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Ship seakeeping in UKC determination – a further study on wave force transfer functions

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Abstract
Modeling of ship motions in waves concentrates in most applications on the response amplitude operator (RAO). This mathematically not demanding method of analysis is very attractive, but loses some essential information in certain situations. The objective of present contribution is to establish and investigate preliminary foundations for a seakeeping model as optimal for under keel clearance (UKC) estimation. A special attention was devoted to transients of motions, stationary harmonic motions, coupling between degrees of freedom, and the wave force transfer functions – all in the aspect of shallow water environment.

Introduction

The reduction of under keel clearance (UKC) due to the wave action, and the ship motions resulting thereof, beside some other motion phenomena (the squat, a roll during turning) as also affecting the UKC, is a serious problem for the safe operation of ports. The literature and science has just started to cover such aspects – e.g. [1].

Response amplitude operator (RAO) or, in other words, transfer function approach while simulating ship motions has its own advantages and disadvantages, even then, when we additionally keep track of phase shifts between the waves and the resulting motions. Assuming, one is only interested in the stationary (steady-state) harmonic oscillations, or the linear combination (superposition) of them, under external and also harmonic waves, the response operator is very useful in a lot of application areas, and is easily to be computed. There is of course disregarded the transient stage (the so-called memory effect). The computations of RAOs are algebraic for usually linear motion differential equations (e.g. [2, 3]). They depend on the coefficients of underlying equations.

For instance, the RAOs allow efficient investigations on the static response characteristics of a ship, as in case of the hull design optimization or ship operation optimization in heavy weather. It is exactly a very fast way to compare competitive designs from the "long-term performance" point of view, and is a perfect concept for formulating / estimating various statistical (probabilistic) properties of ships motions in irregular waves. Besides, the RAOs (but expanded to accommodate the phase angles of motions), if combined with a sea spectrum, are capable of simulating the steady-state motions in the time-domain by providing a time series of random motions. The major drawbacks consist of difficulties with storing RAO values for finitely spaced absolute frequencies in the regions of resonance or zero encounter frequencies, and subsequently with computing the exact motion response in such regions for irregular waves. This could be resolved by additional recalculation of RAOs within these regions from the viewpoint of given purpose.

However, RAOs are not the latest "state-of-the--art" in the ship motions analysis, especially if the dynamic response is of primarily concern. For harmonic excitations (according to the Fourier theory, any arbitrary function can be decomposed into a linear combination of harmonic functions) the motion output of the linear system starts to be harmonic (expressed as sine or cosine) only after some time, dependent on the so-called time constant of the system. RAOs concept gives a full description of the system response in this asymptotic, steady--state case. The time constants can be deduced from the coefficients of equations.

The coefficients of ship linear ODEs are generally the motion frequency-dependent (i.e. not being the pure time function) with particular regard to the added mass and damping terms. This results from the surrounding water action. The water flow velocity potentials, especially in the free surface, are widely used to calculate such effects. The direct, normal integration of the linear equations in such conditions can not be performed. This only can be done if the coefficients of such equations are assumed practically constant for the given application, or some special integrals are pre-calculated – see e.g. [4, 5, 6, 7, 8].

The bottom of a ship's hull can be treated as a plane of the rectangular shape, the corners (vertices) of which can be fixed according to the three motions (precisely their displacements) – heave, roll, and pitch – from the total six degrees of freedom (DOFs). It shall be remembered that these motions are connected with restoring force (the so--called spring terms in the mass-damping-spring systems). Due to the ship's symmetry of port and starboard side, the motions are to some extent decoupled in that, the surge, heave, and pitch (denoted as 1, 3, 5, the so-called symmetric motions), and the sway, roll, and yaw (referred to as 2, 4, 6, named asymmetric motions), are being both developed independently. Within each group, the magnitude of coupling (interaction) between particular motions considerably varies – a lot factors are involved here. Having the heave, pitch, and roll characteristics, in terms of RAOs, one can evaluate the probabilistic properties of extreme motions of the hull bottom points e.g. for the risk calculation. But this is only one side of the problem.

The other aspect is strongly connected with the transient (dynamic) behavior of motions. The necessity of fully dynamic approach arises when the ship motions (oscillations) are being studied with the aim of determining the under keel clearance, where a single touch of the seabed could really be dangerous. Since the governing equations are decoupled into two groups – two sets of three second order equations – any elementary motion in the time domain theoretically can be resolved into a linear differential equation of sixth order with reference to the motion displacement. Because a particular motion is coupled with additional two motions, the reduction of only one coupling (from the total two) decreases the equation order by two, here up to the fourth order. Without any coupling, the motion equation is of the second order. There are a lot of algebraic (analytic) methods in the mechanical or electrical engineering science to study the transient stage of a single degree of freedom oscillations, having essentially lower (mostly second) orders – e.g. [9]. In a lot of situations, 1DOF models are really and surprisingly a good approximation of more complex motion phenomena. It is sometimes difficult to find the intensity of coupling just on the basis of equation coefficients. Switching off artificially some degrees of freedom in the general ship motions, the resulting RAOs can serve to establish simpler and more effective dynamic models for particular, narrower application.

The research on the performance of impulse response techniques for solving the linear equations with frequency-dependent parameters, in view of the motion transients, is scheduled in the future as the second step of current research. It can provide additional dynamic effects to those known from the constant coefficient equations. In the present paper the classical full 2^{nd} order model with constant coefficients is presented and studied for various dynamic (memory) effects, allowing a proper understanding and judgment of the situation.

The problems with UKC are obviously attributed to the ship operation in shallow water (generally pertaining to a port area), in only which the motion displacements of the hull extreme points can reduce the under keel clearance down to zero. In such conditions one should take a proper consideration of various shallow effects while computing the wave excitation generalized forces (in terms of forces and moments) and the hydrodynamic generalized forces (called the radiation forces, and consisting of the mentioned added masses / inertia and damping effects) [10, 11, 12]. Of special importance and interest seems to be the knowledge on wave spectrum in the investigated shallow water (SW) region, and additionally the validation limitations of traditional linear (superposition) theory of waves and ship motions – see e.g. [9]. In this context, much more simpler matter is the effect of shallow water on the wave encounter frequency.

The shallow water conditions in our UKC concern, expressed as depth-draught ratio h/T, are much lower than 1.5 – the values 1.1÷1.2 must be here the reference points.

The following topics are studied hereafter correspondingly: the transient significance in simulating seakeeping motions, the change of encounter velocity in shallow water due to the wave length (wave number), performance (accuracy) of various theoretical methods for estimating the hydrodynamic parameters of motion linear equations in waves for

SW, and finally the role of coupling between motions in 6DOFs and the importance of wave force transfer functions in the economy of developing the seakeeping models.

The magnitude of transient in ship motions

Although in the following the notations refer to the roll, the same applies to other motions – heave and pitch – since the equations are quite the same (linear and second order), so it is the solution. The difference only exists in the interpretation and particular values of coefficients. The roll is treated independently from other motions (in general the roll is coupled with the sway and yaw – see above) and is referred to as the pure (plain) roll.

For the both, linear restoring and damping moment, the roll equation (φ in [rad]) reads:

$$\left(J_x + m_{44}\right)\ddot{\varphi} + 2N_{44}\dot{\varphi} + m \cdot g \cdot GM_0 \cdot \varphi = M_x(t) \quad (1)$$

where:

- J_x – moment of inertia [kg·m^2];
- m_{44} – added mass (inertia) [kg·m^2];
- N_{44} – damping coefficient [kg·m^2/s];
- m – ship's displacement (mass) [kg];
- g – gravity acceleration, 9.806 [kg·m/s^2];
- GM – initial metacentric height [m];
- M_x – external excitation moment [Nm].

The above expression can be rewritten in the more useful form:

$$\ddot{\varphi} + 2\nu_{44}\omega_{0\varphi}\dot{\varphi} + \omega_{0\varphi}^2\varphi = m_x(t) \quad (2)$$

or

$$\ddot{\varphi} + 2\delta_\varphi\dot{\varphi} + \omega_{0\varphi}^2\varphi = m_x(t), \quad \delta_\varphi = \nu_{44}\omega_{0\varphi}$$

$$\omega_{0\varphi} = \sqrt{\frac{mg \cdot GM_0}{J_x + m_{44}}} \quad (3)$$

where:

- ν_{44} – non-dimensional damping coefficient [–], positive;
- $\omega_{0\varphi}$ – frequency of free (natural) undamped oscillations [1/s], positive;
- m_x – external excitation moment function [1/s^2] (having the meaning of angular acceleration, hence called the forced acceleration).

The Laplace transform $\Phi(s)$ of the response $\varphi(t)$, given by Eq. (3), for the general case of $m_x(t)$ takes the form of:

$$\Phi(s) = \frac{L\{m_x(t)\}}{s^2 + \left(2\nu_{44}\omega_{0\varphi}\right)s + \omega_{o\varphi}^2} + \frac{s\varphi_0 + 2\nu_{44}\omega_{0\varphi}\varphi_0 + \dot{\varphi}_0}{s^2 + \left(2\nu_{44}\omega_{0\varphi}\right)s + \omega_{o\varphi}^2} \quad (4)$$

where:

- φ_0 – initial roll angle (displacement) [rad];
- $\dot{\varphi}_0$ – initial roll angular velocity [rad/s].

Introducing the harmonic excitation as follows:

$$m_x(t) = m_{x0}\cos\omega t\,,\; L\{m_x(t)\} = m_{x0}\frac{s}{s^2 + \omega^2} \quad (5)$$

where m_{x0} is the amplitude of the excitation function, and ω denotes the excitation frequency, one can finally get the below relation:

$$\varphi(t) = e^{-\delta_\varphi \cdot t}\left[\frac{m_{x0}}{\sqrt{D}} \cdot \frac{\omega_{0\varphi}}{\omega_\varphi} \cdot \cos\left(\omega_\varphi t - \varepsilon_{1\varphi}\right) + \right.$$
$$\left. + \varphi_0\frac{\omega_{0\varphi}}{\omega_\varphi}\cos\left(\omega_\varphi t - \varepsilon_\varphi^*\right) + \frac{\dot{\varphi}_0}{\omega_\varphi}\cos\left(\omega_\varphi t - \frac{\pi}{2}\right)\right] + \quad (6)$$
$$+ \frac{m_{x0}}{\sqrt{D}} \cdot \cos\left(\omega t - \varepsilon_\varphi\right)$$

where ω_φ is the frequency of free damped oscillations according to:

$$\omega_\varphi = \omega_{0\varphi}\sqrt{1 - \nu_{44}^2} \quad (7)$$

and

$$\sqrt{D} = \sqrt{\left(\omega_{0\varphi}^2 - \omega^2\right)^2 + 4\delta_\varphi^2\omega^2} \quad (8)$$

The term $\sqrt{D}^{-1}$ just represents the RAO, i.e. the ratio of response amplitude to the excitation amplitude. In the frequency domain, the plot of RAO assumes different image as dependent upon the level of damping. A detailed graphical presentation and analysis of the RAO behavior for various frequencies (the so-called amplitude-frequency or amplification characteristics), also including the phase characteristics, can easily be found in classical manuals on physics and/or mechanical engineering science under the topic of forced oscillations (vibrations) of damped mechanical systems. One should keep in mind that the reported RAO plots generally refer to the constant coefficients of linear equations, that is not true for ship motions.

The three phase angles in the particular cosine functions of (6) yield as follows (the information is here given in terms of sine and cosine, since the usual tangent function is ambiguous)

$$\begin{cases} \cos\varepsilon_{1\varphi} = -\left(\omega_{0\varphi}^2 - \omega^2\right)\dfrac{1}{\sqrt{D}} \cdot \dfrac{\omega_\varphi}{\omega_{0\varphi}} \\ \sin\varepsilon_{1\varphi} = -\left(\omega_{0\varphi}^2 + \omega^2\right)\dfrac{1}{\sqrt{D}} \cdot \dfrac{\delta_\varphi}{\omega_{0\varphi}} \end{cases} \quad (9)$$

$$\begin{cases} \cos\varepsilon_{\varphi} = \left(\omega_{0\varphi}^2 - \omega^2\right)\dfrac{1}{\sqrt{D}} \\ \sin\varepsilon_{\varphi} = 2\delta_{\varphi}\dfrac{1}{\sqrt{D}} \end{cases} \tag{10}$$

$$\begin{cases} \cos\varepsilon_{\varphi}^{*} = \dfrac{\omega_{\varphi}}{\omega_{0\varphi}} \\ \sin\varepsilon_{\varphi}^{*} = \dfrac{\delta_{\varphi}}{\omega_{0\varphi}} \end{cases} \tag{11}$$

The mentioned in the previous chapter time constant of the system is defined by:

$$T = \frac{1}{\delta_{\varphi}} \tag{12}$$

and is understood as the elapsed time when the natural (inherent) oscillations are being reduced in the amplitude by ca. 63%.

For the below simulations of transient, the consecutive four components of equation (6) have been marked accordingly:

$$\varphi(t) = f_1(t) + f_{1a}(t) + f_{1b}(t) + f_2(t) \tag{13}$$

where f_1 stands for natural damped oscillations (with zero initial conditions), f_2 symbolizes the forced oscillations, and f_{1a} depicts the free damped oscillations as started from the non-zero initial roll angle φ_0. The term f_{1b} for non-zero roll velocity will not be tested hereafter, thus the value $\dot{\varphi}_0$ equal to zero is assumed.

Figure 1 shows the simulation of (13) for three different values of the initial roll angle.

The amplitude of forced steady-state oscillations is abt. 2.4° (the solid thick line). The time constant, ref. to Eq. (12), equals approx. 28 s. In the worst case of $\varphi_0 = -2°$, the resulting (combined) roll motion during the transient (initial) stage is almost twice as large as the steady-state oscillation. Also, the image of roll variation is much far from the pure harmonic one. The practical convergence of $\varphi(t)$ to $f_2(t)$ occurs after some 50 s. The functions f_1 and f_{1a} have nearly identical plot, hence the natural oscillations f_1 can be considered as being doubled. The situation is better for $\varphi_0 = 0°$, since f_{1a} vanishes. Hence, the contribution of natural damped oscillation is much lower under such conditions.

The best case from the transient point of view is achieved for $\varphi_0 = +2°$, in which f_1 and f_{1a} cancel each other, thus giving no memory effect. Therefore, by a proper adjustment of the initial conditions "the memory effect" can be suppressed. So, the system starts the harmonic oscillations from the beginning of time. Such an adjustment directly follows from the required phase shift in the system response – namely, the initial condition for the displacement (roll angle) shall be set to this phase angle (together with the corresponding value of initial velocity). This rarely happens in real life, mostly accidentally, because there is generally no correlation of the initial conditions and the excitation itself.

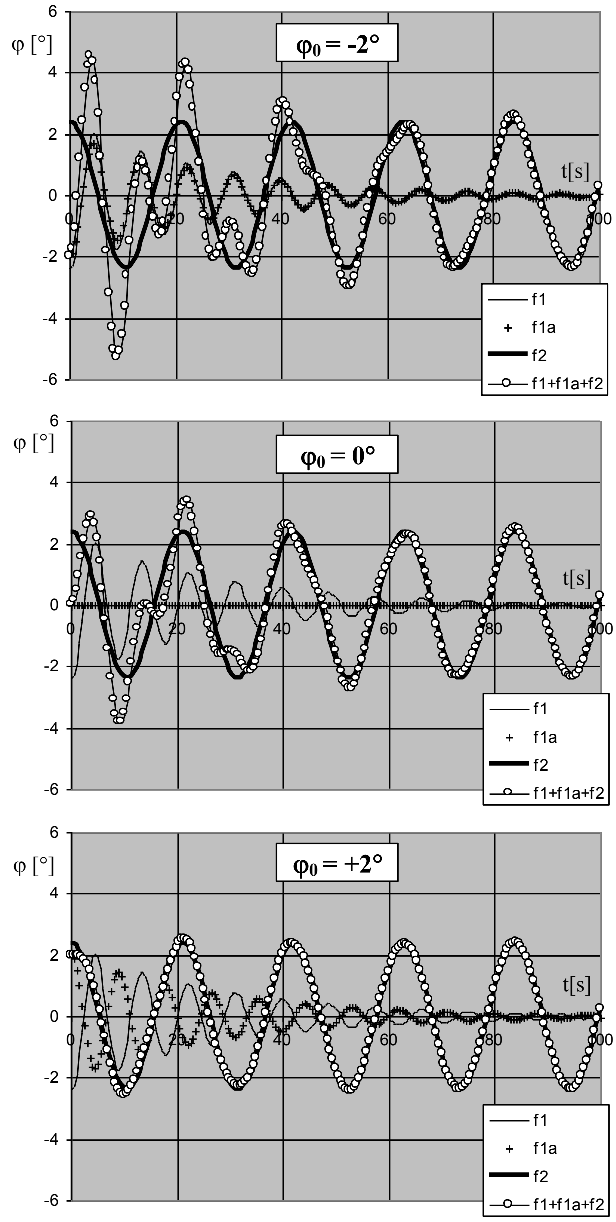

Fig. 1. The memory effect (transient) in roll oscillation development

Other variants (combination) of the forced oscillations amplitude and the initial roll angle are also worthwhile to be studied. The investigated problems of possible increase of "dynamic" roll angle over the steady-state amplitude is analog to the well known dynamic transverse stability issues for small and large heel (roll) angles, in which the first maximum roll angle ("dynamic angle") under constant heeling moment can be related to the static angle. A relatively very low damping is usually assumed in that context, but the background governing equations for the dynamical stability are still the same as Eq. (6).

Wave dispersion relation in shallow water

The basic notion in studying the effects of wave on ship motions is the encounter frequency ω_E characterized by:

$$\omega_E = \omega - k \cdot v \cdot \cos\gamma\,,\ k \overset{\text{df}}{=} \frac{2\pi}{\lambda} \tag{14}$$

where:

- ω – wave absolute circular frequency [rad/s];
- k – wave number [1/m];
- λ – wave length [m];
- v – ship's forward velocity [m/s];
- γ – wave incidence angle [rad], [°], 0° for stern wave, 180° for head wave (i.e. traveling from the bow to the stern).

The specific value of k (and wave length) is dependent on the wave propagation condition in the area, defined by the water depth h [m], and the wave frequency:

$$k = f(\omega,h),\quad \lambda = f(\omega,h) \tag{15}$$

Also useful in some applications is the knowledge of the wave profile velocity (celerity):

$$c \overset{\text{df}}{=} \frac{\lambda}{T}\text{, where the wave period } T \overset{\text{df}}{=} \frac{2\pi}{\omega} \tag{16}$$

Hence:

$$c = \frac{\omega\lambda}{2\pi} = f(\omega,h) \tag{17}$$

The dependencies in (15) and (17) arise from the fundamental dispersion relation of the implicit non-linear form:

$$k \cdot g \cdot \tanh(k \cdot h) = \omega^2 \tag{18}$$

where g is the gravity acceleration.

Figure 2 presents some numerical values of (15) and (17) for the conditions of shallow water. The water depths included (the visual identification of appropriate curve in figure 2 shall not pose a difficulty) are 5÷10 m every 1 m, 15÷30 m every 5 m, and the deep water as the reference ($h = \infty$, DW).

Ship motion RAOs in SW

In the literature, there exist various formulations, based on the potential strip theory (sometimes including 3D flow effects), for estimating hull hydrodynamic forces (of reaction / passive nature) and wave exciting (active) hydrodynamic forces. The hull forces are represented by introducing added masses and damping terms. Both types of coefficients, as mentioned before, are functions of the motion frequency (the concept is valid for harmonic oscillations only) and forward speed. The motion frequency and ship speed affect the water potential flow field, and this influence is strongly emphasized in the shallow water conditions. The wave force for a harmonic wave is a function of the encounter frequency, also seriously dependent (through the wave potential) on the water depth, and in the steady-state of oscillations this results in ship motions of the same encounter frequency. The transient still keeps being unknown.

Reverting to the considered motion response amplitude operators, these magnitudes comprise the SW effects on both hull forces and wave forces, they can be resolved into the transfer functions

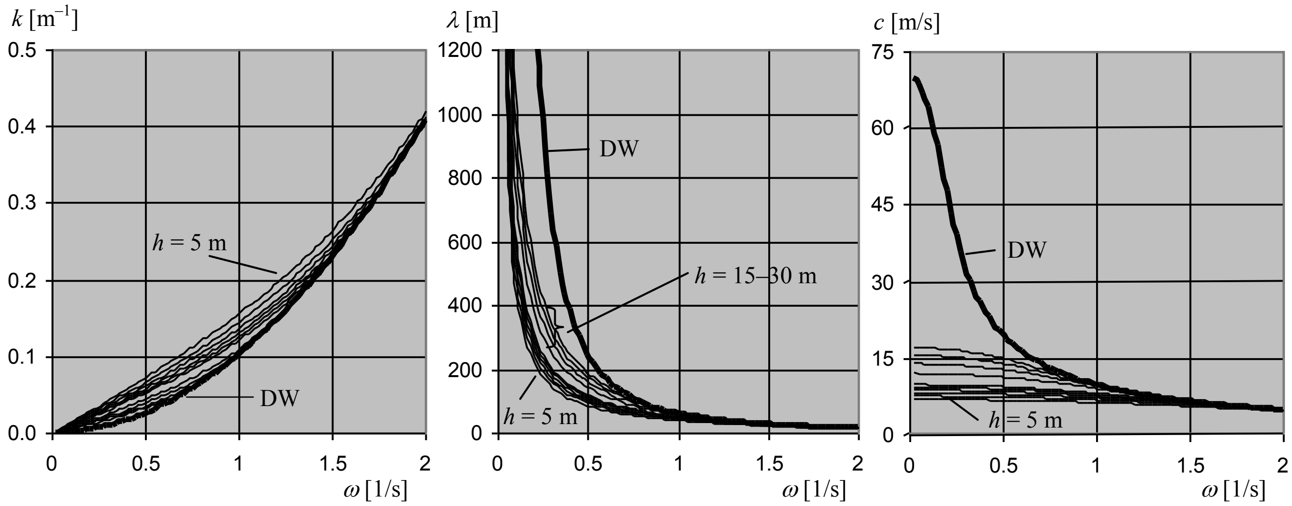

Fig. 2. Wave dispersion relation in SW range

from the wave to its force, and further from the wave force to the resulting motion.

Figures 3 to 5 show computations of RAOs in roll-φ, heave-z, and pitch-θ performed by means of Journee's SEAWAY software, see the website, for the reference container ship S-175 (L_{BP} = 175 m, B = 25.4 m, T = 9.50 m, c_B = 0.57, m = 24 700 t, GM = 0.98 m, GM_L = 203 m, C_{WP} = 0.72) in the water depth ratio h/T = 1.5. The latter shallow water condition is limiting for the Ursell-Tasai method, as assuming (similarly to Frank's method) no SW impact on the hydrodynamic coefficients of added masses and damping coefficients – the only SW effect is regarded in the wave potential. The Ursell--Tasai method is compared with more sophisticated Keil's method, where the full regard of SW effects is reflected, that predestines this method for much lower h/Ts (down to 1.05).

The RAOs in figures 3 to 5 are plotted against the wave absolute frequency and are defined by:

$$\text{RAO}z = \frac{\xi_{0z}[\text{m}]}{\xi_{0WV}[\text{m}]}$$

$$\text{RAO}_{\varphi,\theta} = \frac{\xi_{0\varphi,\theta}[°]}{k \cdot \dfrac{180}{\pi} \xi_{0WV}[\text{m}]} \tag{19}$$

where:

ξ_{0WV} – wave amplitude;

ξ_{0z}, $\xi_{0\varphi}$, $\xi_{0\theta}$ – motion amplitudes.

The lack of systematic, very reliable validations of the theoretical methods, due to low availability

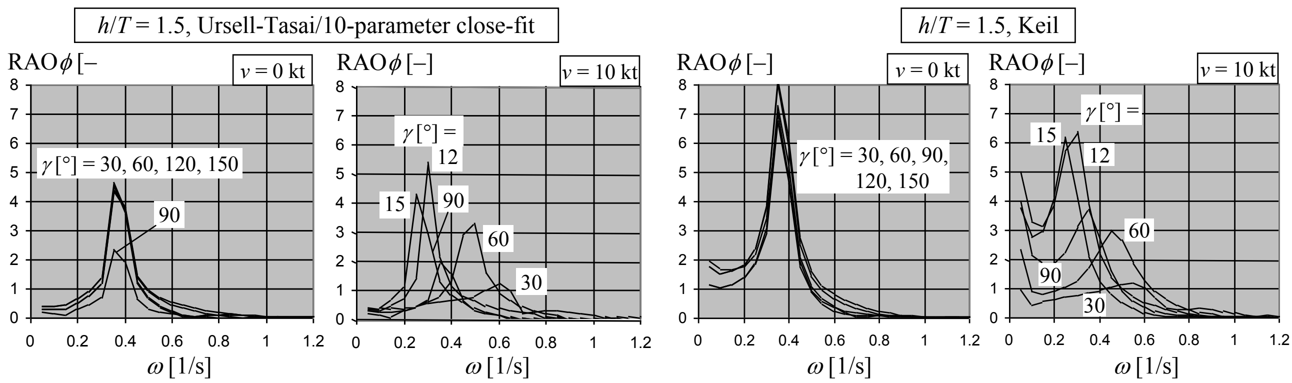

Fig. 3. Roll RAOs in medium SW by different methods

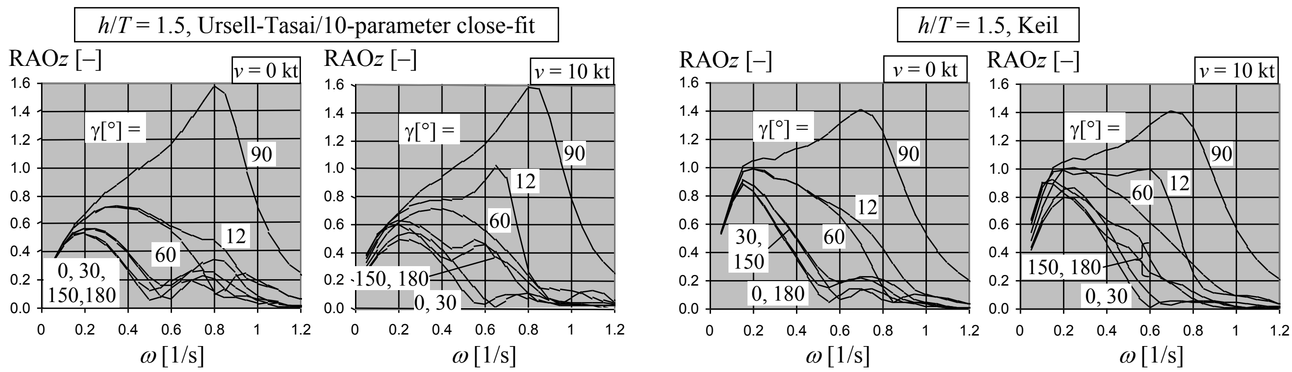

Fig. 4. Heave RAOs in medium SW by different methods

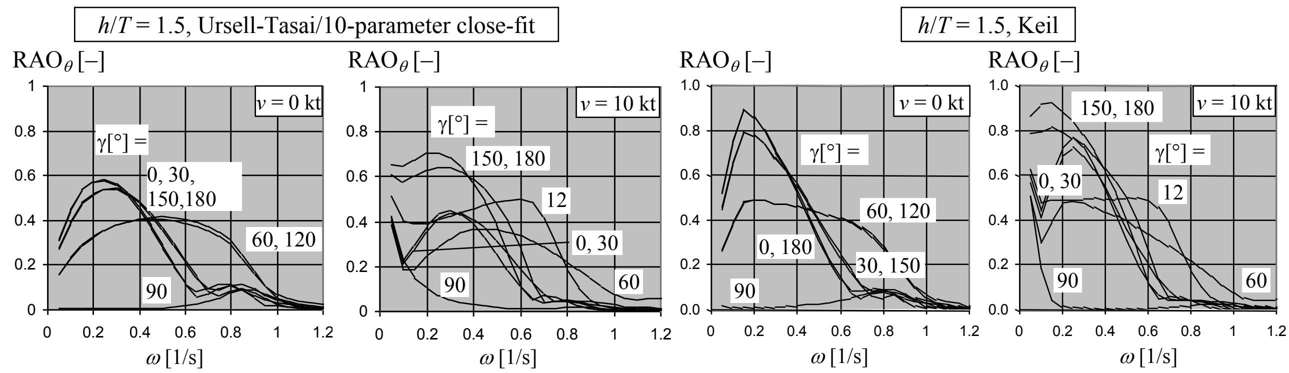

Fig. 5. Pitch RAOs in medium SW by different methods

of experimental results, especially in SW, shall be properly accounted for in decision-making on the acceptable risk in a ship's navigation under low UKC. Further research is recommended in this area.

Both above theoretical methods nearly exactly converge at deep water. However, within the speed range of 10 knots (rather usual in shallow waterways) there is a considerable discrepancy between the methods.

Coupling of motions in SW – sensitivity analysis

In this chapter the effect of coupling with other motions (degrees of freedom) are studied with regard to the fundamental (for UKC) roll, heave, and pitch.

The roll "4" is theoretically coupled with sway "2" and yaw "6" within the asymmetric motions. The heave "3" and pitch "5" are mutually dependent, and additionally linked to the surge motion "1" within the set of symmetric motions. However, the equation of surge is not standard for classical strip methods for theoretical calculations of linear motions. Another theory and/or algorithm for surge motion linear coefficients is often combined. This is also valid for SEAWAY software.

Figure 6 presents the roll RAO in the situations, based on SEAWAY computations, when other contributing motions are forced to be zero i.e. having constraints superimposed upon. This is equivalent to keeping zero values for the corresponding coupling coefficients in the set of linear motion equations. The following cases, both for DW (the general reference) and SW ($h/T = 1.2$), are included:

"246" – full roll coupling with sway and yaw;
"4" – pure roll (no coupling);

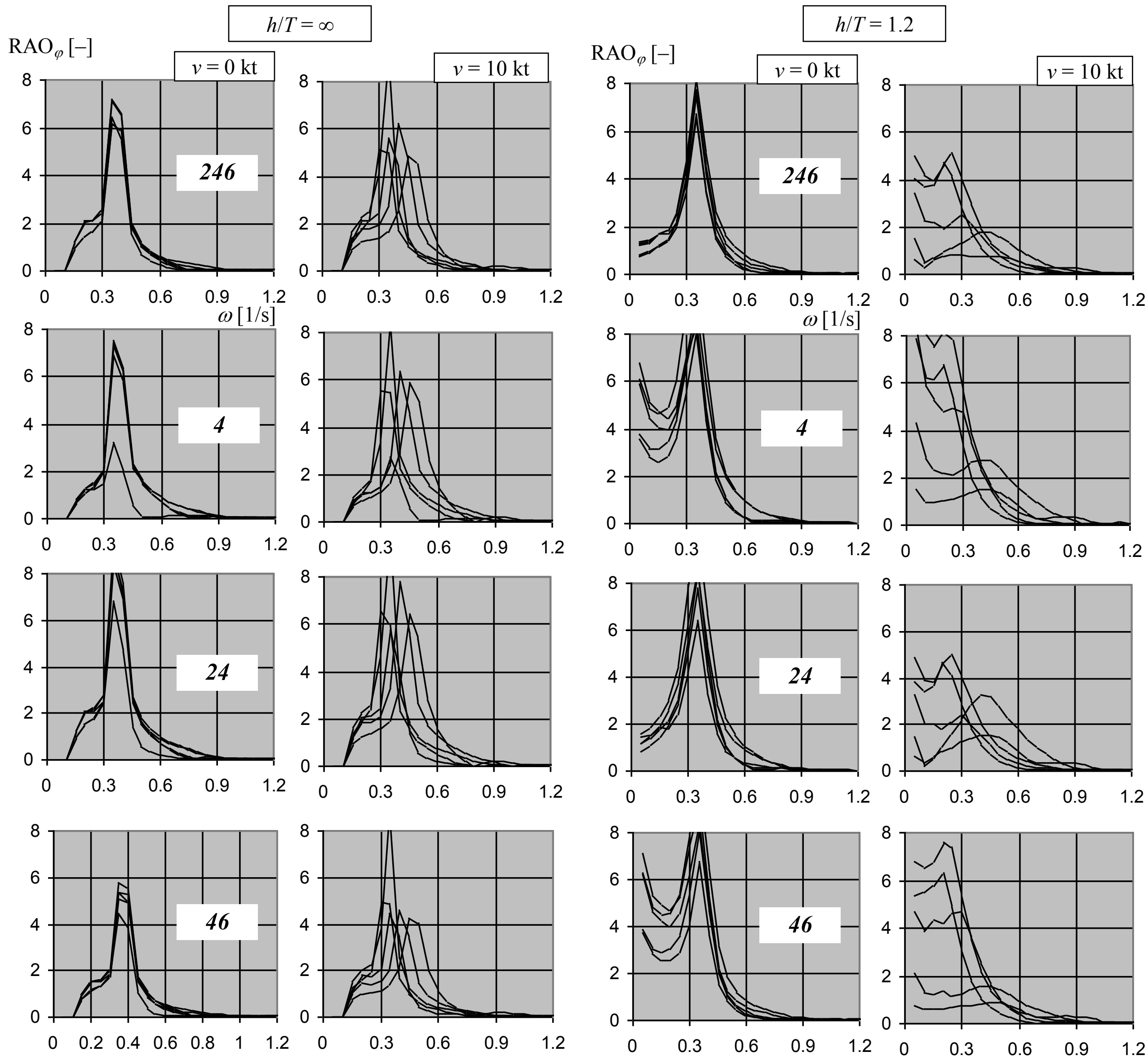

Fig. 6. Effect of motions coupling on roll RAO (DW and SW) – Keil's method

“24” – roll affected by sway only;
“46” – roll affected by yaw only.

The wave incidence angles examined are from 0° to 180° with increment of 30°, the markings of which have been removed from figure 6 for the purpose of readability and rough evaluation of the effects.

For DW, the case “46” gives lower RAOs, while on the other hand the variant “24” provides higher RAOs, so the sway and yaw cancel each other their effects on roll. That is why pure roll (the case “4”) is nearly identical to the fully coupled case. This would be a straight way to adopt the nonlinear model of pure roll. The situation with the roll RAO behavior changes in $h/T = 1.2$. The “24” case is almost the same as “246”, moreover, “46” and “4” also agree well. The latter leads to a conclusion that yaw coupling is not essential in simulation of roll in SW. The impact of sway shall be anyhow considered, because of possible accuracy problems at lower frequencies.

The coupling within symmetric motions is also very interesting. The investigations carried out for the same S-175 container vessel, and operational conditions as above, also covered all combinations of coupling. The computational results have surprisingly revealed absolutely no dependence of heave (being coupled only with pitch) on the surge motion. For the heave RAO, the case “135” against “35” does not exhibit any difference. The same is valid for RAO in pitch (being coupled only with heave) – “135” is also identical to “35” from the viewpoint of pitch motion. This specific behavior can be of course assigned to the surge equation algorithm implemented in SEAWAY.

The mutual coupling of heave and pitch in all conditions of speed and water depth is however rather slight. The effect of pitch upon heave is practically negligible – the case “35” for heave well compares to pure heave “3”. Additionally, one should keep in mind that the speed and water depth effects are rather small in heave, at least for the SEAWAY output.

When it comes to the behavior of pitch, the speed and water depth effects are more remarkable, but not too high. The impact of heave upon pitch is noticeable in the region of very low wave absolute frequencies (below 0.3). But for the rest of frequencies it can be concluded that “35” also converges to pure pitch (the “5” case). In the SW condition, the dependence of pitch on heave is even lower.

Since the results with regard to heave and pitch are not attractive for visual presentation – the plots of RAOs are similar to each other as stated above – they have been omitted in this paper.

Force RAOs

One of important aspects in an adequate modeling of seakeeping behavior is to properly measure or theoretically compute (the latter approach is more common for some reasons) the wave exciting forces. The wave forces, in terms of force or moment, consist of the Froude-Krylov (hydrostatic) and the diffraction (hydrodynamic) components. A good evaluation of wave forces, in addition to rather well recognized role of added masses and damping coefficients, essentially contributes to the seakeeping model quality. In view of the UKC problem, such wave forces are needed to be accurately assessed under definite shallow water conditions and combined with forward speed.

While a lot of available computational methods for wave exciting forces well converge in deep water, some remarkable differences (due to internally incorporated assumptions and/or approximations) arise in shallow water. This really requires further investigations from the viewpoint of achievable accuracy or asks for an appropriate judgment of the motion simulation output. There are major problems with the availability of experimental validation data.

The usually adopted linear theory for the wave calculations results in the linear dependence of the wave force amplitude (in any mode of motion) on the wave amplitude. Such proportionality coefficients (they are functions of the wave encounter frequency) are often referred to as the wave force transfer functions, or simply the wave force RAOs.

Figure 7 presents some output from the mentioned SEAWAY software for the same container vessel as in the previous chapters – only the forward speed 10 kt case is included. The results are however plotted (for a legibility and a possible further referencing) versus the wave absolute circular frequency, where the wave incidence angle (consciously not shown) is a parameter to this family of curves. The presented wave forces relating to heave, roll and pitch modes of motion are made dimensionless as follows:

$$\begin{aligned} \mathrm{RAO}_{Fz}[-] &= \frac{F_z}{\rho A_{WL} g \xi_{0WV}} \\ \mathrm{RAO}_{Mx}[-] &= \frac{M_x}{mgL \cdot k\xi_{0WV}} \\ \mathrm{RAO}_{My}[-] &= \frac{M_y}{mgFM_L \cdot k\xi_{0WV}} \end{aligned} \tag{20}$$

where:

F_z – exciting wave heave force [N];
M_x – exciting wave roll moment [Nm];

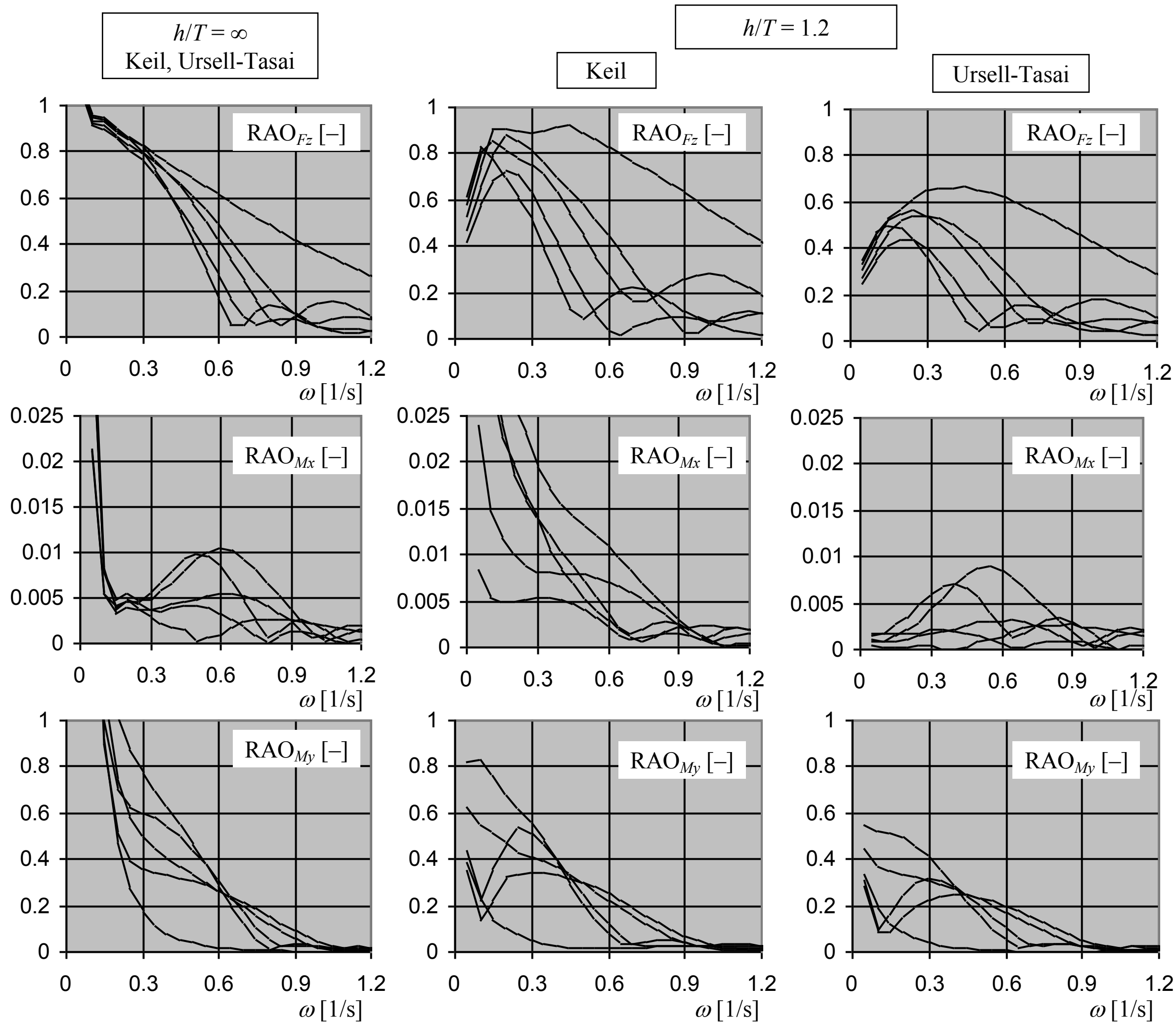

Fig. 7. Nondimensional wave force RAOs for speed 10

M_y – exciting wave pitch moment [Nm];
ρ – water density [kg/m^3];
A_{WL} – waterplane ares [m^2];
L – ship's length (between perpendiculars) [m];
FM_L – longitudinal metacentric radius [m].

The analyzed methods of Keil and Ursell-Tasai for the computation of the wave forces themselves are identical in deep water conditions ($h/T = \infty$). However, they significantly differ if "shallow water corrections" are to be applied. The studied in figure 7 condition of $h/T = 1.2$ is very representative, since the variation in behavior for other water depths (1.05 and 1.5), as also investigated, is very similar both, in shape and magnitude. The shallow water heave force by Keil is very close to the deep water instance, while the corresponding force by Ursell-Tasai is almost twice lower. In contrast, the wave excited roll moment in SW by Ursell-Tasai is reaching the value of DW. The Keil's roll moment in SW is much higher and is assuming a quite different shape, particularly in the low frequency range. Finally, the pitch moment by Ursell-Tasai is similarly much lower (here by 50%). Up to now, it cannot be decided which method is better suited for SW calculations, though the greater potential seems to be possessed by the method of Keil. Anyhow, further studies are recommended in this field, especially with other not mentioned procedures, as to gain the most comprehensive knowledge of the phenomena.

It also shall be kept in mind that even the wave heave force itself can induce the pitch motion due to the mentioned coupling between degrees of freedom.

Introducing in the present chapter the concept of wave force RAOs is very beneficial in discussing or supplementing the usual meaning/interpretation of the mechanical RAO, denoted by $\sqrt{D}^{-1}$ in chapter 1. Strictly speaking, the latter is essentially the transfer function between the excitation (of the force or moment nature) and ship motion itself.

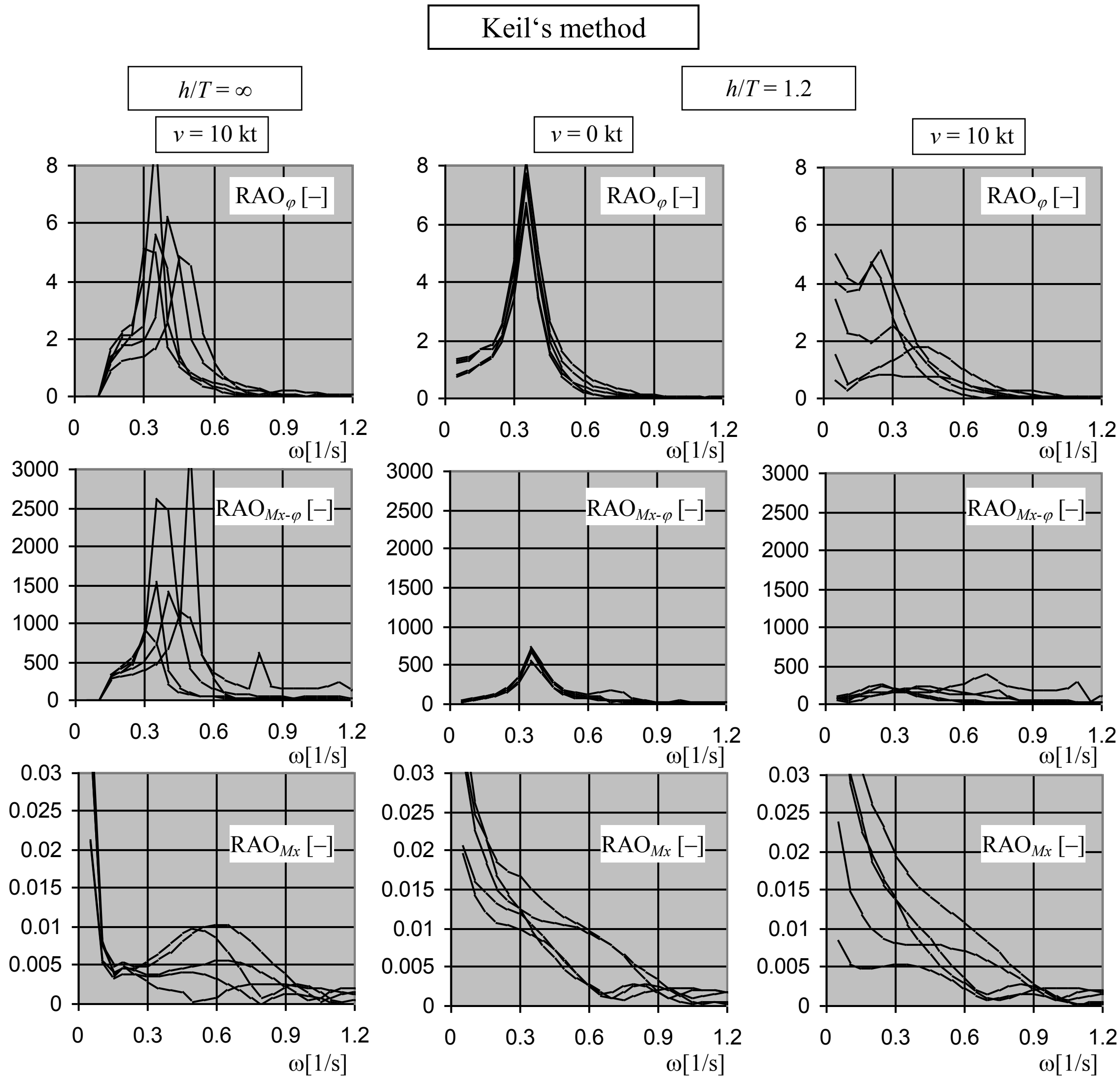

Fig. 8. Demonstration of the wave "force-to-motion" RAOs

Multiplying such a force-to-motion RAO (at discrete frequencies) by the wave force RAO (called the wave-to-force RAO) provides the common understanding of the motion RAO (often named the wave-to-motion RAOs) as widely used in the field of ship seakeeping.

Dividing the motions RAOs into two components, the one related to the wave external (exciting, forcing) action and other connected with the ship inertia dynamics, makes the modeling (tuning) of seakeeping behavior much easier.

Figure 8 illustrates this process of splitting the seakeeping behavior among two items: the force RAO (on the right-hand side of motion equations) and inertia-related RAOs (constituting the left side of the motion equations), with the special focus on shallow water. The Keil's method and the roll behavior were only used as an example. Such an approach can also be considered as a kind of a sensitivity analysis. However, one should keep in mind that the demonstrated force-to-motion RAOs (the middle part of figure 8) include the "noise" from the usual coupling between degrees of freedom, though being rather minor as stated in the previous chapter. This of course assures that one finally arrives at "the full" standard motion RAOs (i.e. wave-to-motion RAOs).

Presented in figure 8 the "standard" roll RAO (the upper part of this figure) corresponds to figure 6 (the top). The wave roll moment RAO (the bottom of figure 8) coincides with figure 7 (the middle part). Hence the force-to-motion RAO is written in nondimensional form by:

$$\mathrm{RAO}_{Mx-\varphi}[-]=\frac{\mathrm{RAO}_{\varphi}[-]}{\mathrm{RAO}_{Mx}[-]} \tag{21}$$

and presented in the middle part of figure 8.

By comparing all three rows of figure 8, it can be noticed that the most of seakeeping behaviour is taken from the inertia part of the model (added masses and damping terms) for deep water, and from the wave force part for the shallow water condition. However, in the latter case, in the aspect of the involved product of factors (and hence a sensitivity), the force-to-motion RAOs are also important.

Conclusions

Although the presented outcomes in the last three chapters are based on the SEAWAY software and the implementations of theoretical potential strip methods therein, other results in shallow water both, for the same container ship and other type and size of ships are welcome. The focus shall be made in such investigations on the accuracy of RAOs and equation parameters themselves. Both serve as different comparison / assessment indices of the resulting transient motion prediction reliability. A study on the role of coupling between degrees of freedom, from the viewpoint of the UKC seakeeping model simplification (reduction), is of utmost importance. Another question remains with regard to the role of the shallow water wave spectrum on the motion spectrum of ship's bottom points.

The proper (optimal) choice of the motion model structure would certainly allow the efficient numerical integration of the motion differential equations with frequency-dependent coefficients, particularly in real- and fast-time modes of motion simulation. In this context, a full integration of the seakeeping model with the ship manoeuvring (very low frequency) model, as providing the ship's roll dynamic behavior in close turns and squat transients (in terms of sinkage and trim) is just a matter of time for UKC prediction.

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