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Mathematical model and numerical computations of transient pipe flows with fluid-structure interaction

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Abstract

Transient flows in closed conduits are of interest from over a century, but the dynamic interaction between the fluid and the pipe is taken into consideration more thoroughly just from a few decades. A standard model of the phenomenon consists of fourteen first order partial differential equations (PDE), two for a one-dimensional (1D) liquid flow and twelve for 3D pipe motion. In many practical cases however, a simpler four equations (4E) model can be used, where 1D longitudinal pipe movement is assumed. A short description of waterhammer event with fluid-structure interaction taken into account is presented in the article. The 4E mathematical model is presented in detail with the assumptions and main algorithms of computer program that has been developed. Two phase flow is assumed not to take place, but the friction between the liquid and the pipe wall are taken into consideration. A method of characteristics (MOC) with time marching procedure is employed for finding the solutions, but instead of direct solving the resulting finite difference equations (FDE) the "wave method" is proposed. Some other important elements of the algorithm are presented and selected results of numerical computations as well.

Keywords: Waterhammer; Transient pipe flow; Fluid-structure interaction (FSI); Numerical modeling; Method of characteristics

Nomenclature

A_c, A_s	–	fluid and pipe cross section area, m^2
c, c_s	–	fluid and pipe elastic wave celerity, m/s
D	–	inner diameter of a pipe, m
E	–	Young modulus of pipe material, Pa

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e	–	thickness of a pipe wall, m
G	–	shear modulus of pipe material, Pa
g	–	gravity constant, m/s^2
I	–	area moment of inertia, m^4
I_0	–	polar area moment of inertia, m^4
K	–	bulk modulus of liquid, Pa
L	–	length of a pipe, m
M	–	moment of force at pipe section, Nm
p	–	pressure, Pa
v	–	liquid velocity, m/s
w	–	pipe longitudinal velocity, m/s
Q	–	force acting at pipe section, N
t, x	–	time and space coordinates, s, m

Greek symbols

α	–	angle between the pipe and horizontal direction
λ	–	Darcy-Weisbach friction factor
ν	–	Poison ratio of pipe material
ρ	–	liquid density, kg/m^3
ρ_s	–	pipe material density, kg/m^3
σ	–	longitudinal stresses in pipe material, Pa
τ_s	–	shear friction stresses, Pa
ω	–	angle velocity of a pipe, 1/s

1 Introduction

Unsteady flow in a pipe that may appear in certain circumstances, as sudden valve closure, may cause undesired effects that are well characterized by the alternative name of the process – waterhammer (WH). These effects are strictly connected with the interaction between the liquid and the pipe and its supports. Though the problem of WH has been defined and investigated [1–4] for over a hundred years the classical approach does not take into consideration the dynamic interaction [5–9] between the liquid and the pipe. Instead, not moving structure is assumed with the fluid-pipe interaction being modeled as quasi-static.

The important formula applied for thin-walled pipe circumferential stresses determination as a function of pressure is defined by

$$\sigma_c = p\frac{D}{2e} \; . \tag{1}$$

This formula is still valid in the case of dynamic FSI as usually radial inertia of the liquid and the pipe is neglected. The pressure increase Δp as a result of liquid velocity change Δv in a simple WH event is given by Joukovsky formula

$$\Delta p = -\rho c \Delta v \; . \tag{2}$$

It is worth noticing the similarity of the above equation with that one between the pressure and velocity of a fluid media particle in the acoustic plane wave. Obviously the elastic waves produced by WH events are more complex as the fluid-in-pipe medium is. This fact is visible in the classic formula for the celerity c of elastic disturbances propagation in the liquid, that takes into account the pipe wall elasticity in the shape of the denominator of expression

$$c = \frac{\sqrt{\frac{K}{\rho}}}{\sqrt{1+\psi\frac{KD}{Ee}}} \,, \tag{3}$$

where ψ is a non-dimensional parameter equal or slightly less than 1. Its discussion in different quasi-static cases was made in [1] and one of the formulas developed there and still valid in the dynamic FSI case [7] is given by equation

$$\psi = 1 - \nu^2 \,. \tag{4}$$

In general case of WH to determine p, v as a function of position x and time t one should solve the partial differential equations (PDE) governing the process. This makes also possible taking into account other effects like the friction between the liquid and the pipe wall. It was done at the very early stage of WH theory development and the quasi-steady model of energy losses was employed. This model does not fit the physics of the transient well, but is still quite popular though many unsteady models were developed [2,4], used and examined [10,11]. In fact friction is one of the three elements of FSI pointed out in literature [6,9], though the weakest one. It is also a standard that if the pipe wall movement is taken into account the relative velocity of the liquid and the pipe wall is considered in the friction shear stresses formulas. The shear stresses due to pipe wall friction for quasi-steady approximation are then defined as

$$\tau_s = \lambda\rho\frac{(v-w)\,|v-w|}{8} \,. \tag{5}$$

The second and stronger element of dynamic FSI is the Poisson coupling. Circumferential stresses produce circumferential strains and due to the Poisson effect also the longitudinal ones that propagate as elastic waves in the pipe wall. The reverse process influences the pressure of the liquid and as these changes propagates much faster than the elastic waves in liquid (about 3–4 times in typical situation of water in metal pipe) this effect is known as the precursor wave.

Another element of dynamic FSI occurs when the pipe junctions are able to move what may cause very strong influence on liquid flow parameters. This effect is of great importance especially for non 1D piping as the moving junction may induce coupling, due to boundary conditions, between different modes of pipe vibrations (lateral, longitudinal, torsional).

2 Assumptions and standard model

In Fig. 1 the model of a straight, prismatic pipe element is presented. The main assumptions taken herein are based on the standard model [9].

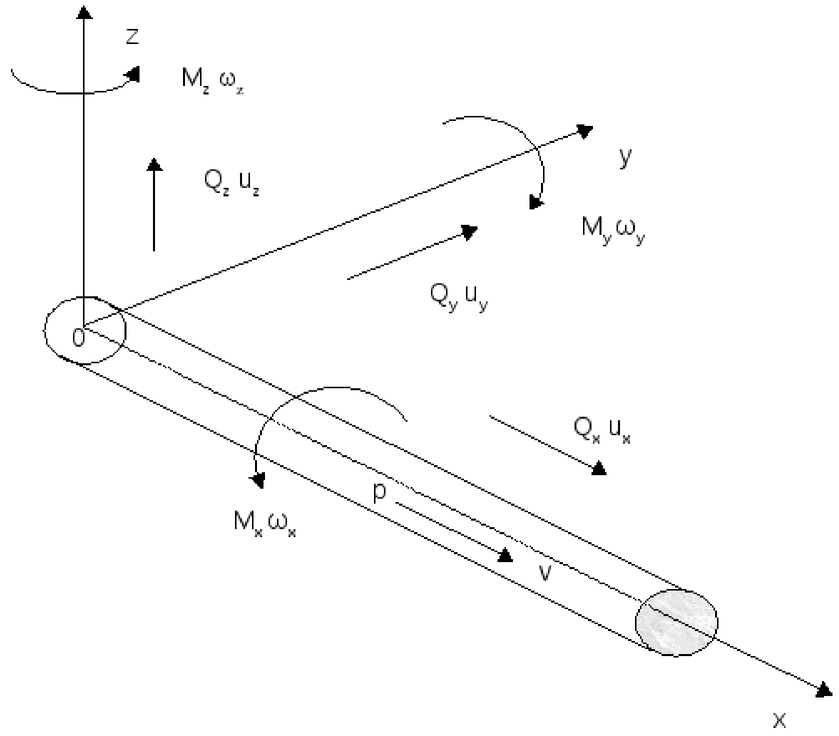

Figure 1. The pipe element and the variables used.

The pipe is assumed being straight, thin and prismatic, of circular cross-section, thin-walled and linearly elastic with buckling not occurring. The flow is one dimensional and relatively slow ($v \ll c$) so the convective terms in fluid equations are neglected. The liquid is weakly compressible, linearly elastic and its density changes are small ($p \ll K$). Low frequency approximation is used what means that the radial inertia of the liquid and the pipe wall are neglected. Friction between the liquid and the pipe wall is taken into account and the quasi-steady model is employed in the present paper. Damping introduced by pipe material [12] and cavitation [13] is assumed not to take place.

One dimensional and one phase flow of the liquid towards the Ox direction is defined by two parameters, the pressure p and the velocity v. The longitudinal pipe movement is coupled with fluid flow due to Poisson effect and is described by the force Q_x and displacement u_x or alternatively, the longitudinal stress σ and velocity w. Torsional rotation of the pipe is assumed not being affected by the liquid. Lateral pipe movement at each of the two planes Ozx and Oyx is described by Timoshenko beam model for the pipe [14,15], with the liquid being accounted as an added mass. The input of liquid moment of inertia and the liquid rotational forces are neglected however. The movement of the system is governed by four groups of equations: 4 equations for longitudinal direction, 2 for torsional and twice 4 equations for lateral vibration of the containing-fluid-pipe. The modes of different groups are uncoupled for an individual straight pipe reach, but the coupling may appear at junctions due to boundary condition. The system movement is than described with 14 variables and governed by 14 linear PDE.

The longitudinal movement of the pipe coupled with the liquid flow is governed by 4 PDE, two equations for the liquid (momentum and continuity)

$$\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} = -g\sin\alpha - \frac{4\tau_s}{\rho D} , \tag{6}$$

$$\frac{\partial v}{\partial x} + \frac{1}{\rho c^2}\frac{\partial p}{\partial t} = 2\nu\frac{\partial w}{\partial x} , \tag{7}$$

and two for the pipe

$$\frac{\partial w}{\partial t} - \frac{1}{\rho_s}\frac{\partial \sigma}{\partial x} = -g\sin\alpha + \frac{\tau_s}{e\rho_s} , \tag{8}$$

$$\frac{\partial w}{\partial x} - \frac{1}{\rho_s c_s^2}\frac{\partial \sigma}{\partial t} = -\frac{\nu D}{2Ee}\frac{\partial p}{\partial t} . \tag{9}$$

The gravity term is also taken into account with α being the angle between the horizontal direction and Ox axis, positive if measured counter-clockwise. The velocity c was defined in Eq. (3) and the velocity of longitudinal elastic waves in pipe wall is

$$c_s = \sqrt{\frac{E}{\rho_s}} . \tag{10}$$

The second group of two equations describe torsional vibrations of the pipe

$$\frac{\partial \omega_x}{\partial t} - \frac{1}{\rho_s I_0}\frac{\partial M_x}{\partial x} = 0 , \tag{11}$$

$$\frac{\partial \omega_x}{\partial x} - \frac{1}{G I_0}\frac{\partial M_x}{\partial t} = 0 . \tag{12}$$

For lateral vibrations at the Oyx plane the Timoshenko beam model is given with the following equations:

$$\frac{\partial w_y}{\partial t} + \frac{1}{m}\frac{\partial Q_y}{\partial x} = 0 \; , \tag{13}$$

$$\frac{\partial w_y}{\partial x} + \frac{1}{T}\frac{\partial Q_y}{\partial t} = \omega_z \; , \tag{14}$$

$$\frac{\partial \omega_z}{\partial t} - \frac{1}{b}\frac{\partial M_z}{\partial x} = -\frac{1}{b}Q_y \; , \tag{15}$$

$$\frac{\partial \omega_z}{\partial x} - \frac{1}{s}\frac{\partial M_z}{\partial t} = 0 \; . \tag{16}$$

The parameters m, b, T, s are respectively mass linear density (mass per unit length), moment of inertia linear density, shear stiffness and bending stiffness:

$$m = \rho_s A_s + \rho A_c \; , \tag{17}$$

$$T = \kappa G A_s \; , \tag{18}$$

$$b = \rho_s I \; , \tag{19}$$

$$s = EI \; . \tag{20}$$

Various formulas can be found in literature for the shear coefficient κ [16,17,18]. Standard approximation for cylindrical thin-walled pipe gives the value of 0.5 [16]. When estimated with the elasticity theory more accurate formulas are obtained [17,18]. They give for κ values slightly greater than 0.5 and can be cast into the following general form:

$$\kappa = 0.5 + \beta \; , \tag{21}$$

where $\beta = \nu/(8+6\nu)$ for [17] and $\beta = \nu/(4+2\nu)$ for [18].
Equations for vibrations at the Ozx plane include also gravity term:

$$\frac{\partial w_z}{\partial t} + \frac{1}{m}\frac{\partial Q_z}{\partial x} = -g\cos\alpha \; , \tag{22}$$

$$\frac{\partial w_z}{\partial x} + \frac{1}{T}\frac{\partial Q_z}{\partial t} = -\omega_y \; , \tag{23}$$

$$\frac{\partial \omega_y}{\partial t} - \frac{1}{b}\frac{\partial M_y}{\partial x} = \frac{1}{b}Q_z \; , \tag{24}$$

$$\frac{\partial \omega_y}{\partial x} - \frac{1}{s}\frac{\partial M_y}{\partial t} = 0 \; . \tag{25}$$

3 The basic model and characteristic equations

In many practical situations the 4E (basic) model is sufficient for the proper description of the system. The Eqs. (6–9) describe coupled and damped elastic waves propagating in fluid-pipe medium and can be transformed according to the method of characteristics (MOC). This transformation results in equivalent equations where only total derivatives with time exist, for the $x(t)$ dependence being the path of the wave. The final CE [5,9,12] can be written in the following form [19].

The "liquid" wave is governed by the compatibility equation C1

$$\frac{d}{dt}(v+Sw)+\frac{\epsilon}{c_1}\frac{d}{dt}\left(\frac{p}{\rho}-S\frac{\sigma}{\rho_s}\right)=-(1+S)g\sin\alpha-(1-R)\frac{4\tau_s}{\rho D}\ , \tag{26}$$

valid for $\epsilon=+1$ (C1+) or $\varepsilon=-1$ (C1-) and $x(t)$ dependence

$$\frac{dx}{dt}=\pm c_1=\epsilon c_1\ , \tag{27}$$

that defines propagation towards opposite directions of the Ox axe. The wave celerity is

$$c_1=\frac{c}{\sqrt{A}}\ . \tag{28}$$

The compatibility equations C2 define the "pipe" wave:

$$\frac{d}{dt}(Rv-w)+\frac{\epsilon}{c_2}\frac{d}{dt}\left(R\frac{p}{\rho}+\frac{\sigma}{\rho_s}\right)=(1-R)g\sin\alpha-(1+S)\frac{\tau_s}{\rho_s e}\ , \tag{29}$$

$$\frac{dx}{dt}=\pm c_2=\epsilon c_2\ , \tag{30}$$

$$c_2=c_s\sqrt{A}\ . \tag{31}$$

The parameters S ($S\geq 0$) and R ($R\geq 0$) are small (in the cases of interest) non-dimensional quantities and they are equal to zero if there is no Poisson coupling. The explicit formulas are:

$$S=\frac{4\nu\gamma}{(1-\gamma+\chi)+\sqrt{(1-\gamma+\chi)^2+4\gamma\chi\nu^2}}\ , \tag{32}$$

$$R=\xi S\ . \tag{33}$$

The form of the CE and formulas for S, R are valid if following condition holds:

$$1-\gamma+\chi\geq 0\ . \tag{34}$$

In the above equations ξ is the ratio of water-to-pipe mass of the same segment, χ is the parameter in the denominator of Eq. (3) and γ the square ratio of longitudinal elastic wave celerities in the open space liquid and in pipe wall:

$$\gamma = \frac{K\rho_s}{E\rho} \ , \tag{35}$$

$$\chi = \frac{KD}{Ee} \ , \tag{36}$$

$$\xi = \frac{D\rho}{4e\rho_s} \ . \tag{37}$$

Values (35) and (36) can be used in Eq. (34) to get the relation

$$\frac{D}{e} + \frac{E}{K} \geq \frac{\rho_s}{\rho} \ , \tag{38}$$

which is seen to be valid for all practical cases.
Parameter A is defined with formula

$$A = \frac{1+\gamma+\chi+\sqrt{(1-\gamma+\chi)^2+4\gamma\chi\nu^2}}{2\,(1+\chi(1-\nu^2))} \tag{39}$$

and in practice is slightly greater than 1 ($A = 1$ if $\nu = 0$).
For the case of water in steel pipe and $D/e = 100$, it can be calculated that $S \approx 0.025$, $R \approx 0.08$, $A \approx 1.05$.

The shape of Eqs. (26) and (29) allows to notice that each of the waves C1 and C2 is not in fact a pure liquid or pure pipe wave but is a coupled wave according to the shape of new variables, generalized velocities $u^{(1)}$, $u^{(2)}$ and stresses $s^{(1)}$, $s^{(2)}$:

$$u^{(1)} = v + Sw \ , \tag{40}$$

$$s^{(1)} = r - \tilde{S}q \ , \tag{41}$$

$$u^{(2)} = Rv - w \ , \tag{42}$$

$$s^{(2)} = \tilde{R}r + q \ . \tag{43}$$

The variables r, q are respectively the normalized (measured in meters per second) pressure and stress according to the following formulas:

$$r = \frac{p}{\rho c_1} \ , \tag{44}$$

$$q = \frac{\sigma}{\rho_s c_2} \ . \tag{45}$$

One can also see that the wave celerities c_1 (Eq. (28)) and c_2 (Eq. (31)) are slightly changed in compartment to the celerities of the original liquid and pipe waves.

Modified S, R parameters are defined with the following equations:

$$\tilde{S} = S\frac{c_2}{c_1} , \tag{46}$$

$$\tilde{R} = R\frac{c_1}{c_2} . \tag{47}$$

Looking at the CE as the equations in new variables (40)–(43) seems to be more convenient as they can be solved in two pairs governing "liquid" and "pipe" waves. The "wave method" requires however the solution of some specific problems which are discussed within Section 5.

4 Physical model of the piping and boundary conditions

Mathematical model presented above can be applied to the real system for simple WH testing. The scheme of the system is presented in Fig. 2 and it consists of pressure tank at the beginning of the pipe, the pipe itself with a number of rigid supports and an instantaneously closing valve rigidly mounted to the foundation at the downstream end.

In such a system three types of boundary conditions (BC) are valid. At the pressure tank the only condition for the structure is

$$w = 0 . \tag{48}$$

Assuming constant pressure p_T of the tank during the transient the condition for the flow is

$$p_T = p + \zeta\frac{\rho v^2}{2} , \tag{49}$$

where ζ is a minor losses coefficient that can also take into account the pressure change due to dynamic pressure difference between the pipe and the tank. At the begining of the pipe, only negative compatibility equations C1–, C2– are valid so with the above BC the four equations can be solved for the four unknowns $p, v, w(= 0)$, σ at the junction. At the other end of the pipe the positive compatibility equations C1+, C2+ are valid and the boundary condition for instantaneously closing valve is

$$w = v = 0 . \tag{50}$$

If the pipe is restrained by a number of rigid supports between its both ends the four BC at each support are the result of equilibrium conditions and are given

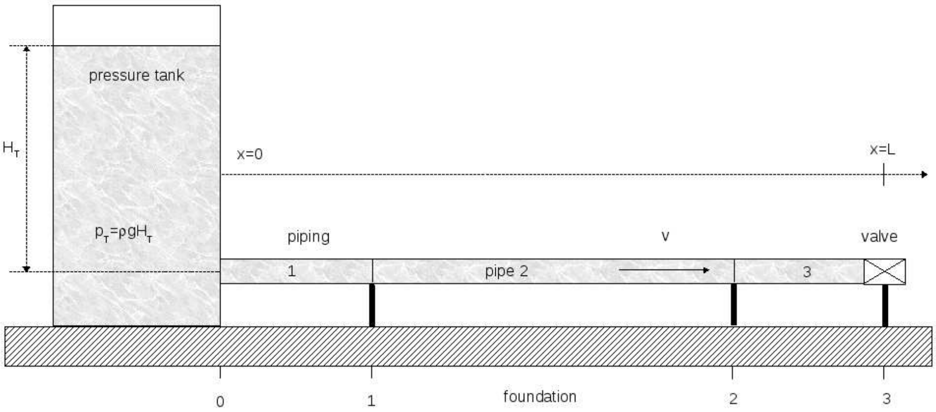

Figure 2. Physical model of the piping.

below for variables of the left (L) and right (R) side of the junction:

$$w_L = w_R = 0\ , \tag{51}$$

$$v_L = v_R\ , \tag{52}$$

$$p_L = p_R\ . \tag{53}$$

The conditions are becoming more complex if there are minor losses, diameter change or elastic mounting at the junctions, but these cases are not considered here. The case of more than two pipes at a junction can be also examined.

5 Numerical method

The goal now is to find the solution for time (t) and position (x) dependence of the variables of the system. The support structure with $(M+1)$ supports $(0\dots M)$ divides the whole pipe into M individual pipes and each of it is solved separately with their variables coupled at the junctions. The standard idea is to use the same time step Δt for the whole piping. The space size Δx of the mesh is determined for each pipe segment and should be the result of time step, pipe parameters and the necessity to fulfil the CFL condition (Courant, Fredrichs, Lewy) for stability and convergence [20,2]

$$C_N = \frac{c\Delta t}{\Delta x} \le 1\ . \tag{54}$$

It is easy to fulfil the above condition for constant celerity and one wave (classic WH), just keeping the Courant number $C_N = 1$, what means the selection of space grid size satisfying the relation

$$\Delta x = c\Delta t \,. \tag{55}$$

In the case of two kinds of waves with different celerities it is not as simple however and a careful method has to be selected, especially if we want to avoid interpolation because of its known disadvantages. As keeping the condition $C_N = 1$ is an optimal method that does not introduce numerical dissipation [2] a wave method of CE integration is proposed. Instead of looking for the four unknowns of four CE directly we can integrate each pair of CE independently and look for solutions in $u^{(1)}$, $s^{(1)}$ and $u^{(2)}$, $s^{(2)}$ variables separately. The true parameters p, v, w, σ can than be found by solving the Eqs. (40)–(43). However it can be done only at such points of t-x plane that are the nodes of both C1 and C2 grids. It is possible only if the ratio of wave celerities is a rational number so

$$\frac{c_1}{c_2} = \frac{m_1}{m_2} \,. \tag{56}$$

The numbers m_1 and m_2 are relatively prime integers and have to be not too large for the method to be practical. To keep them small the celerities of the waves can be slightly adjusted. It is acceptable, as the physical parameters influencing the celerities are measured quantities and can be charged with a certain error. In fact this idea means that we test numerically a slightly different system. But if the difference in parameters does not exceed a few percent the results are expected to be consistent with the true ones. So, to keep the proper shape of Eq. (56) we can slightly change the densities of liquid and pipe material increasing one and decreasing the other. Such changes have to be taken into account also in other formulas (see Section 3). Now if the integers m_1 and m_2 in (56) are known it is possible to construct the C1 and C2 grids. First, let us compute the distance:

$$\delta x = \frac{c_1 \Delta t}{m_1} = \frac{c_2 \Delta t}{m_2} \,. \tag{57}$$

The space sizes of C1 and C2 grids are calculated as:

$$\Delta x_1 = m_1 \delta x = c_1 \Delta t \,, \tag{58}$$

$$\Delta x_2 = m_2 \delta x = c_2 \Delta t \,. \tag{59}$$

So, for each of CE the Courant number is equal to one ($C_{N1} = C_{N2} = 1$). The common nodes for both grids lie at the space separation of

$$\Delta x = m_1 \Delta x_2 = m_2 \Delta x_1 = m_1 m_2 \delta x \tag{60}$$

and the variables p, v, w, σ can be calculated at these points. The length L of individual pipe should be an integer (or even integer for staggered grid) multiplication of Δx distance. Such construction may require approximation, but the error is small and can always be reduced by decreasing the time step Δt.

In Fig. 3 the grid is sketched for the case of $m_1 = 2$ and $m_2 = 3$ (for clarity the paths of the waves are plotted at different time steps) and the common nodes are marked with bold verticals.

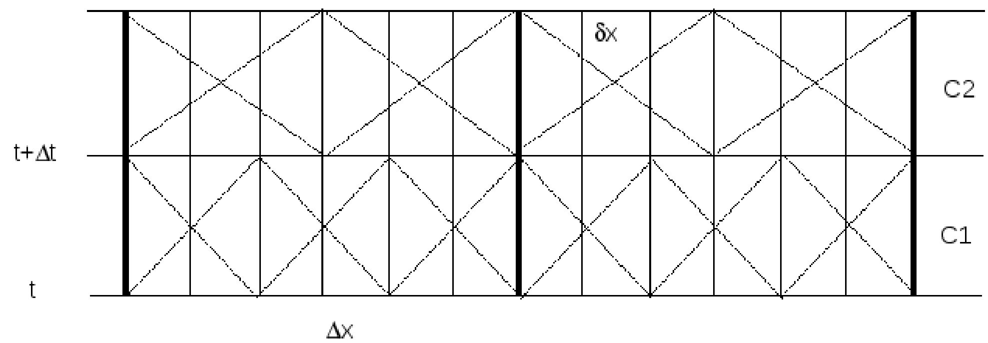

Figure 3. The numerical grid and the paths of C1 and C2 waves for $m_1 = 2$ and $m_2 = 3$.

The solution of each of the CE transformed to finite deference equation (FDE) is simple if there are no non-linear terms. However because of friction such terms may exists. In the classic WH literature [1] this type of FDE are solved analytically for the quasi-steady friction model with a certain scheme of time integration of the losses. It is quite reasonable however to apply a different method. As the friction is small the losses may be computed iteratively with the first iteration taken for the initial velocities. Such method converges quickly and the second iteration is usually a very good approximation [19,21]. It also allows for easy and unified implementation of various, even complex unsteady friction models. There is however another problem to solve. The velocities v and w are both known only at common nodes of C1 and C2 grids, so the interpolation will be required at other points. But this time the disadvantage of interpolation is not crucial as it is used for small friction terms. Moreover, the interpolation of the whole friction term, instead the velocity, may be done which seems to be a good idea from the physical point of view. Iteration method can be also employed for the minor losses determination in BC equations like (49).

6 Computation results

A piping of a physical model presented in Fig. 2 was assumed for computations with various number of rigid supports and the following initial water flow parameters: $p_T = 1$ MPa, $v = 0.66$ m/s. Total length L of a steel pipe, pipe diameter D and pipe-wall thickness e were assumed to be: $L = 74$ m, $D = 37.2$ mm, $e = 2.6$ mm.

At the following figures the computed time dependences of the flow-move variables are presented at the end of the piping (2 m before instantaneously closing valve). All variables (pressures and stresses) on the diagrams are normalized with formulas (44) and (45) to have the units of velocity [m/s]. In Fig. 4 the pressure is plotted for the pipe with no inner supports. One can observe the influence of Poisson effect and the precursor wave as a fast (quasi) rectangular pressure changes of small amplitude interfered with the pure liquid pressure wave (the dashed line is the dependence for the case with no Poisson coupling). The pressure changes have a specific shape which results in higher amplitude at the beginning of each pulse. This effect is much emphasised on the next diagram (Fig. 5), that presents the results for the case of piping rigidly restraint at 48 inner supports. One can see the pressure variations (oscillations) are much faster and the initial increase of it is sharper what was enlarged in Fig. 6. This kind of effects were observed at experiments made at the Hydraulic Machinery Department of the Institute of Fluid-Flow Machinery PAS [10,11]. In the last diagram (Fig. 7) the variation of structural longitudinal velocities and stresses are presented. Analyses of the data from Fig. 7 allows to identify the main oscillations to be the result of a pipe wave travelling and reflecting at boundaries along the ending 4m-long pipe segment. This is strictly consistent with the celerity ratio being $c_2/c_1 = 4$ for this case.

7 Concluding remarks

A description of waterhammer event with fluid-structure interaction taken into account and the standard model (14E) of the phenomenon was shortly presented in the article. The basic model (4E) governed by 4 equations was discussed in a greater detail and the main algorithms of computer program, that has been developed, were presented. The wave method with suitable computational grid construction was proposed for finding the solution of resultant CE. The results of numerical investigations allow to notice that the influence of FSI (Poisson effect) on pressure variation exist even for relatively rigidly restraint piping. The computed high pressure peaks and fast oscillations were also observed at experimental data [10,11].

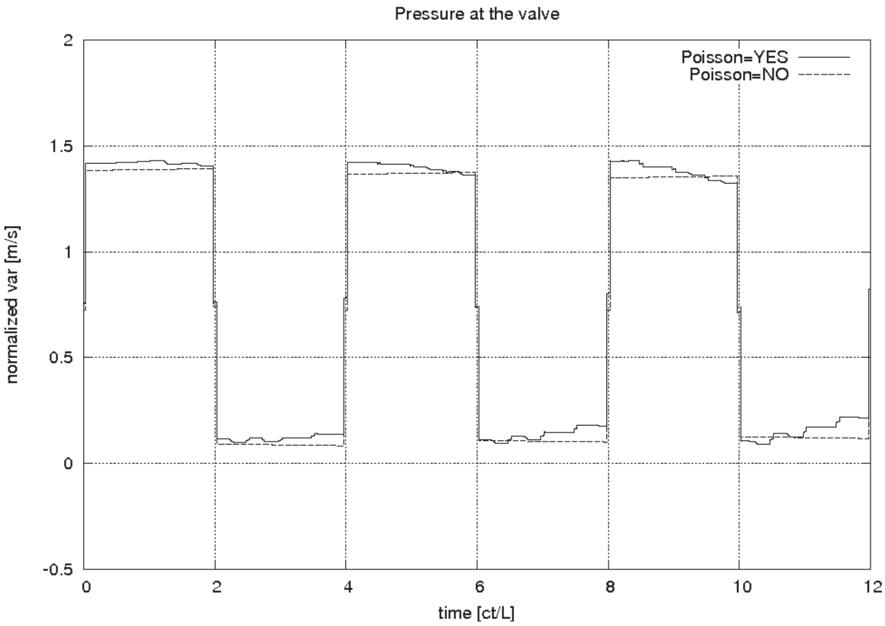

Figure 4. The pressure-time dependence at the end of the pipe with no inner supports.

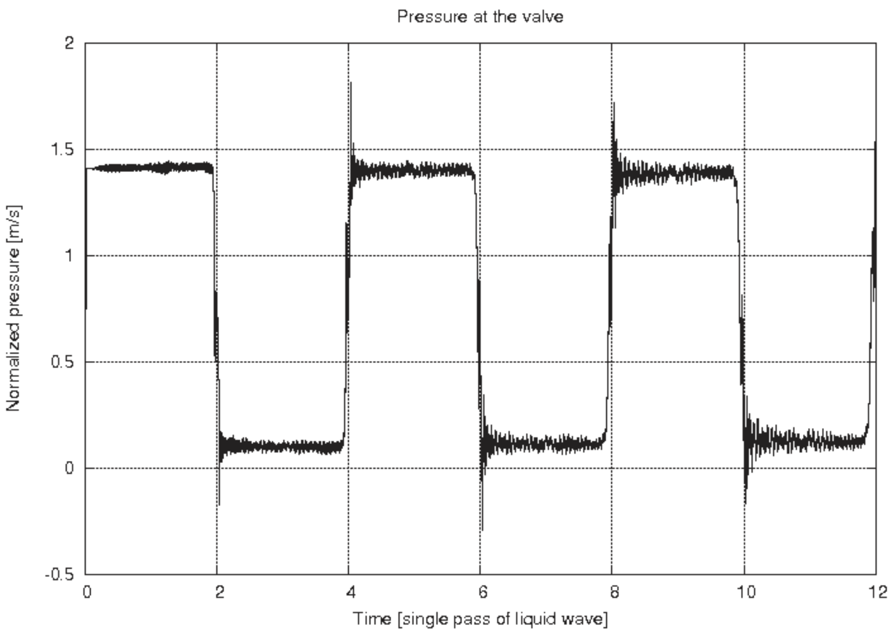

Figure 5. The pressure-time dependence for the pipe with 48 inner supports.

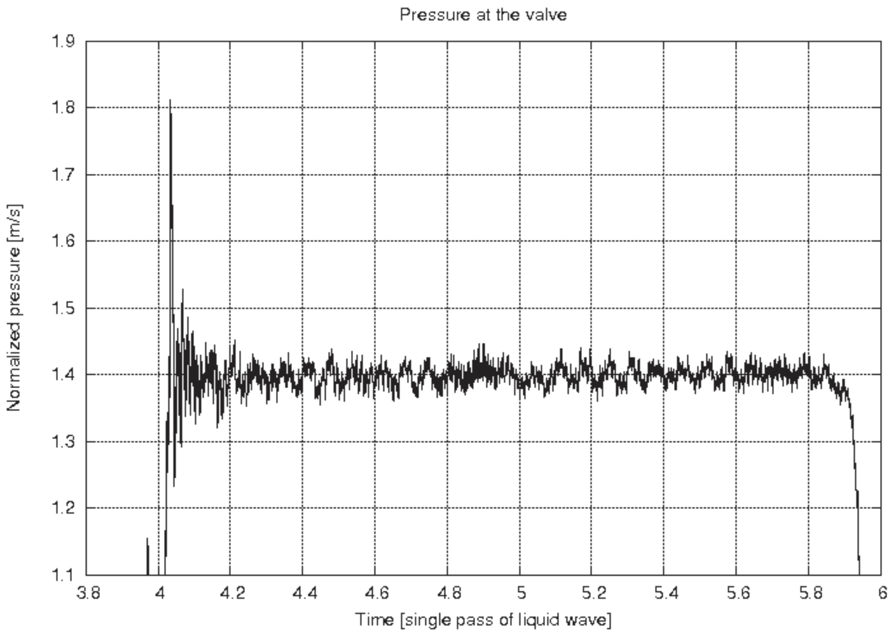

Figure 6. Enlarged part of Fig. 5.

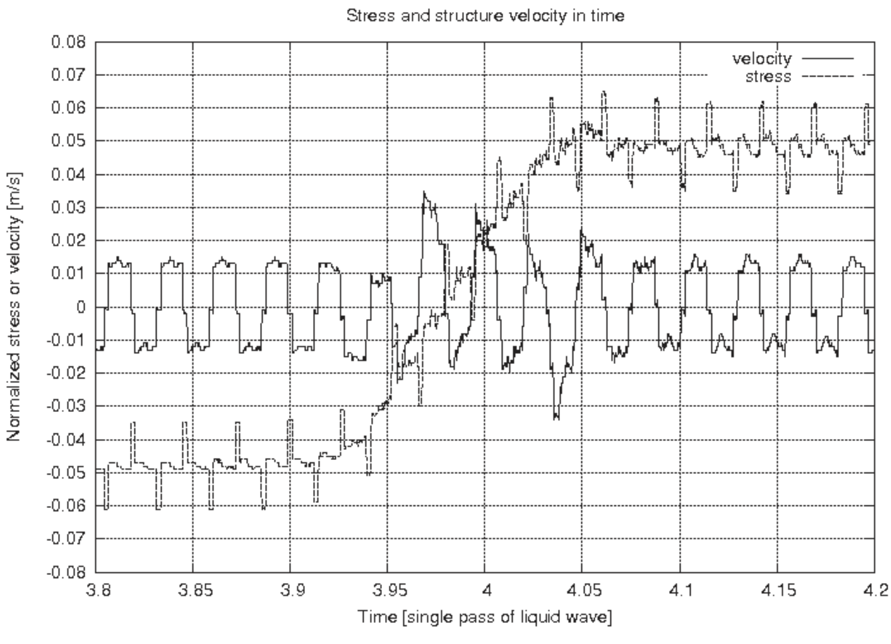

Figure 7. The stress and pipe wall velocity in time for the pipe with 48 inner supports.

The real importance of these effects however has to be appreciated separately taking into account the time scales of these runs.

To make the description of WH-FSI process more accurate some of the assumptions pointed at Section 2 can be relaxed. It is impractical however to do it in one, complex numerical model and they are usually analysed separately for better physical understanding. These problems are discussed in classic WH books [1,2], survey papers [4,6,9] and specialized articles. The way of non-prismatic pipes treatment is the subject of [22]. Thick-walled pipes are discussed in [23] and general analyses of curved pipes can be found e.g. in [16]. Frequency domain approach of WH-FSI process is the subject of [8].

Some other assumptions can be relaxed or generalized for further development of the numerical model and computer program. More adequate models of damping mechanism take into account unsteady liquid – pipe wall friction and structural damping of the pipe and elastic supports. Elastic BC will also allow to simulate the influence of junction coupling on the flow-move parameters, what is especially important when standard model of the phenomenon is implemented. Another goal of intended development is the two-phase treatment of the flow [2,13,24] during the transient.

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References

[1] Wylie E.B., Streeter V.L.: *Fluid Transients.* McGraw-Hill, 1978.

[2] Almeida A.B., Koelle E.: *Fluid Transient in Pipe Networks.* Comp. Mech. Publ, Southampton – Boston & Elsevier AS, London 1992.

[3] Adamkowski A.: *Transients in fluid-flow systems of rotor water machines.* Scientific Journals of the IFFM (Institute of Fluid-Flow Machinery PAS) 534/1493/2004, Gdańsk 2004 (in Polish).

[4] Ghidaoui M., Ming Zao, McIlnnis D., Axworthy D.: *A review of water hammer theory and practise.* Applied Mechanics Review **58**(2005), 49–76.

[5] Wiggert D.C., Hatfield F.J., Stuckenbruck S.: *Analysis of liquid and structural transients in piping by the method of characteristics.* Trans. ASME Journal of Fluids Engineering **109**(1987),161–165.

[6] Tijsseling A.S.: *Fluid-structure interaction in liquid-filled pipe systems: a review.* Journal of Fluids and Structures **10**(1996), 109–146.

[7] Wang Z.M., Tan S.K.: *Coupled analysis of fluid transient and structural dynamic responses of a pipeline system.* Journal of Hydraulic Research **35**(1997), 1, 119–131.

[8] Zhang L., Tijsseling A.S., Vardy A.E.: *FSI analysis of liquid filled pipes.* Journal of Sound and Vibration **224**(1999), 69–99.

[9] Wiggert D., Tijsseling A.: *Fluid transients and fluid-structure interaction in flexible liquid-filled piping.* Applied Mechanics Review **54**(2001), 455–481.

[10] Adamkowski A., Lewandowski M.: *Experimental examination of unsteady friction models for transient pipe flow simulation.* Trans. ASME Journal of Fluids Engineering **128**(2006).

[11] Lewandowski M.: *Friction models in waterhammer analyses.* PhD thesis, IFFM, Gdańsk 2008 (in Polish).

[12] Budny D.D., Wiggert D.C., Hatfield F.J.: *The influence of structural damping on internal pressure during a transient pipe flow.* Journal of Fluids Engineering – ASME **113**(1991), 424–429.

[13] Bergant A., Simpson A.R., Tijsseling A.S.: *Water hammer with column separation: A historical review.* Journal of Fluids and Structures **22**(2006), 135–171.

[14] Timoshenko S., Young D.H.: *Vibration Problems in Engineering*, 3^{rd} ed. Nostrad Comp., 1955.

[15] Meirovitch L.: *Analytical Methods in Vibrations.* The Macmillan Company, New York 1967.

[16] Gutowski R., Swietlicki W.: *Dynamics and Vibrations of Mechanical Systems.* PWN, Warsaw 1986 (in Polish).

[17] Cowper G.R.: *The shear coefficient in Timoshenko's beam theory.* Journal of Applied Mechanics – ASME (1966), 335–340.

[18] Hutchinson J.R.: *Shear coefficients for Timoshenko beam theory.* Journal of Applied Mechanics – ASME **68**(2001), 87–92.

[19] Henclik S.: *Mathematical description and numerical algorithms of computer program for transient flows and FSI modelling* (not published). IFFM internal report No. 369/09 (also 694/08), Gdańsk 2009 (in Polish).

[20] Quarteroni A., Valli A.: *Numerical Approximation of Partial Differential Eaquations.* Springer-Verlag, Berlin – Heidelberg 1994.

[21] Bjoerck A., Dahlquist G.: *Numerical Methods.* PWN, Warsaw 1987 (in Polish). (Also English ed. Prentice Hall, 1974).

[22] Adamkowski A.: *Analysis of transient flow in pipes with expanding and contracting sections*. Journal of Fluids Engineering – ASME **125**(2003).

[23] Tijsseling A.S.: *Water hammer with fluid structure interaction in thick-walled pipes*. Computer and Structures **85**(2007), 844–851.

[24] Adamkowski A., Lewandowski M.: *A new method for numerical prediction of liquid column separation accompanying hydraulic transients in pipelines*. Journal of Fluids Engineering – ASME **131**(2009).