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Laser range measurement filtration for PNDS purposes

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Abstract
PNDS (Pilot Navigation and Docking System) [1] utilizes the range measurement between laser head and ship's side to determine the ship's outline presented on the screen. The noisy measurements and dynamic process noise affect the accuracy of determined parameters and propagate to ship's heading and position. To improve the performance of PNDS system there was a need to apply the data filtering technique. In the paper the theoretical basis and algorithm of discrete Kalman filter designed for range optimal estimation in PNDS were described. The real data collected during berthing operation of motor vessel Navigator XXI were filtered and compared to row unfiltered data. The conclusions contain evaluation of filter capabilities and its potential application in PNDS system.

Theoretical assumption of range measurements filtering

The problem of determining the distance and movement parameters of ship's hull from the certain points of the berth during docking operation considers the estimation of dynamic discrete time controlled process. For this purposes it is necessary to assume that measurement and process itself are corrupted by the noise and the process that is measured must be able to be described by linear system. In order to solve the above problem, the Kalman filter recursive algorithm was applied [2, 3, 4]. The measurement and process noise is assumed to be independent, white and with normal distribution. In such case the range measurements by means of laser sensor comes down to estimation the state $x \in \Re^n$ of a discrete time controlled process that is governed by the linear stochastic difference equation:

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \tag{1}$$

With a measurement model $z \in \Re^m$ that is:

$$z_k = H_k \cdot x_k + v_k \tag{2}$$

where:

- A_{k-1} – transition matrix ($n \times n$) relates the state at the previous time step to the state at the current step, in the absence of either a driving function or process noise;
- B_{k-1} – matrix ($m \times n$) relates to the optional control input $u \in \Re^m$ to the state at the current step;
- u_{k-1} – known input to the system;
- H_k – matrix ($m \times n$) relates the state vector $\hat{x}_k^+$ to the measurement z_k in a time k;
- w_k, v_k – the random variables represents the process and measurement noise (respectively) with normal distribution:

$$p(w_k) \approx N(0, Q_k), \quad p(v_k) \approx N(0, R_k)$$

and probability functions are equal:

$$f(w_k) = \frac{1}{\sigma_{wk}\sqrt{2\pi}} \exp\left[-\frac{(w_k - \mu_{wk})^2}{2\sigma_{wk}^2}\right]$$
$$f(v_k) = \frac{1}{\sigma_{vk}\sqrt{2\pi}} \exp\left[-\frac{(v_k - \mu_{vk})^2}{2\sigma_{vk}^2}\right] \tag{3}$$

where:

- Q_k – process noise covariance, defined by variable w_k, for normal distribution, equals variance σ_{wk}^2;
- R_k – measurement noise covariance, defined by variable v_k, for normal distribution, equals variance σ_{vk}^2;

μ_{wk}, μ_{vk} – expected values of noise and process errors;

σ_{wk}, σ_{vk} – standard deviations of noise and process errors.

Let's assume that $\hat{x}_k^- \in \Re^n$ denotes a priori estimate of system state at step k, given knowledge of the process prior to step k, and $\hat{x}_k^+ \in \Re^n$ denotes a posteriori state estimate at step k, given measurement z_k at step k:

$$\hat{x}_k^- = A_k \hat{x}_{k-1}^+ \tag{4}$$

Hence in base of above it is possible to define a priori and a posteriori estimates errors and respective covariance matrixes a P_k as:

– a priori estimates:

$$e_k^- \equiv x_k - \hat{x}_k^-,\ P_k^- \equiv E\left[e_k^- \cdot e_k^{-T}\right] \tag{5}$$

– a posteriori estimates:

$$e_k^+ \equiv x_k - \hat{x}_k^+,\ P_k^+ \equiv E\left[e_k^+ \cdot e_k^{+T}\right] \tag{6}$$

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state $\hat{x}_k^-$ at some time and then obtains feedback in the form of (noisy) measurements z_k. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations [4, 5]. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain a priori estimates for the next time step. The measurement update equations are responsible for the feedback – i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate $\hat{x}_{k-1}^+$.

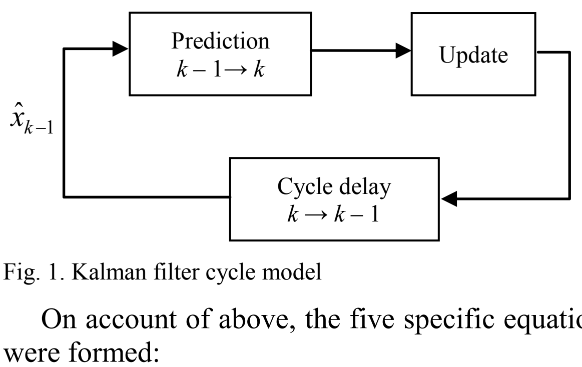

Fig. 1. Kalman filter cycle model

On account of above, the five specific equations were formed:

– projection the state ahead (4):

$$\hat{x}_k^- = A_k \hat{x}_{k-1}^+ \tag{7}$$

– projection the covariance ahead:

$$P_k^- = A_k P_{k-1}^+ A_k^T + Q_k \tag{8}$$

– gain:

$$K_k = P_k^- H_k^T \left[H_k P_k^- H_k^T + R_k\right]^{-1} \tag{9}$$

– update state estimate:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k\left[z_k - H_k \hat{x}_k^-\right] \tag{10}$$

– update the covariance:

$$P_k^+ = \left[I - K_k H_k\right] P_k^- \tag{11}$$

where: I – identity matrix P_k, and index T denotes matrix transposition.

Simulation of range measurement filtration

The simulation of range measurement filtration algorithm is based on assumption that laser distance describes one dimensional position measurement, where position and velocity of the object is estimated. In this particular case there is no control input hence $u = 0$ and state vector equation takes the form:

$$x_k = Ax_{k-1} + w_{k-1}.$$

State vector can be described as:

$$x_k = \begin{bmatrix} s_k \\ v_k \end{bmatrix} \tag{12}$$

where:

$$s_{k+1} = s_k + tv_k \tag{13}$$

$$v_{k+1} = v_k \tag{14}$$

hence transition matrix is as follows:

$$A = \begin{bmatrix} 1 & \mathrm{d}t \\ 0 & 1 \end{bmatrix} \tag{15}$$

Matrix relating state and measurement is:

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{16}$$

Taking into consideration that ship tracked remained in motion, the covariance matrix Q is supposed to take high values:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \sigma^2 w_k \tag{17}$$

R is a 1×1 size matrix which is equal to measurement variance.

The measurement of ship's hull movement was simulated by section of matrix A parameters and process error vector. A matrix is a transition matrix form previous state to current in "dt" time period. Process error vector is a vector of momentary acceleration taking values according to normal distribution function by means of "randn" (mean 0, standard deviation 0.01).

Figure 2 presents the simulation of filter built in MATLAB® software environment for parametres:

- simulation time equals 60 s;
- dt = 0.5 s;
- initial range equals 10 m;
- standard deviation of range measurement equals 1 m;
- standard deviation of acceleration equals 0.01 m/s^2;
- range and acceleration variables generated according to normal distribution.

The true ranges were denoted by red colour and measured ranges by blue colour. The ranges estimated by the filter by green colour.

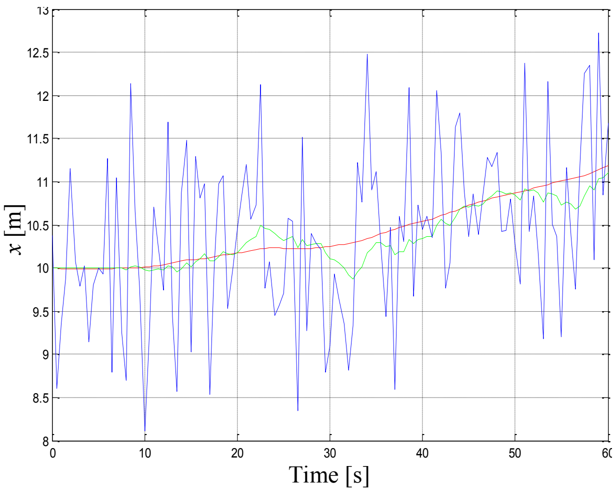

Fig. 2. The simulation of laser range measurement filtration

Filtration of real ranges coming from experiment

In the algorithm the simulation of the process and ranges measurements was replaced by real ranges data. Figure 3 presents the performance of filter worked out in MATLAB® environment for parameters:

- simulation time equals 60 s;
- dt = 1;
- initial range equals 10 m;
- standard deviation of range measurement equals 1 m;
- standard deviation of acceleration equals 0.01 m/s^2;
- range measurement increased 10 times for assumed standard deviation.

Ship's velocity at point of range measurement was determined as derivative of ship's trail relative to time in each filter loop iteration.

At the figure 3 the blue colour indicates measured ranges, the green colour indicates the ranges estimated by the filter.

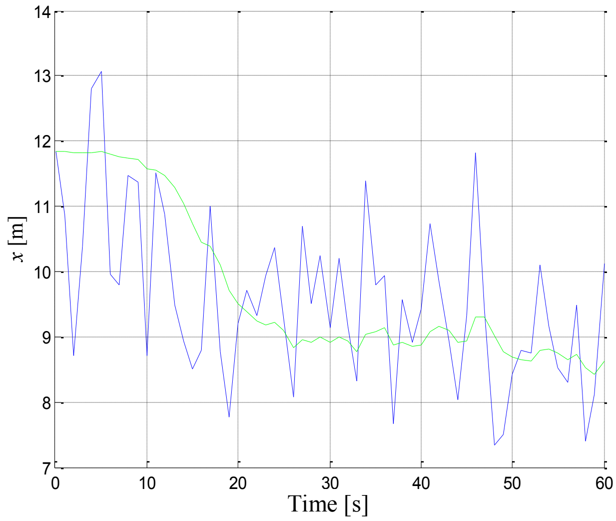

Fig. 3. Filtration of real laser range measurements – version I

Tracking results were found as unsatisfactory. Additionally phase shift of signal was observed. That led to filter modification what resulted in adding acceleration as a quantity to be estimated.

After modification of the filter state vector was as follows:

$$x_k = \begin{bmatrix} s_k \\ v_k \\ a_k \end{bmatrix} \quad (18)$$

transition matrix A formed as:

$$A = \begin{bmatrix} 1 & \mathrm{d}t & \frac{\mathrm{d}t^2}{2} \\ 0 & 1 & \mathrm{d}t \\ 0 & 0 & 1 \end{bmatrix} \quad (19)$$

Covariance matrix of process noise Q:

$$Q = \begin{bmatrix} \frac{\mathrm{d}t^5}{20} & \frac{\mathrm{d}t^4}{8} & \frac{\mathrm{d}t^3}{6} \\ \frac{\mathrm{d}t^4}{8} & \frac{\mathrm{d}t^3}{3} & \frac{\mathrm{d}t^2}{2} \\ \frac{\mathrm{d}t^3}{6} & \frac{\mathrm{d}t^2}{2} & \mathrm{d}t \end{bmatrix} * \sigma^2 w_k \quad (20)$$

Matrix relating state and measurement is:

$$H = [1 \quad 0 \quad 0] \quad (21)$$

In such shape filtration algorithm was found in accordance with reality.

Figure 4 presents the modified filter performance for the following parameters:

- simulation time equals 60 s;
- dt = 1;
- initial range equals 10 m;

- standard deviation of range measurement equals 1 m;
- standard deviation of acceleration equals 0.01 m/s^2;
- range measurement increased 10 times for assumed standard deviation,
- velocity measurement were obtained directly from values approximated in laser head by means of its filtration algorithm.

At figure 4 the blue colour indicates measured ranges while green colour indicates ranges estimated by the Kalman filter.

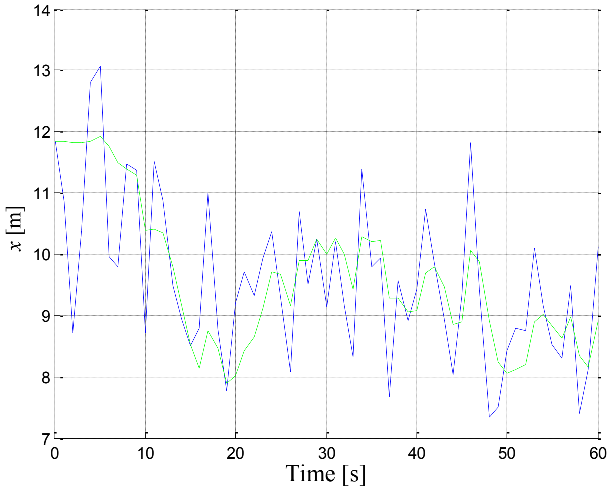

Fig. 4. Filtration of real laser range measurements – version II

Conclusions

The worked out algorithm of filtration can be implemented directly on the laser head processor or as an element of navigation data visualization system integrated in PNDS. The presented versions of laser range measurements filtration algorithm enable to correct momentary measurement errors in different variants of measuring systems. The first version is based solely on measured ranges and calculated velocities. Second version takes into consideration accelerations. In the future work on range measurement filtration for the use of PNDS, the fusion of data received from laser, gyrocompass and GNNS sensors is planned. This will improve the performance of the filter and enable to achieve the higher degree of accuracy of the ship's position estimation.

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