PRESSURE DISTRIBUTION AND BEAM PATTERN OF THE PARAMETRIC ACOUSTIC ARRAYS

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The aim of this paper was analysis of the acoustic pressure distribution along sound beam and theoretical investigations of the beam pattern of the parametric acoustic arrays. The changes of pressure amplitude for different frequency waves along the sound beam were investigated. The mathematical model was built on the basis of the KZK equation. The problem was considered as an axial symmetric one. To solve the problem the finite-difference method was applied. The mathematical model and some results of numerical investigations are presented.

INTRODUCTION

The finite amplitude waves interaction problem is considered in both experimental and theoretical investigations. The formation of the different frequency waves, another that primary one is the effect of waves interaction [1, 2]. Especially appearance of the difference frequency wave has very important practical application. The generation of it has application in construction and investigations of the parametric acoustic arrays. Narrow beam pattern with low frequency for high frequencies of primary waves and small size of the transducer are the most important characteristics of them.

The mathematical model of the finite amplitude waves interaction problem is often built on the basis on the KZK equation. The equation describes the acoustic pressure changes along the sound beam. This equation allows including nonlinearity, dissipation of medium and sound beam diffraction. Assuming axial symmetry of the source it is comfortably to solve this equation in cylindrical coordinates.

The main aim of the paper was numerical analysis of beam pattern of the parametric acoustic arrays. The paper presents mathematical model and some examples of theoretical investigations. The pressure amplitude changes for different frequency waves as a function of distance from the source and pressure amplitude distribution at horizontal section were studied. The beam pattern changes for parametric acoustic array were investigated, too.

1. MATHEMATICAL MODEL

We assume that the circular transducer with radius equal a[m] is placed in plane yOz and two different frequency waves are propagated in the x direction (Fig. 1). The problem is solved in the cylindrical coordinates (x,r).

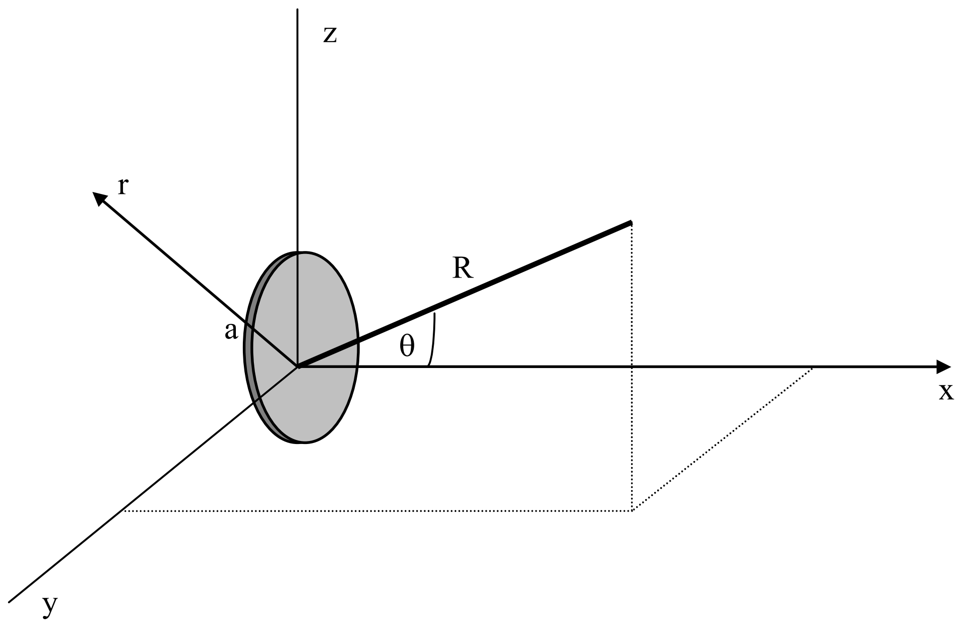

Fig. 1. The geometry of the problem

Pressure distribution on the source is defined as a sum of two harmonic functions with the same amplitudes and frequencies f_1 and f_2 respectively:

$$p'(x=0,r,\tau)=\begin{cases}-p_o\sin 2\pi f_1 - p_o \sin 2\pi f_2 & for \quad r\le a\\ 0 & for \quad r>a\end{cases} \tag{1}$$

The mathematical model is built on the basis of the KZK equation:

$$\frac{\partial}{\partial\tau}\left(\frac{\partial p'}{\partial x}-\frac{\varepsilon}{\rho_o c_o^3}p'\frac{\partial p'}{\partial\tau}-\frac{b}{2\rho_o c_o^3}\frac{\partial^2 p'}{\partial\tau^2}\right)=\frac{c_o}{2}\left(\frac{\partial^2 p'}{\partial r^2}+\frac{1}{r}\frac{\partial p'}{\partial r}\right) \tag{2}$$

where $p'=p-p_o$ denotes an acoustic pressure, variable $\tau=t-x/c_o$ is the time in coordinate system fixed in the zero phase of the propagating wave, ρ_o – medium density at rest, c_o – speed of sound, ε – nonlinearity parameter, b – dissipation coefficient of the medium.

The solution of Eq. (2) is looked for inside the cylinder with radius r_{max} for distance from the source to x_{max} , i.e. in space:

$$S=\left\{(x,r)\in R^2:\ x\in[0,x_{\max}],\ r\in[0,r_{\max}]\right\}$$

This radius must be suitably big for investigated distances from the source.

The finite-difference method is used to solve the problem numerically. To solve Eq. (2) the rectangular net is constructed. In this way function p'(x,r,τ) is discretized in both space and time. The net is defined in following form:

$$x_n = n\Delta x, \quad r_k = k\Delta r, \quad \tau_m = m\Delta\tau$$
$$\Delta x = \frac{x_{\max}}{N_x}, \Delta r = \frac{r_{\max}}{N_r}, \Delta\tau = \frac{\tau_{\max}}{N_\tau} \tag{3}$$

where $n=0,1,\ldots,N_x-1$, $k=0,1,\ldots,N_r-1$, $m=1,2,\ldots,N_\tau$.

The pressure changes along the sound beam are the result of computer calculations. The spectrum changes which are equivalent to acoustic pressure changes are calculated using the fast Fourier transform (FFT algorithm).

Knowledge of the pressure amplitude distribution along the sound beam allows calculating the beam pattern of these waves. The beam pattern for fixed frequency wave is calculated as a ratio:

$$D_f = \frac{p_f(R,\theta)}{p_f(R,\theta=0)} \tag{4}$$

where $p_f(R,\theta)$ denotes value of pressure of the fixed spectrum component at the distance R from the source and an angle θ is measured from the x axis (Fig. 1)

2. NUMERICAL INVESTIGATIONS

The numerical calculations were carried out using own computer program that was worked out on the basis of obtained algorithm.

The formation of the different frequency waves, another that primary one, is the result of waves propagation in the same direction. Figure 2 presents normalized on-axis pressure amplitude for different frequency waves as a function of distance from the source. Curve number 1 shows the pressure amplitude for f_1 frequency wave. Curve number 2 presents analogous result obtained for f_2 frequency wave. The result of calculations obtained for sum frequency wave presents next curve. The last curve presents the on-axis pressure amplitude changes for difference frequency wave. The calculations were carried out assuming that the circular transducer with radius a=25 mm was the source of two finite amplitude waves of pressure p_o=150 kPa and frequencies f_1=600 kHz and f_2=800 kHz respectively.

Transverse normalized pressure distribution of the difference frequency wave at distance x=0.75 m and x=1 m from the source is presented at Fig. 3.

Figures 4 and 5 show normalized pressure amplitude distribution at horizontal section for f_1 and difference frequency wave respectively.

The pressure amplitude changes along the sound beam for different frequency waves were presented till now. Beam pattern was calculated on the basis of these results. Figure 6 shows the beam pattern of the f_1 and difference frequency wave as a function of angle θ. Calculations were carried out for two fixed distances from the source.

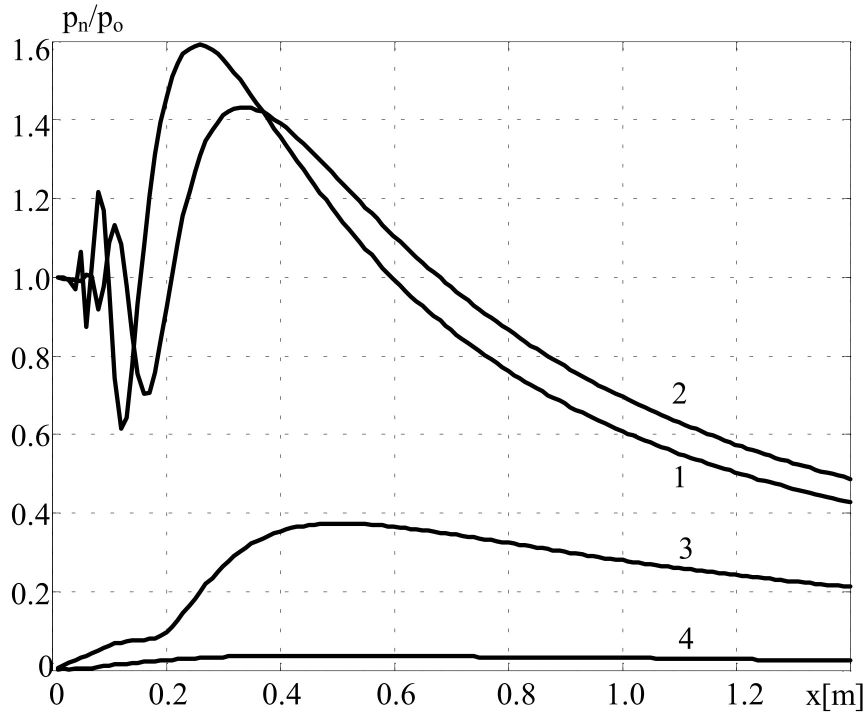

Fig. 2. Normalized on-axis pressure amplitude for f_1 (1), f_2 (2), sum (3) and difference (4) frequency waves as a function of distance from the source

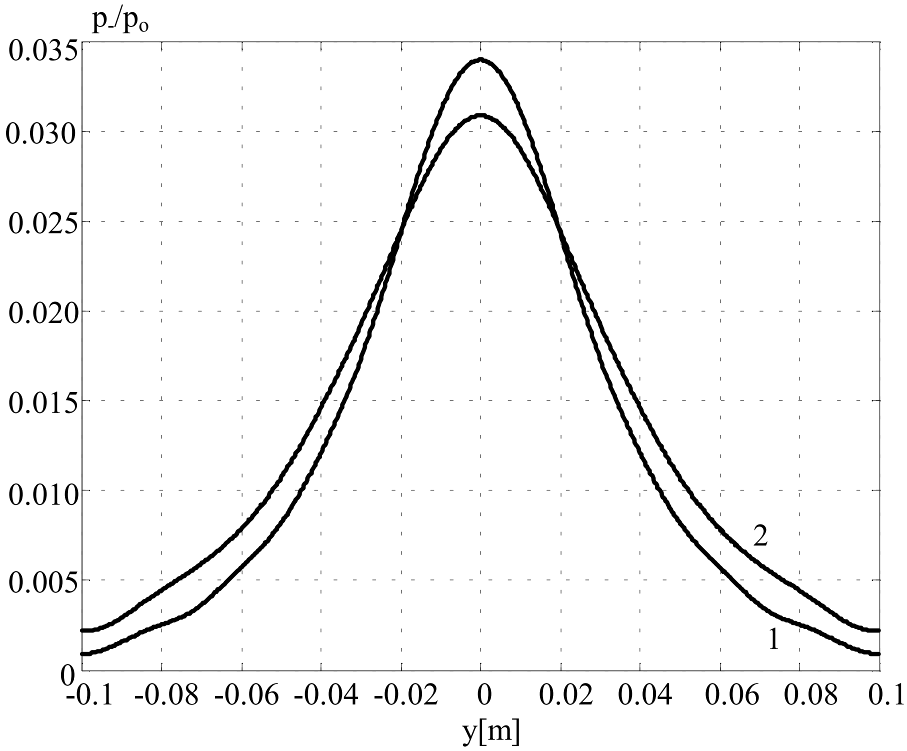

Fig. 3. Transverse normalized pressure distribution of the difference frequency wave at distance x=0.75 m (1) and x=1m (2) from the source

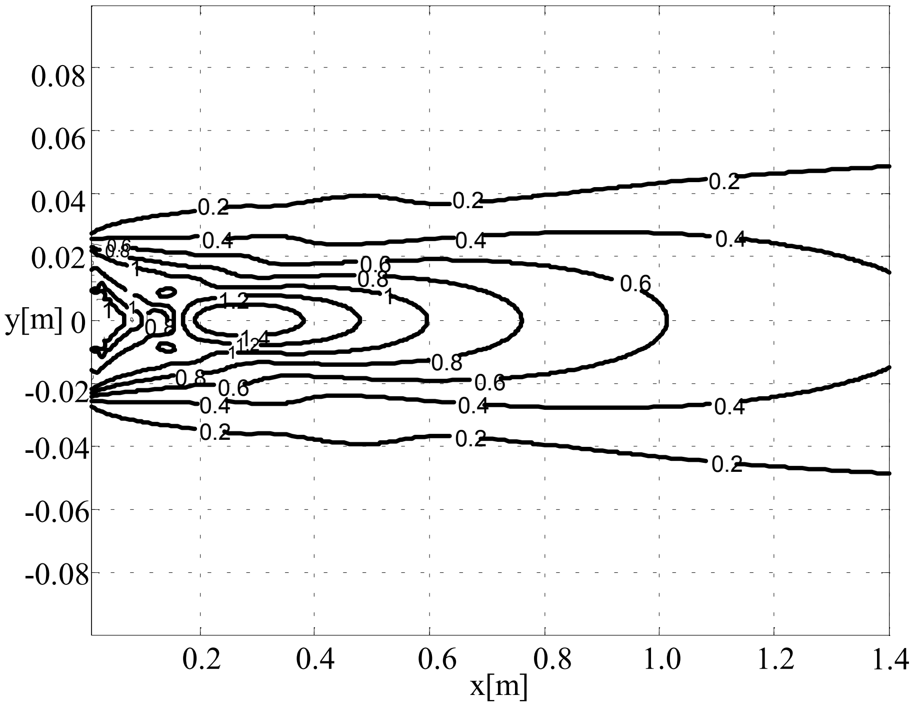

Fig. 4. Normalized pressure amplitude distribution for f_1 frequency wave at horizontal section

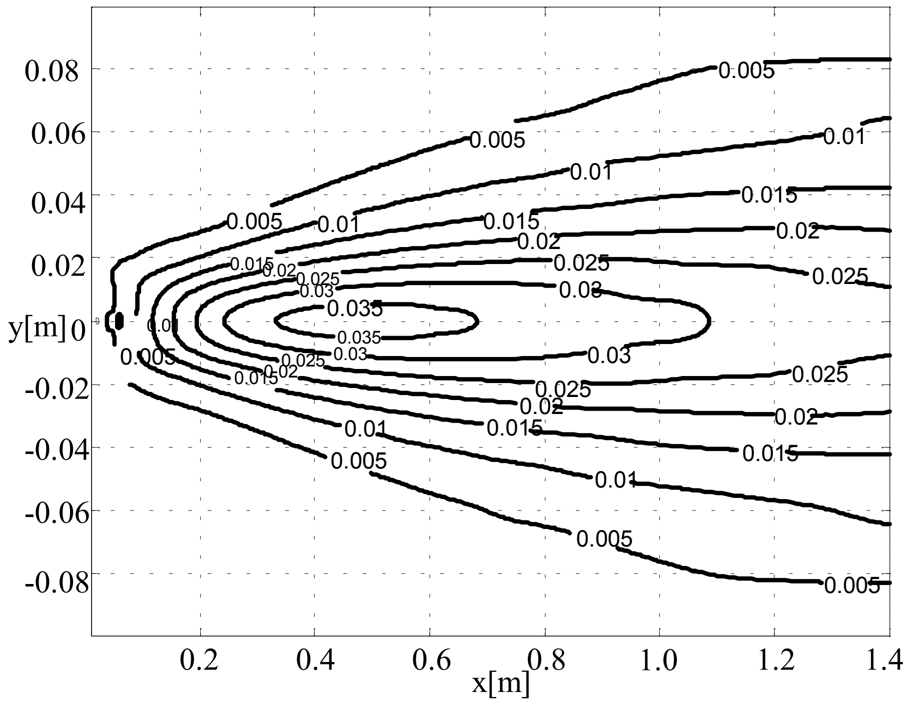

Fig. 5. Normalized pressure amplitude distribution for difference frequency wave at horizontal section

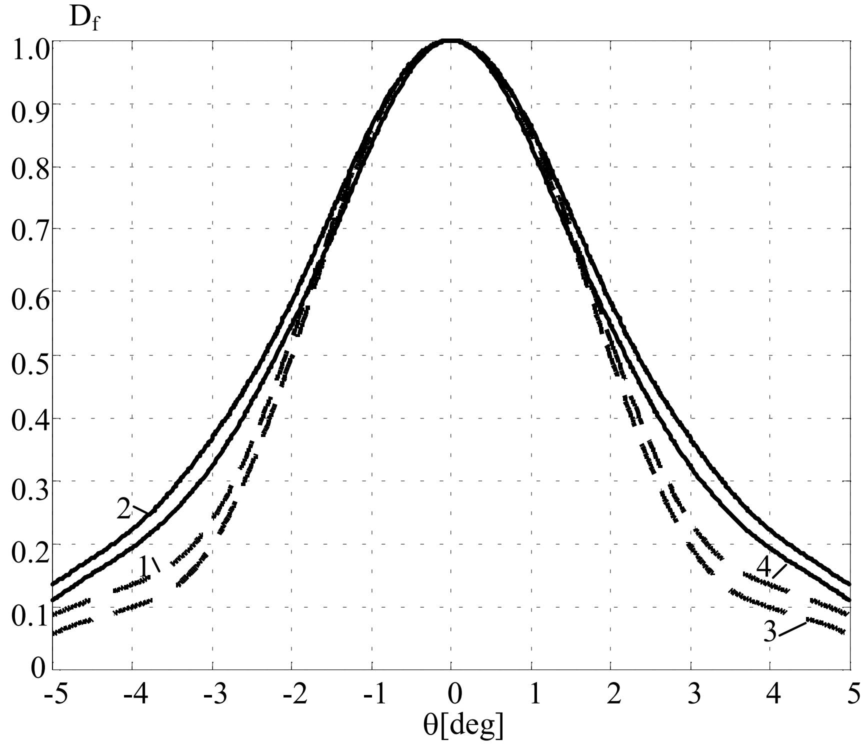

Fig. 6. Beam pattern of the f_1 and difference frequency wave: 1 - f_1, R=0.75 m; 2 - difference, R=0.75 m; 3 - f_1, R=1 m; 4 - difference, R=1 m

3. CONCLUSIONS

The problem of pressure distribution and formation of beam pattern for parametric acoustic arrays was considered. Mathematical model and some examples of numerical calculations were presented. Mathematical model was built on the basis on the KZK equation. The finite-difference method was used to solve the problem numerically.

Proposed method can be used to analyse the beam pattern changes for different values of source and medium parameters.

Correct choice of the numerical parameters (step sizes and values of space parameters) is very important during numerical calculations. Exact analysis of the results of numerical calculations shows that values of the step sizes have greater influence for accurate value of pressure amplitude than for beam pattern one.

ACKNOWLEDGEMENTS

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