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Generalized non-Newtonian flow of ice slurry through bends

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Abstract

The present paper covers the flow of ice slurry made of the 10.6% ethanol solution through small-radius bends pipes. The study presents the results of experimental research on ice slurry flow resistance in 90° bends in laminar and turbulent flow ranges. The research has made it possible to derive a set of criterial relationships which determine the local resistance coefficients of ice slurry, which is a Bingham fluid and whose flow is treated as a generalized flow of a non-Newtonian fluid.

Keywords: Ice slurry; Flow resistance; Bends, Non-Newtonian fluid

Nomenclature

D – bend diameter of bend pipes, m
d_i – inner pipe diameter, m
K – consistency index, Ns^n/m^2
L – pipe length, m
n – characteristic flow-behaviour index, $n = d(\ln \tau_w)/d(\ln \Gamma)$
p – pressure, Pa
v_m – mean flow velocity in pipes, m/s
De – Dean number: $De_L = Re \left(\frac{d_i}{D} \right)^{0.5}, \quad De_T = Re \left(\frac{d_i}{D} \right)^2$
Re – Reynolds number according to Metzner-Reade, $Re = \frac{\rho_B v_m^{2-n} d_i^n}{8^{n-1} K}$

Greek symbols

Δ – increment
ε_B – quotient of active shear stresses of a Bingham fluid

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Γ	–	tube shear rate, 1/s
μ_p	–	dynamic plastic viscosity coefficient, Pa·s
ρ	–	density, kg/m^3
τ	–	shear stress, Pa
τ_p	–	yield shear stress, Pa
τ_w	–	shear stress at pipe wall, Pa
ξ	–	local resistance coefficient for bends pipes

Subscripts

B	–	Bingham
C	–	critical value
E	–	bend
in	–	inlet
out	–	outlet
L	–	length, laminar
T	–	total, turbulent

1   Introduction

Ice slurry belongs to a group of heat carriers (secondary refrigerants) used in cooling and air-conditioning. Due to its high specific heat capacity, it can be used not only as a heat carrier but also as a heat-accumulating medium. It is a mixture of ice crystals and either water, or water containing a dissolved substance which lowers the freezing point (salt, glycol, alcohol). Ice slurry is a non-Newtonian fluid. The agent is treated as a rheologically stable fluid. The rheological models most frequently assigned to this fluid are the Ostwald-de Waele power model [1,2], the Bingham model [3–6] and the Casson model [7].

Problems with determining flow resistances for Newtonian fluid flow through curved pipes can be divided into two groups. In the case of small-radius bends ($D/d_i < 6$), both friction resistances and local resistances generated by a local change in flow geometry contribute to the drop in pressure [8]. For $D/d_i < 6$, Idilecik [8] suggests that the total resistance coefficient of bends is a sum of the coefficient of local resistance of the bend and the friction coefficient. In this case, the local resistance coefficient is only a function of geometrical parameters of the bend (angle, relative bend diameter). In [9] Ito proposes that a total local resistance coefficient is determined for the flow of non-Newtonian fluids through strongly curved pipes. For small Dean numbers ($De < 91$), Ito makes the total local resistance coefficient directly dependent on the friction resistance coefficient and flow geometry. For Dean numbers of $De > 91$, the local resistance coefficient is a function of the Reynolds number and the geometric parameters of the bend.

The other type of issues related to the flow of non-Newtonian fluids through curved pipes concerns their flow through spirally curved pipes and large-diameter bends. Ito [8,10] considers the pressure drops in coil pipes and large-diameter

bends ($D/d_i \geq 6$) to be dependent on the friction resistance coefficient, which is a function of the Dean number defined differently for laminar and turbulent flow.

The basic works concerning the flow of non-Newtonian fluids through curved pipes include, among others [11–13], which highlight the results of research on pipe coils. The work [12] presents a generalization of the results of research on friction resistance coefficients for Ostwald-de Waele fluids. The results of research presented in [13] concern the flow resistances of a Bingham fluid in coil pipes. The generalized formula for the friction resistance coefficient takes on the form of the Ito relationship [10], corrected by implementing a correction function in the Dean number definition. Paper [11] lists the results of research on flow resistances in curved circular and elliptical ducts. An analysis of the results of research covered in this paper indicates that various authors determine the friction resistance coefficient in curved pipes by means of generalizing the Ito formula. The modification of this relationship involves the implementation of correction functions both in the general form of the Ito formula and in the Dean number definition.

The resistances in ice slurry flow through fittings were the main topic of [14–16]. Norgaard et al. [15,16], when analysing the flow resistances of the entire installation fed with ice slurry of 16% propylene glycol water solution, determined the local resistance coefficient of slurry for bends and 90° elbow pipes. The analysed pipe diameters were: $d_i = 0.021$ and 0.027 m. The paper does not mention the bend diameter of the curved elements. The authors carried out experiments for the flow velocity range of 0.5–1.5 ms^{-1} and for the mass fraction of ice of $0\% \leq x_s \leq 30\%$. The intended values of local resistance coefficients decrease with increase in velocity and grow together with the increment in mass fraction of ice. The values of these coefficients for elbows and mass fraction of ice of $x_s \leq 10\%$ are ca. 1.5 to twice as high as in the case of the bends. For higher mass fractions of ice ($x_s = 30\%$), the ratio equals 1.0–1.5. The data included in the study have been presented in a graphic form and have not been generalized.

The results of research on the flow resistance of ice slurry obtained on the basis of the flow of a 10–12% solution of ethyl alcohol through a bend with a diameter of $d_i = 0.015$ m has been presented in [14]. Flow velocities of the slurry changed within the range of 0.2–0.9 m/s and the mass fraction of ice oscillated within the range of $x_s = 0$–25%. Like Norgaard, also Bel and Lallemand [14] confirm the nature of the changes in the local resistance coefficient, occurring together with the changes in velocity and the mass fraction of ice. Both studies show that for bends, flow velocities of 0.5 m/s and the mass fraction of ice of $x_s = 25\%$; 30%, the measured values of local resistances were respectively 3 and 6 times greater than the values obtained from calculations. A lack of generalizations or information on the bend radius makes it difficult to use the results presented in [14].

Turian et al., in [17], deal with the flow resistance of suspensions (being Ostwald-de Waele fluids: 3.6–12.7% laterite suspension and 10.7–30% gypsum suspension) in various types of fittings (bend, tees, diffusers, etc.). The usefulness of this work lies in the fact that it indicates the variability of the local resistance coefficient in a generalized Reynolds number function according to Metzner-Read. The observations presented in [17] concern pipe diameters of $d_i = 0.025$–0.05 m and relative bend diameters of $D/d_i = 0$–25 for $90°$ elbows. The relationships suggested in the aforementioned work are quite peculiar, as they present an individualized form of the formulas for the total local resistance coefficient for each D/d_i relation. In the work, the authors make a reference to the Ito formula [9], pointing out that the resistances of the flow of water through bends are higher than the values of pressure drops obtained from the Ito formula.

The present paper concerns the flow of ice slurry made up of a 10.6% ethanol solution through the pipe bends. It contains the results of experimental research on flow resistances of the slurry through bends in laminar and turbulent flow regime. The result of experimental research is a series of criterial relationships describing the local resistance coefficient for bend pipes and for ice slurry being a Bingham fluid, whose flow is treated as a generalized flow of a non-Newtonian fluid.

2   Flow resistance measurement method for small-radius bends

The research programme concerning flow resistances in $90°$ bends included measurements for:

- Bends pipes diameters of $d_i = 0.01$; 0.016; 0.02 m, with the bend diameter-pipe diameter ratio of $D/d_i = 1$; 2.

- Mean flow velocities of $0.1 \leq v_m \leq 4.5$ m/s, with the corresponding Reynolds number values of $200 < Re < 13700$,

- Mass fraction of ice in the slurry of $0 \leq x_s < 30\%$.

The study of ice slurry flow encompassed a rheological identification of the ice slurry [6], measurements of pressure drops of the ice slurry in straight pipes [6,18] as well as measurements of flow resistances in bends. A detailed description of the measurement stand (Fig.1) has been provided in [6] and [18].

The measurement of ice slurry temperature is based on data obtained from highly precise resistance sensors with large dimensions Pt100(7013) (HART Scientific, sensor mantle diameter – ca. 5 mm, active length – min. 0.02 m). The flux of the ice slurry has been measured by means of a Danfoss MASSFLO 2100 mass

flow-meter, which also allows for a continuous measurement of the density (and hence also of the mass fraction of ice) of the flowing medium. Pressure drops have been measured using Fuji FKCV pressure-change transducers, with two measurement ranges: 0–32 kPa and 0–1 kPa (accuracy: 0.07% of the measurement range).
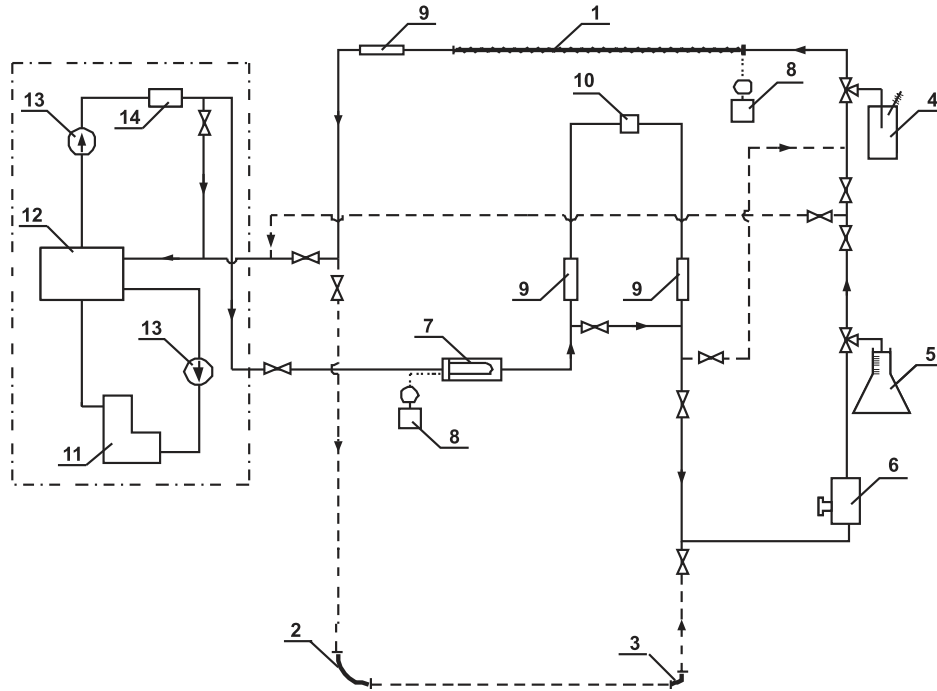

Figure 1. Schematic diagram of the test stand: 1 – heated measuring segment, 2 – bend (D/d_i=2), 3 – bend (D/d_i=1), 4 – calorimetric measurement, 5 – measurement of density, volume change, ice and air content, 6 – mass flow-meter, 7 – heater, 8 – wattmeter and autotransformer, 9 – flow visualization, 10 – air-escape, 11 – ice generator, 12 – accumulation container, 13 – pump, 14 – volume flow-meter.

Pressure drop in the fitting (Δp_E) is made up of pressure losses caused by inlet disturbances of the flow of the agent, pressure losses resulting from the agent's flow through a bend, as well as outlet losses. Separate measurements of each of these components are difficult but not necessary, as for design purposes, it is enough to know the overall pressure drop in a piece of fitting. Flow resistance measurements for bends have been performed alongside investigations of pressure drops in straight-line sections with a length of L [6]. The entire measurement system (Fig.2) used was comprised of a straight-axis inlet and outlet sections (L_{in}, L_{out}), as well as a piece of fitting (bend).

In order to determine the pressure drop in bends, the adopted method was similar to the one presented in [17]. Pressure drop measurements encompassed:
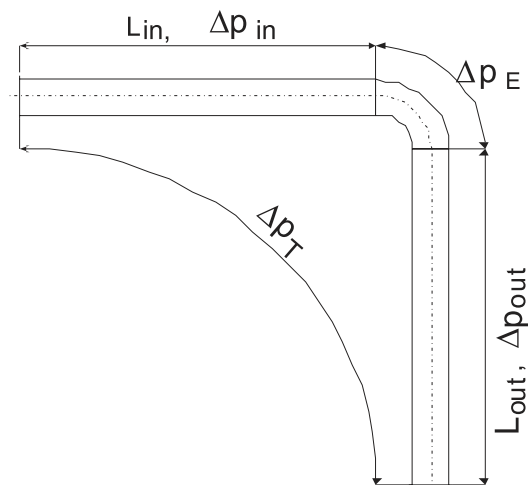
Figure 2. Outline of a measurement stand for investigating flow resistance in bends.

the total pressure drop Δp_T in element under study, as well as friction-induced pressure drop in the straight-axis measurement sections with a length of L. Pressure drops in a bend were calculated using the following formula:

$$\Delta p_E = \Delta p_T - \Delta p_L \frac{L_{in} + L_{out}}{L} \ . \tag{1}$$

Regardless of pipe diameter, the lengths of the inlet (L_{in}) and outlet (L_{out}) sections used in the measurement of pressure drops in bends were ten times the diameter of the pipe. It has to be noted that the entire measurement system was preceded and followed by straight pipes, which stabilized the flow of the agent.

In the research, specific attention was paid to the repeatability of the obtained measurement results, which guarantees the minimum number of random errors. That is why, in the analysis of measurement uncertainties, it was assumed that the determined values correspond to systematic errors resulting from the accuracy of the measurement devices and methods. Systematic uncertainties of intermediate values were calculated as their total differential equation, with respect to all parameters determining their value. Hence the relative systematic uncertainty for mean velocity fits within the range of 0.9–2.1%. In the case of local resistance coefficient for bends, the uncertainty belongs to the range of 2.8–8%.

3   Measurement results

Experimental research on pressure drops in bends has been performed according to a predetermined experiment plan, in which flow resistances have been made

dependent on the velocity of the agent's flow v_m and the mass fraction of ice x_s. Figure 3 presents variation of pressure drops in the investigated bends ($D/d_i = 2$) and ($D/d_i = 1$), for a pipe diameter of $d_i = 0.016$ m. The results of the measurements indicate that for $x_s \leq 10\%$, the influence of the mass fraction of ice on pressure drops in both bends is negligible. In the case of a flow through bends, the effect occurring in straight pipes has not been observed. For pipe flow it has been possible to observe a decrease in the flow resistance of the ice slurry in comparison with the flow resistance of the carrying liquid. For ice slurries with a mass fraction of ice of $x_s \geq 10\%$, there are velocities at which the flow resistance of the slurry is lower than the flow resistance of the carrying liquid.
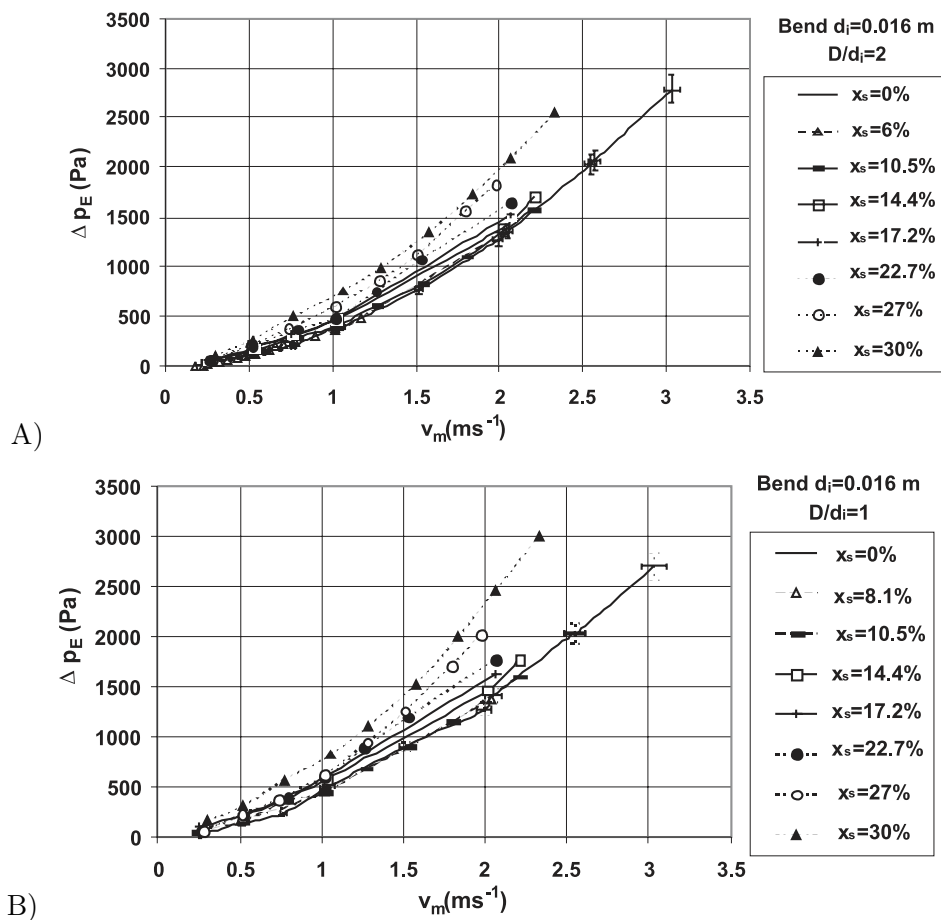

Figure 3. Flow resistance (Δp) as a function of the mean velocity of the slurry for: A) bend $d_i = 0.016$ m, $D/d_i = 2$; B) bend $d_i = 0.016$ m, $D/d_i = 1$.
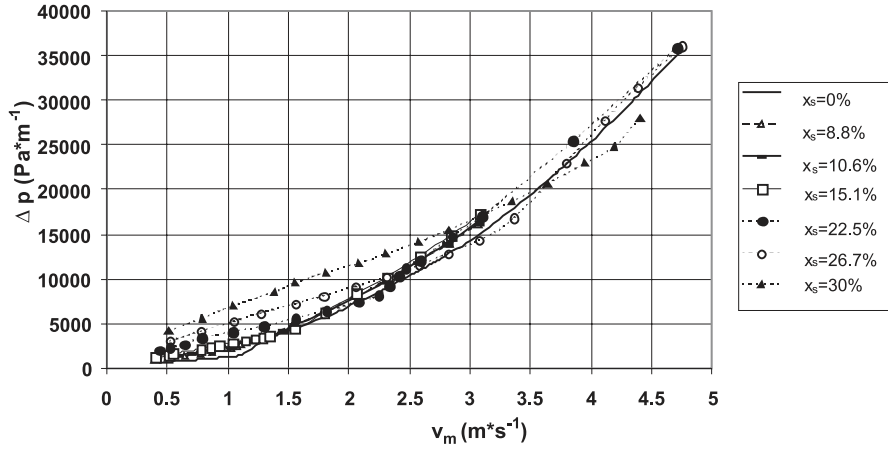
Figure 4. Pressure drop ($\Delta p/L$) as a function of the mean velocity of the ice slurry flow for
$d_i = 0.01$ m.

This effect has been observed for all the examined pipe diameters, as well as for
rectangular cross-section ducts [18]. The phenomenon of the carrying liquid's flow
resistance curve being crossed by the ice slurry resistances curve (Fig. 4), can be
explained by the fact that together with the changing mass fraction of ice, the
physical properties of the suspensions also change, leading to a change in the type
of the movement of the ice slurry.

For the same flow ranges, the flow resistances of the carrying liquid are al-
ways lower than the flow resistances of the slurry. Higher mass fractions of ice
contribute to the fact that in certain velocity ranges, for the same mean velocities,
the ice slurry is only in laminar flow areas, whereas ethanol has already reached
the turbulent flow area, where flow resistances are generally higher. For higher
mass fractions of ice, the change in the movement of the slurry occurs at higher
values of the Reynolds number. The presence of solid particles in the homoge-
neous flow of the slurry makes ice crystals absorb part of the kinetic energy of
the turbulences from the carrying liquid. Flow laminarisation process occurs in
the slurry, as well as a "delay" in the loss of the stability of laminar flow [6,18].
The aforementioned phenomenon has not been observed for low mass fractions of
ice. Ice slurry flow through bends reduces the occurrence of this phenomenon in
the examined fittings, leading to a situation in which, irrespective of the type of
movement, for the mass fraction of ice of $x_s \geq 10\%$, pressure drops in bends are al-
ways higher than the corresponding pressure drops of the carrying liquid (Fig. 3).
Figure 5 presents the influence of pipe diameter, the mass fraction of ice and the
Reynolds number (mean velocity) on pressure drops in bend pipes.

In order to calculate flow resistances in bends, it is necessary to know the substitute local resistance coefficient (ξ_E), defined by the following formula (2):

$$\xi_E = \frac{2\Delta p_E}{\rho_B v_m^2} \ .$$

(2)

For a Newtonian fluid flow through curved pipes, Ito's empirical formulas are the ones most frequently quoted: [8,9,11] and [17]. Ito [9] determines the substitute local resistance coefficient, referred to the overall flow resistance of the medium through a bend.
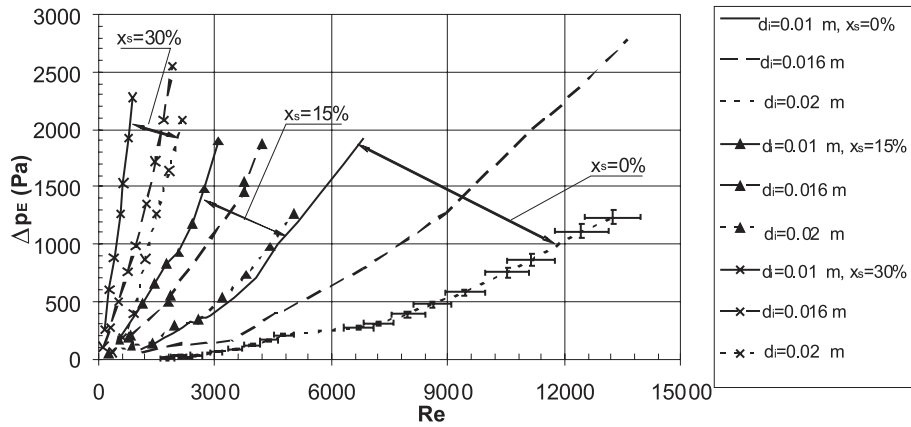

Figure 5. Influence of a generalised Reynolds number, pipe diameter and ice content on pressure drops in a bend ($D/d_i = 2$).

This method of determining the substitute local resistance coefficient is prone to the smallest measurement error. It results from the fact that the coefficient of local resistances caused by the ripping off the near-wall layer, is burdened with an error of determining the total pressure drop, as well as the pressure drop resulting from friction resistances. When determining the local resistance coefficient for bends, Ito's assumptions have been considered as correct. For $D/d_i < 6$, the substitute local resistance coefficient is dependent on the Dean number ($\xi = f(De)$), which for laminar and turbulent flow is defined by Eqs. (3) and (4), respectively:

$$De_L = Re\sqrt{\frac{d_i}{D}} \ ,$$

(3)

$$De_T = Re\left(\frac{d_i}{D}\right)^2 .$$

(4)

In this case it has been assumed that the Reynolds number (Re) is a generalised Reynolds number according to Metzner-Read:

$$Re = \frac{\rho_B v_m^{2-n} d_i^n}{8^{n-1} K} \ . \tag{5}$$

The generalised parameters n and K depend on the rheological properties of the fluid, the mean shear stress at the wall, mean flow velocity, as well as on the dimensionless geometrical constants determined individually for various flow geometries of vertical flow cross-sections. For Bingham fluids and circular cross-section geometry, these parameters are calculated on the basis of the following formulas [19]:

$$n = \frac{1 + \frac{1}{3}\varepsilon_B^4 - \frac{4}{3}\varepsilon_B}{1 - \varepsilon_B^4} \ , \tag{6}$$

$$K = \left(\frac{1}{4}\mu_p\right)^n \tau_w^{1+3n} \left[\frac{1}{4}\tau_w^4 - \frac{1}{3}\tau_w^3 \tau_p + \frac{1}{12}\tau_p^4\right]^{-n} \ . \tag{7}$$

The introduction of the generalised Reynolds number Re Eq. (5) enables to eliminate the direct impact of ice content on the local resistance coefficient, in the $\xi_E(Re)$ dependency for bends. Moreover, such a definition of the Reynolds number allows for a comparison of the authors' own research results with the results of experimental research carried out for other non-Newtonian fluids. One of the basic works concerning pressure drops during the flow of suspensions through bends and other types of fittings is [17]. It lists the formulas for the local losses coefficient as a function of the Reynolds number for standard $D/d_i = 2$ bends, as well as for elbows ($D/d_i \approx 0$). Figure 6 features the results of the authors' own research on bend pipes compared with the results of research presented in [9,17].

Figure 6 indicates that the local resistance coefficients for ice slurry are higher than in the case in which this medium is treated as a Newtonian fluid (Ito formulas [9]), but at the same time they are lower than in the case of coefficients determined according to Turian et al. [17]. Higher local resistance coefficients of ethanol ice slurry, being a Bingham fluid, in comparison to Newtonian fluids are quite obvious.

The local resistance coefficients presented in [17] apply to aquatic laterite and gypsum suspensions. Considering the flow of suspensions as a multiphase one, it is possible to explain the discrepancies between the ξ coefficients for ice slurry, laterite and gypsum by means of the lower values of the dynamic solid particles viscosity coefficients for ice slurry (different interactions of the solid particles and the carrying fluid during their movement, as well as viscosity caused by mutual friction between solid particles).
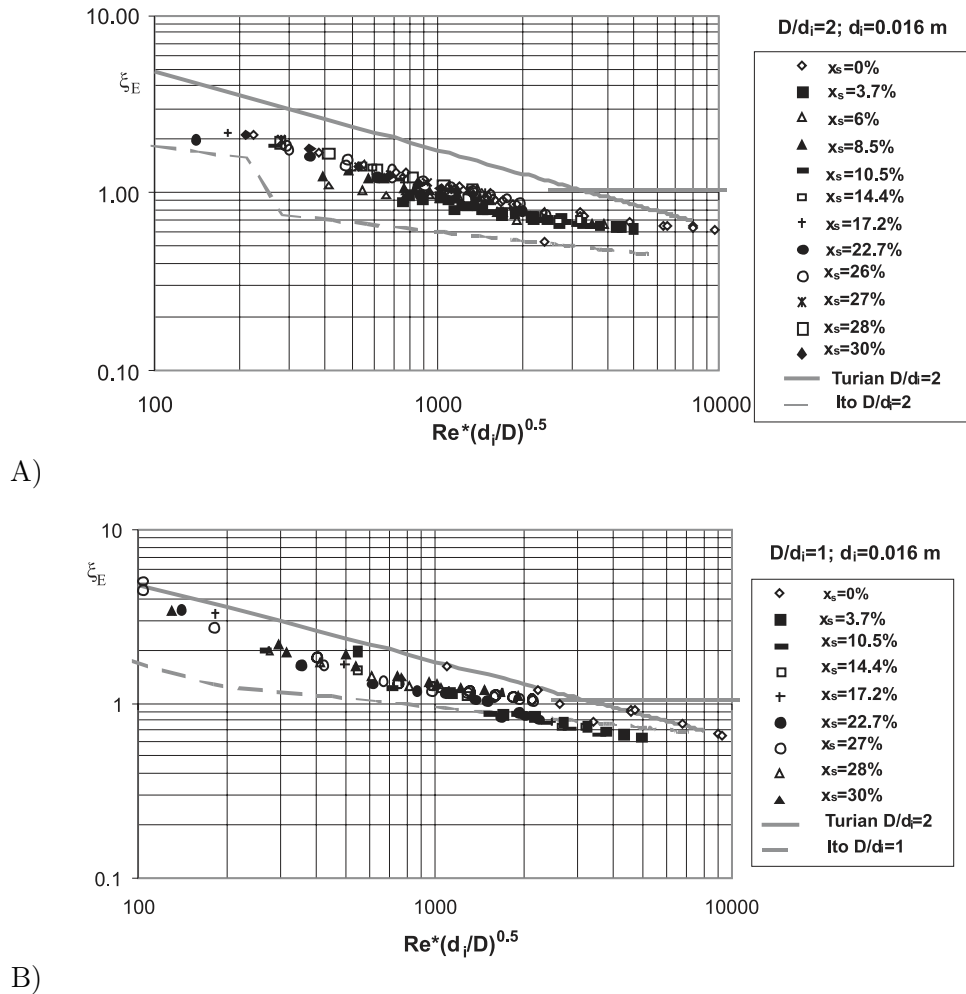
Figure 6. Results of research compared with the results obtained by Turian et al. [17] and Ito [9]: A) bend d_i=0.016 m, D/d_i=2; B) bend d_i=0.016 m, D/d_i=1.

Figure 6 indicates that in laminar flow ranges, regardless of pipe diameter and bend radius, the local resistance coefficients are decreasing functions of the Dean number. In the intermediate and turbulent flow ranges, the monotonicity of the ($\xi_E = f(De)$) function is maintained; however, it is possible to observe a greater stabilization of the local resistance coefficient in comparison to the values of ξ_E for the laminar range.

It needs to be noted that the superposition of the phenomena occurring within the inlet and outlet sections, as well as in the bend element itself, causes instability of the pressure signal, impeding the observation of, e.g. the nature of the changes in ice slurry movements. In this case, stabilization of the local resistance

coefficients for laminar flow has been assumed as the criterion of the movement change. It has been assumed that the Dean number, $De_{CL} = Re(d_i/D)^{0.5} = 2500$, is in this case the criterion of the change in the character of the movement for ice slurry flow through bends.

Both results of research presented in [17] and the authors' own measurements of the flow resistance coefficient for ice slurry in bends presented in Fig. 6, indicate that the substitute local resistance coefficient ξ_E depends on the Dean number and the relative bend diameter (D/d_i). When determining the formula for the local resistance coefficient for bends, it has been assumed that the required formula will correspond with the same relationship for bends $(1 \leq D/d_i \leq 2)$, which has the following form:

$$\xi_E = \xi_E \left(De \; ; \; {d_i}/{D} \right) \; . \tag{8}$$

Moreover, the research on flow resistances in bends concerned mainly the laminar flow range. Outside the laminar range, the flow occurred mainly in the intermediate flow range. Only for pipe diameters of $d_i = 0.016$ and 0.02 m, part of the measurement points are located within the turbulent flow range. The following formula determines the relationship describing the local resistances coefficient in laminar flow range for a generalized flow of ice slurry through bends [9]:

$$\xi_E \left(De_L; {d_i}/{D} \right) = \frac{4.6 \left(\frac{d_i}{D} \right)^{0.33}}{[0.87 + 0.1 * \log \left(De_L \right)]^{8.1}} \; . \tag{9}$$

In the turbulent flow range, the substitute local resistance coefficient for a generalised flow of ice slurry can be derived from the following formula:

$$\xi_E \left(De_T; {d_i}/{D} \right) = \frac{\left(\frac{d_i}{D} \right)^{-0.306} \left[10.6 - 3.45 \Big/ \left(\frac{d_i}{D} \right) \right]}{15.2 De_T^{0.204}} \; . \tag{10}$$

Figures 7 and 8 present a comparison between the measured and calculated local resistance coefficients for ice slurry in bends for laminar and turbulent flow.

With a 98% probability, it can be assumed that the confidence intervals for Eqs. (9) and (10) equal 14% and 15%, respectively.

A)
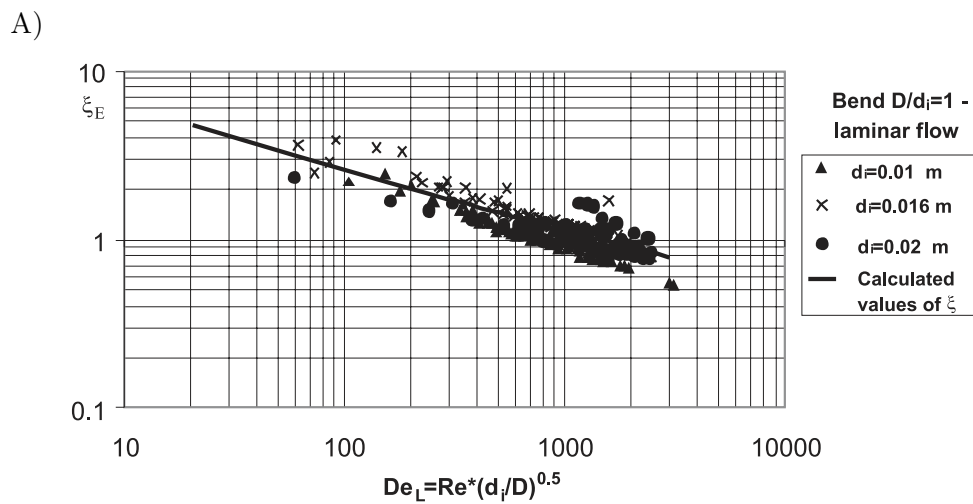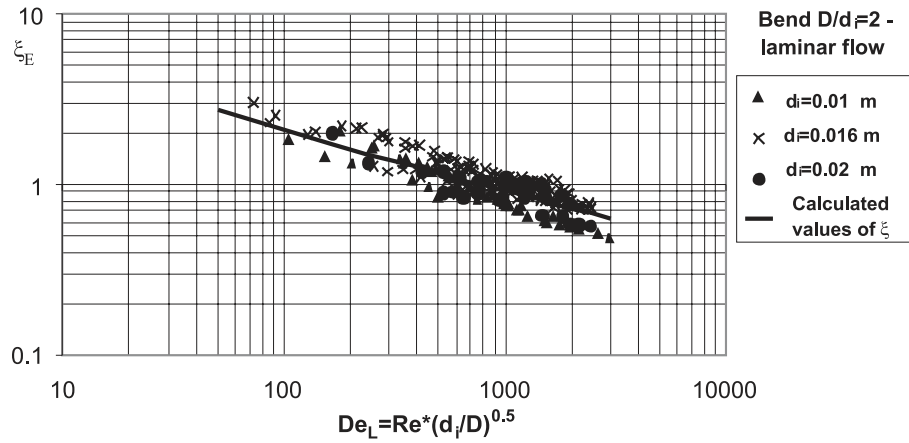
B)

Figure 7. The dependency of the substitute local resistance coefficient on the generalised Dean number for laminar flow of ice slurry through bends: $D/d_i = 2$; B) $D/d_i = 1$.
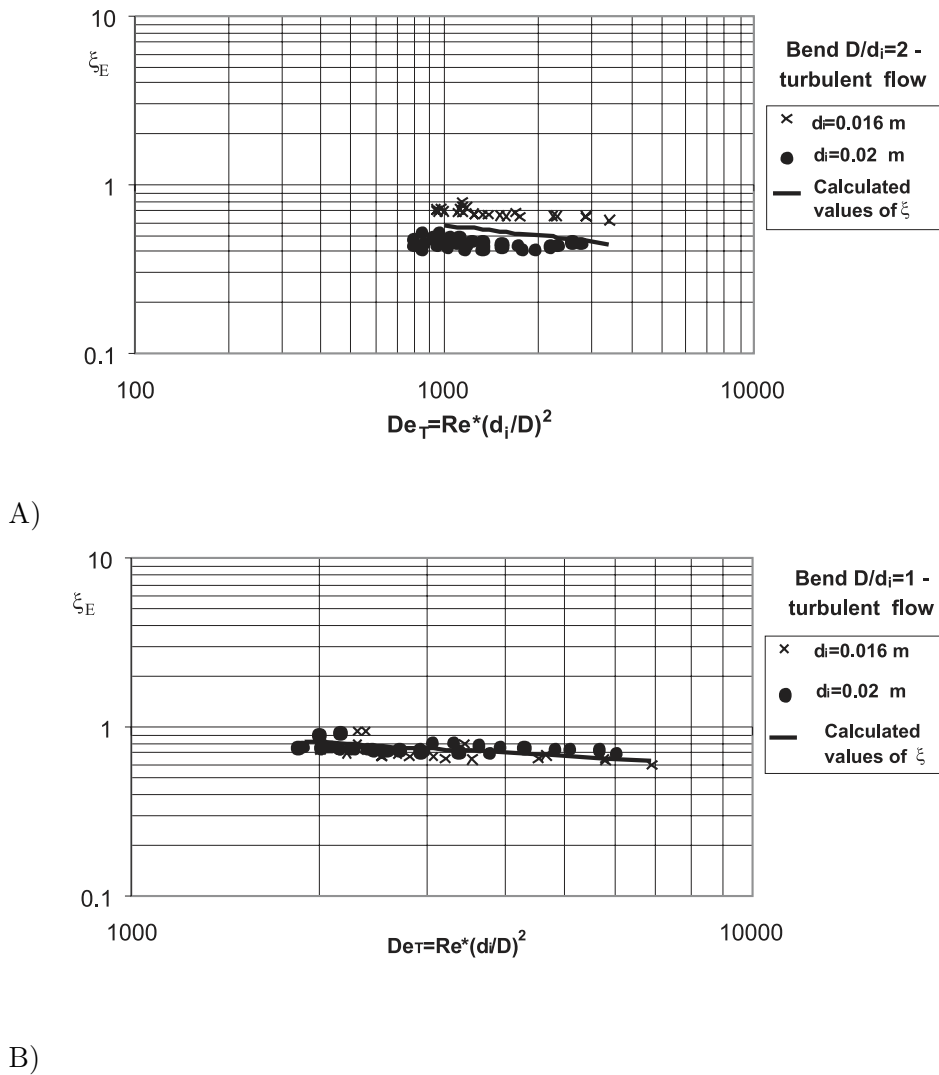
A)

B)

Figure 8. The dependency of the substitute local resistance coefficient on the generalised Dean number for turbulent flow of ice slurry through bends: $D/d_i = 2$; B) $D/d_i = 1$.

4 Summary

The paper presents the results of experimental research on the flow resistances of Bingham-fluid ice slurry in bends. The research, performed for three pipe diameters and a relative bend radius of $1 \leq D/d_i \leq 2$, has made it possible to take into consideration the influence of friction resistances as well as the flow geometry

on the total local resistance coefficients. The study attempts to make the local resistance coefficient dependent on the Dean number defined for a generalized Reynolds number according to Metzner-Read. Thanks to the suggested generalization, the dependencies quoted for laminar and turbulent flow may be used in order to determine the flow resistances in bends for other ice slurries, provided that their physical properties, hydraulic and geometrical conditions are similar.

References

[1] Guilpart J.: *Experimental study and calculation method of transport characteristics of ice slurries*. First Workshop on Ice Slurries of the Int. Inst. of Refrigeration, Yverdon-Les-Bains 1999, 74–82.

[2] Sasaki M., Kawashima T., Takahashi H.: *Dynamics of snow-water flow in pipelines, slurry handing and pipeline transport*. Hydrotransport **12**(1993), 533–613.

[3] Christensen K.G., Kauffeld M.: *Heat transfer measurements with ice slurry*. Int. Conf. of the Int. Inst. of Refrigeration (1997).

[4] Egolf P.W., Kitanovski A., Ata-Caesar D., Stamatiou E., Kawaji M., Bedecarrats J.P., Strub F.: *Thermodynamics and heat transfer of ice slurries*. Int. J. Refrig. **28**(2005), 51–59.

[5] Jensen E., Christensen K., Hansen T., Schneider P., Kauffled M.: *Pressure drop and heat transfer with ice slurry*. Purdue University, IIF/IIR (2000), 521–529.

[6] Niezgoda-Żelasko B., Zalewski W.: *Momentum transfer of ice slurries flows in tubes experimental investigation*. Int. J. Refrig. **2**(2006), 418–428.

[7] Kitanovski A., Poredos A.: *Concentraction distribution and viscosity of ice-slurry in heterogeneous flow*. Int. J. Refrig. **25**(2002), 827–835.

[8] Idielcik I.E.: Handbook of Hydraulic Resistance, CRC Press, 1994.

[9] Ito H.: *Friction factors for turbulent flow in curved pipes*. Trans. ASME J. Basic Eng. **82 D**(1960), 131–145.

[10] Ito H.: *Pressure losses in smoth pipe bends*. Trans. ASME J. Basic Eng. **81 D**(1959), 123–134.

[11] Matras Z.: *Hydraulic transport of rheologically complex non-Newtonian fluids in ducts*. Cracow University of Technology, Kraków 2001 (in Polish).

[12] Mishra P., Gupta S.W.: *Momentum transfer in curved pipes. 2: Non-Newtonian fluids.* Ind. Eng. Chem. Process Des. Dev. **18**(1979), 137–142.

[13] Tada T., Fukui Y., Oshima S., Yamane R.: *E ects of Bingham viscosity on flow in curved pipes.* JSME Int. J. **37**(1994), 322–327.

[14] Bel O., Lallemand A.: *Study of two phase secondary refrigerant – 1: Intrinsic thermophysical properties of an ice slurry.* Int. J. Refrig. **22**(1999), 164–174 (in French).

[15] Nørgaard E., Sørensen T.A., Hansen T.M., Kauffeld M.: *Performance of components of ice slurry systems: pumps, plate heat exchangers and fittings.* Int. J. Refrig. **28**(2005), 83–91.

[16] Nørgaard E., Sørensen T.A., Hansen T.M., Kauffeld M.: *Performance of components of ice slurry systems: pumps, plate heat exchangers and fittings.* Proc. of the 3rd IIR Workshop on Ice Slurries, Lucerne 2001, 129–136.

[17] Turian R.M., Ma T.W., Hsu F.L., Sung D.J., Plackmann G.W.: *Flow of concentrated non-Newtonian slurries: Friction losses in bends, fittings, valves and Venturie meters.* Int. J. Multiphase Flow **24**(1998), 243–269.

[18] Niezgoda-Żelasko B., Żelasko J.: *Generalized non-Newtonian flow of ice-slurry.* Chem. Eng. and Proc. **46**(2007), 895–904.

[19] Kozicki W., Chou C.H., Tiu C.: *Non-Newtonian flow in ducts of arbitrary cross-sectional shape.* Chem. Eng. Sci. **21**(1966), 665–679.