DETERMINING THE IMPEDANCE OF ULTRASONIC TRANSDUCERS BY THE ADDED ELEMENTS METHOD

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There are a number of problems involved in measuring complex electrical impedance of ultrasonic transducers. This is because in the frequency function impedance tends to change values rapidly, in particular where resonances are involved. Measuring impedance using accurate bridge methods is a tedious and time-consuming process. While modern automated systems offer shorter test periods and high accuracy, they are very costly. This is why the project aims to implement new methods for computing impedance without these limitations. The article proposes a new, simplified method for measuring the electrical impedance of ultrasonic transducers. In the method complex impedance is calculated from results of two measurements of the transducer's impedance magnitude only: with the known reactance and without it.

To demonstrate the practicality of this method the authors included in the article the value of complex electrical impedance computed using the method and compared these with the results of measurements taken with an automated typical measurement system.

INTRODUCTION

Unlike magnitude measurement, measuring the complex electrical impedance of a transducer is difficult and tedious [1]. For this reason determining complex impedance based on magnitude measurement only comes as an attractive solution. It does not require phase measurements, and voltage measurements are sufficient. The result is a radically lower number of samples [1]. Consequently, inexpensive and easy to buy A/C converters can be used.

The main problem is calculating complex impedance from it's magnitude only. This can be done, but the measurements must be supplemented with additional tests of e.g. a magnitude of transducer impedance with a known reactance. Impedance is calculated using the solutions of two electrical root-like equations. Its solutions may be ambiguous; double and different results can be obtained showing the real and imaginary part of impedance. One of the criteria for eliminating ambiguity is to assume that the real part of impedance cannot be negative. If this criterion is not sufficient, if e.g. both computed real parts of impedance are positive, more measurements of transducer magnitude impedance should be taken with another added element, .e. a different additional reactance.

1. MEASUREMENT METHODS

Typically, the magnitude of electrical impedance of an ultrasonic transducer is measured in a constant current circuit, i.e. where the current does not depend on the impedance measured value (but on the high resistance \mathbf{R}_d only) - Fig.1 – [1]. In this case voltage \mathbf{U}_T – Fig.1a on the transducer is proportional to the magnitude being measured (voltage \mathbf{U}_c is proportional to the magnitude of electrical impedance of the transducer with the added element \mathbf{X}_d -Fig.1b).

$$U_{\rm T} = U_{\rm g} \frac{\left| Z_{\rm T} \right|}{R_{\rm d}} \tag{1}$$

But the condition to ensure constant current circuit ($\mathbf{R}_d >> \mathbf{mag}(\mathbf{Z}_T)$) cannot always be easily met. It may become difficult in the case of unloaded transducers with a high dynamics of changes in impedance value. In this case, calculating the impedance module cannot be done using the simplified linear equation eq.1, which expresses the linear relation between module \mathbf{Z}_T and voltage \mathbf{U}_T (\mathbf{U}_c). The equation takes on a more complicated form with two unknowns: the real part \mathbf{R}_T and imaginary part \mathbf{X}_T of transducer impedance - eq.2.

$$U_{T} = U_{g} \frac{\sqrt{R_{T}^{2} + X_{T}^{2}}}{\sqrt{(R_{T} + R_{d})^{2} + X_{T}^{2}}}$$
(2)

Because it is intertwined with resistance Rd, it cannot be used to calculate the impedance magnitude Z_d . Additional measurements are required of e.g. the voltages U_g and U_T as well as of the phase between them [1]. The article proposes a different solution, where measurements of voltages U_T and U_g as depicted in Fig.1a, are supplemented with analogous measurements (voltage U_c and U_g) for the transducer with the serial connected known inductance L_d (known reactance X_d) – Fig.1b– and there is no need to measure the phase.

It can be easily shown (Fig.2) that in both cases of measuring the transducer impedance magnitude the following two electrical equations can be written:

$$U_{T} = U_{g} \frac{\sqrt{R_{T}^{2} + X_{T}^{2}}}{\sqrt{(R_{T} + R_{d})^{2} + X_{T}^{2}}} \qquad \qquad U_{C} = U_{g} \frac{\sqrt{R_{T}^{2} + (X_{T} + X_{d})^{2}}}{\sqrt{(R_{T} + R_{d})^{2} + (X_{T} + X_{d})^{2}}} \qquad (3)$$



Fig.1 Measuring the impedance of the transducer a) without the added element and b) with the added element $X_{\rm d}$



Fig.2 Image of impedance vectors in the circuit of the transducer with the added element X_d

After substituting in equations 3 following expressions: $a = \frac{U_T^2}{U_g^2}$, $b = \frac{U_C^2}{U_g^2}$ and after the necessary transformations and inclusion of additional parameters A, B, C, D and E, a set of two equations with two unknowns R_T and X_T is obtained:

$$X_{T}^{2} + R_{T}^{2} + AR_{T} + B = 0$$
(4)
(C - A)R_T + DX_T + E - B = 0

where: $A = \frac{2 a R_d}{a - 1}$, $B = \frac{a R_d^2}{a - 1}$, $C = \frac{2 b R_d}{b - 1}$, $D = 2X_d$, $E = X_d^2 + \frac{b R_d^2}{b - 1}$

When they are solved, the following expressions are obtained:

$$X_{T1} = \frac{-n + \sqrt{\Delta}}{2m}, \qquad X_{T2} = \frac{-n - \sqrt{\Delta}}{2m}$$
 (5)

where: $m=(C-A)^2+D^2$, n=2D(E-B)+AD(A-C), $p=(B-E)^2+A(C-A)(B-E)+B(C-A)^2$ $\Delta=n^2-4mp$;

and
$$R_{T1} = \frac{B - E - DX_{T1}}{C - A}$$
 $R_{T2} = \frac{B - E - DX_{T2}}{C - A}$ (6)

The impedance magnitude can be measured a single time in a two-channel system, as depicted in Fig. 3



Fig.3 Block diagram of a two-channel measurement system.

The formulas become slightly modified - 7.

$$U_{T} = U_{g} \frac{\sqrt{R_{T}^{2} + X_{T}^{2}}}{\sqrt{(R_{T} + R_{d})^{2} + (X_{T} + X_{d})^{2}}} \qquad U_{C} = U_{g} \frac{\sqrt{R_{T}^{2} + (X_{T} + X_{d})^{2}}}{\sqrt{(R_{T} + R_{d})^{2} + (X_{T} + X_{d})^{2}}}$$
(7)

after transformations and substitutions, as above:

$$X_{T}^{2} + R_{T}^{2} + AR_{T} + FX_{T} + B' = 0$$
(8)
(C - A)R_T + DX_T + E - B' = 0

where:

$$A = \frac{2 a R_{d}}{a - 1}, \qquad B' = \frac{a R_{d}^{2} + a X_{d}^{2}}{a - 1}, \qquad C = \frac{2 b R_{d}}{b - 1}, \qquad D = 2X_{d}$$
$$E = X_{d}^{2} + \frac{b R_{d}^{2}}{b - 1}, \qquad F = \frac{2 a X_{d}}{a - 1}$$

When they are solved, the following expressions are obtained:

$$X_{T1} = \frac{-n' + \sqrt{\Delta'}}{2m'}, \qquad X_{T2} = \frac{-n' - \sqrt{\Delta'}}{2m'}$$
 (9)

where:

C - A

m'=(C-A)²+(F-D)², n'= 2(F-D)(B'-E) +A(C-A)(F-D) +F(C-A)², p=(B-E)²+A(C-A)(B'-E)+B'(C-A)²

$$\Delta^{2}=n^{2}-4m^{2}p^{2};$$
and R_{T1} = $\frac{B'-E + (F-D)X_{T1}}{C_{1}-A_{1}}$
R_{T2} = $\frac{B'-E + (F-D)X_{T2}}{C_{1}-A_{1}}$
(10)

2. MEASUREMENT RESULTS

C - A

A series of measurements was taken of the transducer impedance module without the additional element X_d and with the element added in series X_d (L_d inductance). The measurements were taken for L_d 0.922mH and 200 μ H and R_d 10k and 330 Ω . Next, analogous measurements were taken in the two-channel system. The results were verified with measurements of the magnitude and phase of impedance on a standard computer set [1] – Fig.4.



Fig.4 Transducer impedance: a – as determined from module measurements, b – as measured in the HP circuit analyser

These figures presents the results for $R_d=10k\Omega$ and $L_d=0.922mH$.

Figure 5 shows a circular chart of impedance and the visible effects of ambiguity on the results. Apart from the actual circular chart, on the left-hand side is a similar circular chart – in the second and third quarter of rectangular coordinates R, X.



Fig.5 Transducer impedance – a) ambiguous results of calculations

After numerous measurements and calculations of transducer resistance \mathbf{R}_T and reactance \mathbf{X}_T it was found that apart from the difficulty that comes with the ambiguity of the results, the proposed method required to known exact values of \mathbf{R}_d and \mathbf{X}_d . They have an important influence on the accuracy of the impedance being determined.

3. CONCLUSIONS

Because it is simple and inexpensive (the elements, equipment), the proposed measurement method is very attractive. It comes with a serious limitation, however, because the results are ambiguous. Because of this, the measurements may have to be checked several times. It is important to develop criteria for eliminating wrong results. The proposed method puts emphasis on the calibration process, which helps to precisely determine the added elements of **Rd** and **Xd**.

REFERENCES

- [1] Zenon Jagodziński "Przetworniki ultradźwiękowe" WKŁ Warszawa 1997.
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