BUBBLE MOTION IN AN ACOUSTIC RESONATOR WITH FLOWING LIQUID

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This work describes results of theoretical study of bubble dynamics in an acoustic field. It is known that there is a radiation force on a bubble in an acoustic resonator. This force can provide conditions of bubble levitation. We consider forces acting on a bubble in a resonator with flowing liquid. It is described in the paper bubble motion in such a condition for different parameters. Results of calculations are discussed.

INTRODUCTION

Gas bubbles have attracted the acousticians' attention almost for a century. A number of books has been dedicated this topic (e.g. [1]). In the recent years, however, the investigations of processes involving gas bubbles in acoustic fields have rather expanded, than declined. The gas bubbles can play an important role in different processes such as ones developing in nuclear reactors and heat power plants [1], as well as by sonochemical changes [2]. The sonoluminescence is also associated with bubbles [3,4]. Generally, all these cases involve strong acoustic fields. It is well known, that a radiation force is exerted on a bubble in an acoustic field. This force can cause bubble motion or levitation. In particular, the last behavior occurs in single bubble sonoluminescence experiments [3]. The previous papers mainly considered bubble motion and levitation in a resting fluid acoustic field. The primary objective of the current work is to investigate a bubble motion in a resonator with flowing liquid.

1. RADIATION FORCE EXERTED ON A BUBBLE

It is known, that a ponderomotive force is exerted on an oscillating particle in an applied nonuniform field. In the simplest case neglecting the liquid viscosity we can obtain the following expression for this force exerted on a gas bubble in an acoustic field. For a small bubble ($R<<\lambda$) in an acoustic pressure field $p = p(\vec{r})e^{i\omega t}$, the small-amplitude oscillation equation is [1]:

$$\Delta\ddot{R} + \omega_0^2\Delta R + \varepsilon\Delta\dot{R} = -\frac{p(\vec{r})}{\rho R_0}e^{i\omega t}, \qquad \Delta R = R - R_0, \tag{1}$$

where $\omega_0^2 = \dfrac{3\gamma P_0}{\rho R_0}$ is the monopole resonance frequency for a bubble with the equilibrium radius R_0, γ is the ratio of specific heats for the gas inside the bubble, and ε is to account for energy losses by the bubble oscillations. The instantaneous force exerted on the bubble is $(1/2)W\ \nabla p^*$, where $W = \dfrac{4\pi}{3}R_0(1+\dfrac{3\Delta R}{R_0})$ is the bubble volume. The averaged over the oscillation period force is

$$\vec{F}_a = \frac{\pi R_0^3}{3\gamma P_0(1-\omega^2/\omega_0^2 - i\delta)}\cdot\nabla(p)^2 , \tag{2}$$

where $\delta = 1/Q = \varepsilon/\omega_0$ is the damping coefficient, while Q is the Q-factor.

If the acoustic field frequency is less than the bubble resonance frequency ($\omega<\omega_0$), the radiation pressure force is proportional to the pressure gradient, accordingly such bubbles are drawn into the stronger field region. At frequencies above the resonance one ($\omega>\omega_0$) the radiation force pushes a bubble to the weaker field region. The equilibrium (levitation) condition in the acoustic field of a vertically aligned acoustic resonator will imply the equality of the radiation force and the buoyancy force. According to experiments in a vertical resonator the levitation condition can be held for different bubble size at different locations inside the resonator that is to say bubble stratification.

2. A BUBBLE IN A RESONATOR WITH FLOWING LIQUID

In case of a flow in the resonator, some new remarkable effects may develop. Here the equilibrium condition shall additionally comprise a flow force due to the liquid flow. Hereinafter we are providing a more detailed analysis.

Let an acoustic wave has been excited in a resonator with the length L and with acoustically perfectly rigid walls (Fig.1). Let that a steady flow with a constant velocity $\boldsymbol{V}$ directed along the x-axis is pumped through the resonator. The bubble buoyancy effects related to the Archimedean force shall not be taken into account. Such a scenario is possible if we assume for instance, that the acoustic resonator is at zero-gravity condition or is aligned horizontally as shown in Fig. 1 or lastly the acoustic radiation force and flow force are far above the Archimedean force.

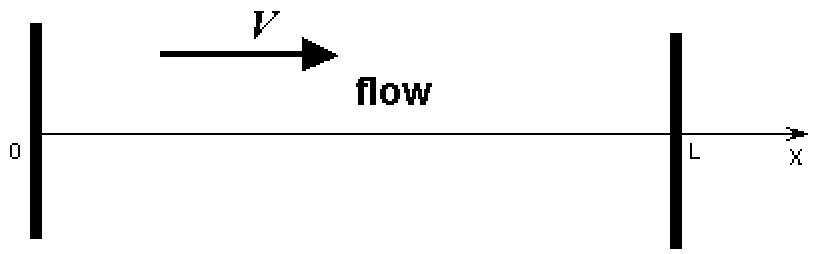

Fig. 1. Acoustic resonator with a liquid flow.

The acoustic field can be presented as two contradirectional plane waves, that is to say $p = 2p_0 \exp(i\omega t) \cos kx$, where p_0 is the plane wave amplitude and $k = \omega/c$ is the wavenumber, while c is the liquid acoustic speed. From the boundary conditions a resonance condition for this system can be obtained. $\lambda/2 \cdot N$ wavelengths (N=1,2,3…) shall be placed on the resonator length.

A bubble in the flow is affected by two forces, that is to say the acoustic radiation force and the flow force (the Archimedean force and gravity force are neglected). The flow force at low Reynolds numbers can be written as follows [5]:

$$\boldsymbol{F}_h = 6\pi\eta R_0 \boldsymbol{V}, \qquad (Re \ll 1), \tag{3}$$

where η is the liquid viscosity.

Equating the forces, determined by (2) и (3):

$$\vec{F}_a + \vec{F}_h = 0,$$

we can easily obtain the bubble equilibrium condition for one point (a counterpart of levitation). After simple conversions this condition takes on the following form (for low Reynolds numbers):

$$x = (-1)^n \frac{1}{2k} \arcsin\left[\frac{3\eta V (R_0^2 \rho\omega^2 - 3\gamma P_0)}{2 p_0^2 k R_0^2}\right] + \pi n, \tag{4}$$

where n is an integer.

If the maximal value of the radiation force is not sufficient to hold the bubble in the flow, the bubble will be carried along the resonator axis with a constant velocity, which shall differ from the flow velocity.

To describe this case we shall pass to the bubble related reference system. Then the equation of motion will take the form of:

$$m\vec{a} = \vec{F}_h + \vec{F}_a.$$

Assuming, that the accelerated bubble motion can not last long, since a steady motion shall be reached soon, we can arrive to the following equation of motion for a bubble in the flow:

$$m\vec{a} = \vec{F}_h + \vec{F}_a = 0.$$

Let denote by V_b the bubble velocity, then

$$\alpha(V - V_b) + A\sin 2kx_b = 0\,. \qquad (5)$$

We have denoted by α and A the factors of the flow force and the acoustic force respectively. If the radiation force is absent, the bubble will apparently move with the flow velocity.

After appropriate conversions we can obtain an expression for the bubble velocity:

$$V_b \equiv \frac{dx_b}{dt} = V + \frac{A\sin 2kx_b}{\alpha}\,. \qquad (6)$$

By integrating equation (6) we can obtain the time dependence of the bubble coordinate x_b. Let the bubble is at $x = x_0$ at the moment $t = 0$. Then the expression for x_b can be presented in a dimensionless form:

$$x_b(t,x_0) = \frac{1}{k}arctg\left\{\sqrt{1-\xi^2}\cdot tg\left[kVt\sqrt{1-\xi^2} + arctg\left(\frac{tg(kx_0)+\xi}{\sqrt{1-\xi^2}}\right)\right] - \xi\right\}, \qquad (7)$$

where $\xi = A/\alpha V$.

Let us consider the function $f(t,x_0) = dx_b/dx_0$. This function has the following context: if $f(t,x_0) < 1$, the bubbles are accumulated along the resonator axis, while if $f(t,x_0) > 1$ rarefication occurs. In other words, we assume that at the initial time two bubbles are introduced into the resonator at x_0 and $x_0 + dx_0$. At the moment t the distance between this bubbles dx is obtained. Using equation (7) plots of $f(t,x_0)$ have been made.

Fig. 2 shows the field distribution. The labeled points are the ones, where the observations of the bubbles at times selected for the numerical example were carried out.

Fig. 3 shows $f(t,x_0)$ vs bubble introduction coordinate for different bubble observation points and by acoustic force to flow force relation = 1/2.

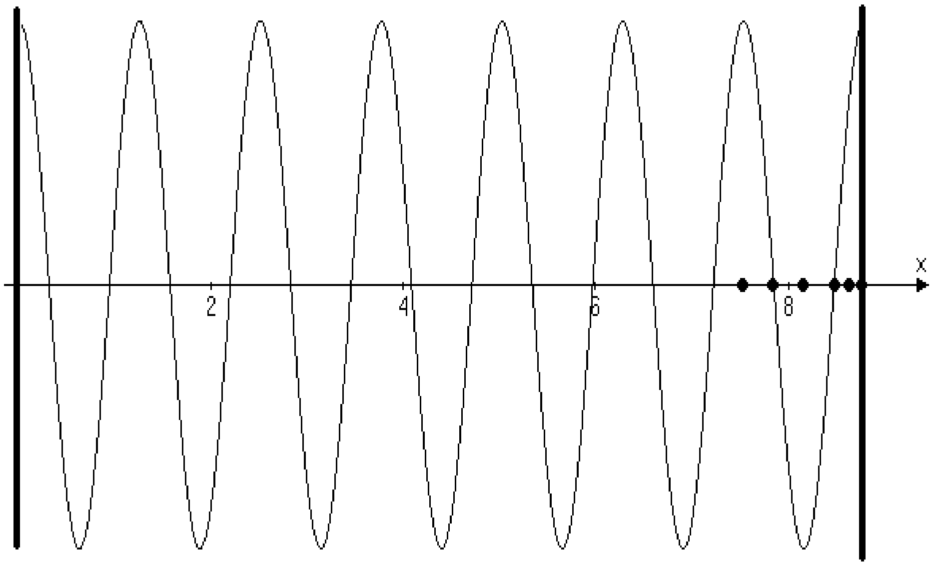

Fig. 2. Sound pressure distribution in a resonator.

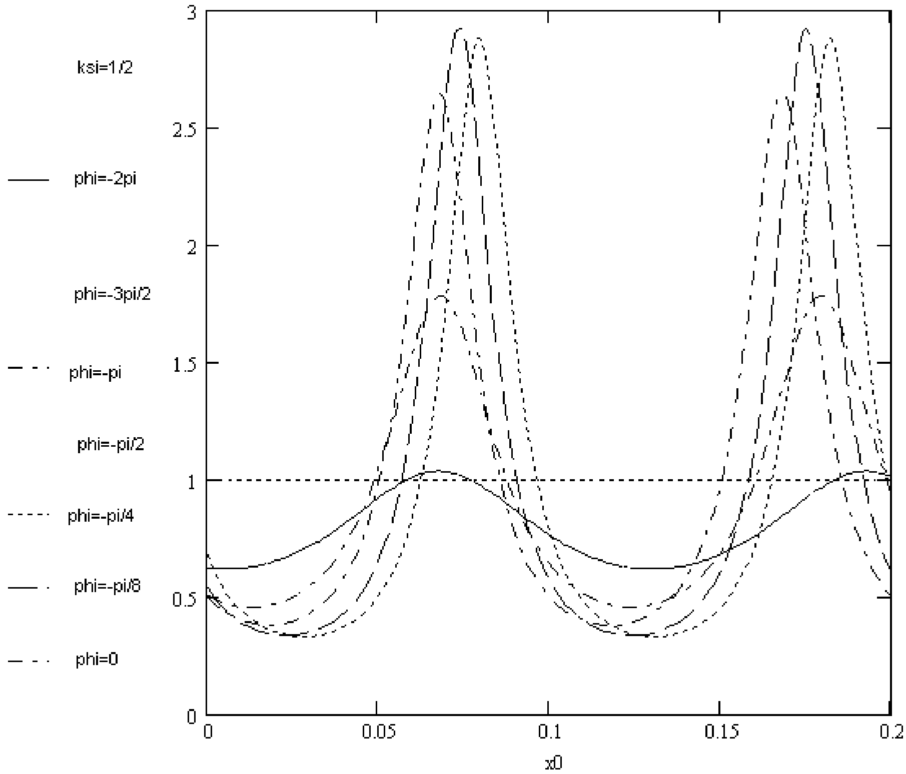

Fig. 3. A plot of $f(t,x_0)$ for severeal observation points in the resonator, ξ=0.5.

Fig. 3 shows clearly, that the bubble introduction at different resonator phases can lead either to bubble accumulation or to rarefication.

CONCLUSION

The effects of gas bubble introduction into an acoustic resonator containing moving medium has been considered in this paper. The equilibrium (levitation) condition for a bubble in the resonator has been obtained. It gas been shown, that in case when the acoustic radiation force is low as compared to the flow force, the bubbles are being dragged by the flow, while bubble accumulation or rarefication depending on the bubble introduction point are possible.

This work was supported by RFBR (04-02-16562, 04-02-17187) and by RS (RS-1641.2003.2, RS-838.2003.2).

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