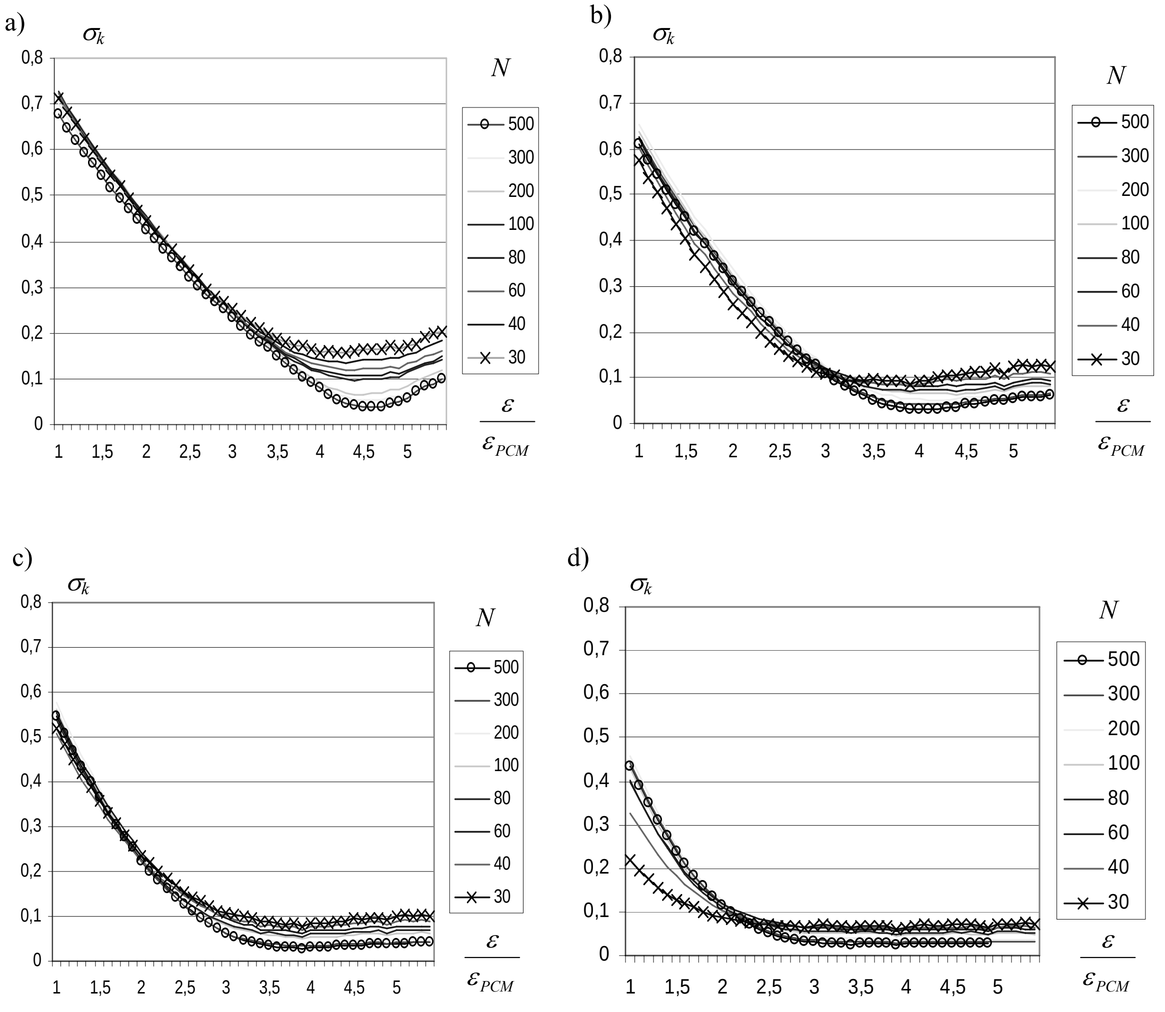

Fig.3 MSE σ_k of CA with the use of MDPCM2 in relation to the normalized quantization step $\frac{\varepsilon}{\varepsilon_{PCM}}$ and sequence length N for: a) α = 1, b) α = 2, c) α = 3, d) α = 5.

Next conclusions we can obtain on base an analyzes of Figures 4 and 5. In Figure 4 is shown the relation between the optimal quantization step ε_{op} and the coefficient α for sequence length N=300. We mean the optimal quantization step ε_{op} as value $\frac{\varepsilon}{\varepsilon_{PCM}}$ for which the error σ_k is minimal.

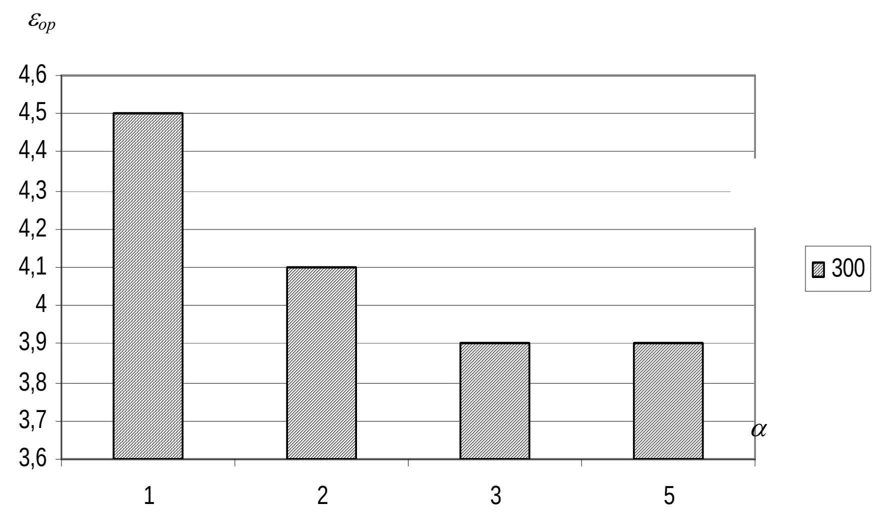

Fig.4 Relation between the optimal quantization step size ε_{op} and coefficient α.
The sequence length N=300

In Figure 4 we can clearly see that the optimal quantization step size ε_{op} is decreased when the coefficient α is increased.

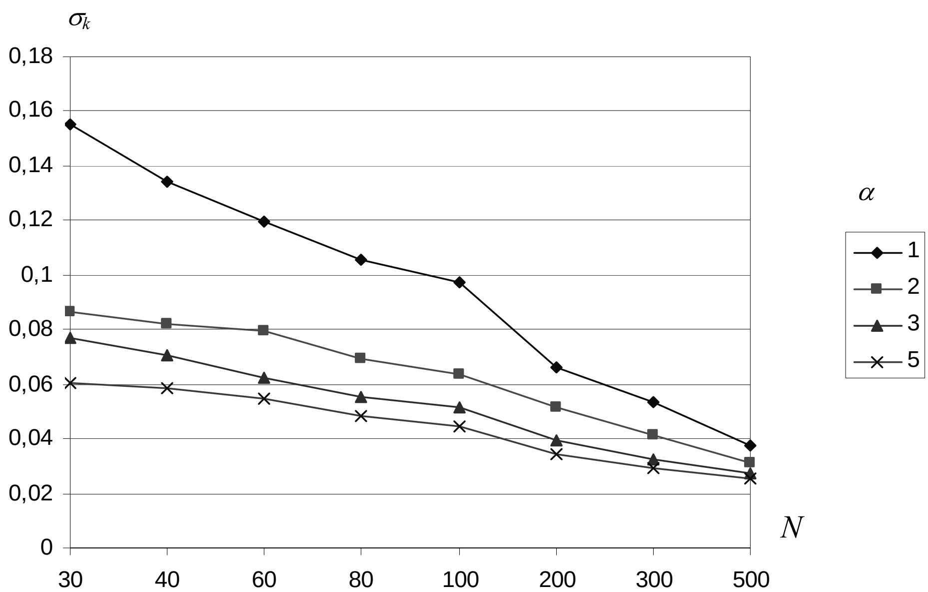

Fig.5 Correlation function estimator error MSE σ_k in the relation to the sequence length N and coefficient α

In Figure 5 we showed the relation between the correlation function error MSE and sequence length N by different coefficients α. It is seen a quasi linear character of the relations for all coefficients α.

5. CONCLUSION

The use of MDPCM in correlation analyses allows to exchange multi-bit arithmetic operations with the logical ones. It leads to essential improve of the fast-acting of that CA in relation to one based on PCM. Additionally, when recognition of periodical noisy signals realizes in the real time CA in the DPCM format ensures better resolution than that in the PCM format. Apart from it, the structures of the correlators with the use of MDPCM are simpler than ones in PCM format.

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