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Application of sequence adjustments in inland navigation position determination

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Abstract

The aim of the article was to evaluate accuracy in determining a vessel position's coordinates in respect to permanent signs, with sequence adjustments method applied. General theory of geodesic classical adjustment, based on a least square method and of sequence adjustments has been presented in its first part. The main part of the paper contains description of the research. It was conducted based on a vessel going from the seaport of Świnoujście to the seaport of Szczecin, determining the positions in respect to navigational structure made by aids in navigation located in Szczecin Lagoon. It consisted in simulating results of measurements in respect to marks of the structure (bearings and distances) and calculating position's coordinates and its accuracy based on the results. The calculations were conducted with classical geodesic adjustment and sequence adjustments methods applied. Considerations regarding results obtained from the research have been performed and general conclusions drawn in the end part.

Description of adjustment problem

Navigational situation described in this paper refers to the vessel run along the Świnoujście–Szczecin fairway. It is assumed that navigation conducted on boards of vessels is based on measurements made in respects to inland beacons providing highly accurate coordinates [1, 2]. Selection of navigational aids provides for new navigational marks taken into consideration during determination of the vessel's position in the next stages of navigation. Measurements of distances and bearings are used in this example.

Geometric system in the first adjustment stage, consisting with k – aids in navigation ($k = 1,...,n$) and a vessel going along the fairway to the seaport of Szczecin, enables to take down the following system of correction equations [3]:

$$\mathbf{V}_i = \mathbf{A}_{Z_i} \cdot \hat{\mathbf{d}}x_{Z_i} \tag{1}$$

$\mathbf{V}_i$ – corrections' vector of j observations in i position of the vessel; $\hat{\mathbf{d}}x_{Z_i}$ – vector of increments in respect to expected coordinates $(X^o_{Z_i}, Y^o_{Z_i})$ of Z vessel in i position;

$$\mathbf{A}_{Z_i} = \begin{bmatrix} \frac{\partial NR_1}{\partial x_{Z_i}} & \frac{\partial NR_1}{\partial y_{Z_i}} \\ \frac{\partial NR_2}{\partial x_{Z_i}} & \frac{\partial NR_2}{\partial y_{Z_i}} \\ \frac{\partial d_1}{\partial x_{Zi}} & \frac{\partial d_1}{\partial y_{Z_i}} \\ \frac{\partial d_1}{\partial x_{Zi}} & \frac{\partial d_1}{\partial y_{Zi}} \end{bmatrix} \quad \mathbf{L}_i = \begin{bmatrix} NR_1^0 - NR_1^{obs} \\ NR_2^0 - NR_2^{obs} \\ d_1^0 - d_1^{obs} \\ d_2^0 - d_2^{obs} \end{bmatrix}$$

Entering the designation:

$$\mathbf{A} = \lfloor \mathbf{A}_{Z_i} \rfloor \quad \text{as well as} \quad \hat{\mathbf{d}}_{x_i} = \left[\hat{\mathbf{d}}_{x_{Z_i}}\right]^{\mathrm{T}}$$

the equation (1) may be written down in the following form:

$$\mathbf{V}_i = \mathbf{A}_i \hat{\mathbf{d}}x_i + \mathbf{L}_i \tag{2}$$

Finally, the adjustment problem obtains the following form in the first stage of the calculations [4, 5, 6]:

$$\begin{cases} \mathbf{V}_i = \mathbf{A}_i \hat{\mathbf{d}}x_i + \mathbf{L}_i & \leftarrow \text{functional model} \\ \hat{\mathbf{C}}_{\mathrm{x}} = m_o^2 \mathbf{Q}_i = m_o^2 \mathbf{P}_i^{-1} & \leftarrow \text{statistical model} \\ \mathbf{\Phi}(\hat{\mathbf{d}}x_i) = \mathbf{V}_i^T \mathbf{P} \mathbf{V}^T = \min & \leftarrow \text{objective function} \end{cases} \tag{3}$$

where:

$$\mathbf{Q}_i = \begin{bmatrix} m_{NR_1}^2 & 0 & 0 & 0 \\ 0 & m_{NR_2}^2 & 0 & 0 \\ 0 & 0 & m_{d_1}^2 & 0 \\ 0 & 0 & 0 & m_{d_2}^2 \end{bmatrix}$$ – cofactors' matrix

P – is a matrix of executed observations.

Solution of this equations' system is as the following:

$$\begin{cases} \hat{\mathbf{d}}x_i = \mathbf{\Theta}_i \mathbf{I}_i \\ \mathbf{Q}_i = \mathbf{\Theta}_i \\ \hat{\mathbf{X}} = \mathbf{X}^0 + \hat{\mathbf{d}}x_i \end{cases} \tag{4}$$

where:

$$\hat{\mathbf{d}}x_i = \begin{bmatrix} \mathrm{d}\,x_{Z_i} \\ \mathrm{d}\,y_{Z_i} \end{bmatrix}, \quad \mathbf{\Theta}_i = \left(\mathbf{A}^T\mathbf{PA}\right)^{-1}, \quad \mathbf{I}_i = -\mathbf{A}^T\mathbf{PA},$$

$$\mathbf{X}^0 = \begin{bmatrix} X_{Z_i}^0 \\ Y_{Z_i}^0 \end{bmatrix}, \quad \hat{\mathbf{X}} = \left[\hat{\mathbf{X}}_{Z_i}\right]$$

$\hat{\mathbf{X}}$ – vector of adjusted coordinated of Z vessel's position.

As a result of the performed adjustment in the first stage, coordinates of the vessel in position $Z_i(\hat{X}_{Z_i}, \hat{Y}_{Z_i})$ have been obtained. Second stage of the adjustment starts the moment the vessel reaches Z_{i+1} position. Then, navigator present on board of the ship determines j observations in respect to visible navigational marks. The following system of correction's equations may be created for this navigational situation [6, 7]:

$$\mathbf{V}_{i+1} = \mathbf{A}_{Z_{i+1}} \hat{\mathbf{d}}x_{Z_{i+1}} + \mathbf{L}_{i+1} \tag{5}$$

$\hat{\mathbf{d}}x_{Z_{i+1}}$ – increments' estimator of Z vessel position's coordinates in position $i+1$ as a sum of previous evaluation of $\hat{\mathbf{d}}x_{Z_i}$ increments' vector and $\mathbf{V}_{Z_{i+1}}$ corrections under determination:

$$\hat{\mathbf{d}}x_{Z_{i+1}} = \hat{\mathbf{d}}x_{Z_i} + \mathbf{V}_{Z_{i+1}} \tag{6}$$

where:

$\hat{\mathbf{d}}x_{Z_i}$ – stands for absolute term in second stage.

Determining the $\mathbf{V}_{Z_{i+1}}$ vector of corrections from the (6) dependence, one may form the following system of corrections' equations for the $i+1$ stage:

$$\begin{cases} \mathbf{V}_{i+1} = \mathbf{A}_{Z_{i+1}} \hat{\mathbf{d}}x_{Z_{i+1}} + \mathbf{L}_{i+1} \\ \mathbf{V}_{Z_{i+1}} = \hat{\mathbf{d}}x_{Z_{i+1}} - \hat{\mathbf{d}}x_{Z_i} \end{cases} \tag{7}$$

$$\widetilde{\mathbf{V}}_{i+1} = \begin{bmatrix} \mathbf{V}_{i+1} \\ \mathbf{V}_{i(II)} \end{bmatrix}, \quad \mathbf{A}_{i+1} = \begin{bmatrix} \mathbf{A}_{Z_{i+1}} & \mathbf{A}_{Z_i(II)} \\ 0 & \mathbf{I} \end{bmatrix},$$

$$\hat{\mathbf{d}}x_{i+1} = \begin{bmatrix} \hat{\mathbf{d}}x_{Z_{i+1}} \\ \hat{\mathbf{d}}x_{Z_{i(II)}} \end{bmatrix}, \quad \widetilde{\mathbf{L}}_{i+1} = \begin{bmatrix} \mathbf{L}_{i+1} \\ -\hat{\mathbf{d}}x_{Z_i} \end{bmatrix}$$

As a result of the above, the (7) system of equations obtains the following form:

$$\widetilde{\mathbf{V}}_{i+1} = \mathbf{A}_{i+1} \hat{\mathbf{d}}x_{i+1} + \widetilde{\mathbf{L}}_{i+1} \tag{8}$$

Ultimately, the adjustment problem in the second stage becomes as the following [5, 8, 9]:

$$\begin{cases} \widetilde{\mathbf{V}}_{i+1} = \mathbf{A}_{i+1} \hat{\mathbf{d}}x_{i+1} + \widetilde{\mathbf{L}}_{i+1} \\ \widetilde{\mathbf{P}}_{i+1} = \mathbf{Q}_{i+1}^{-1} \\ \widetilde{\mathbf{V}}_{i+1}^T \widetilde{\mathbf{P}}_{i+1} \widetilde{\mathbf{V}}_{i+1} = \min \end{cases} \tag{9}$$

where:

$$\widetilde{\mathbf{Q}}_{i+1} = \begin{bmatrix} \mathbf{Q}_{i+1} & \vdots & \underline{0} \\ \cdots & \vdots & \cdots \\ \underline{0} & \vdots & \mathbf{Q}_{Z_i} \end{bmatrix} =$$

$$= \begin{bmatrix} m_{NR_3}^2 & 0 & 0 & \underline{0} & 0 & \vdots & 0 \\ 0 & \ddots & 0 & 0 & 0 & \vdots & \underline{0} \\ 0 & 0 & m_{NR_{j+1}}^2 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & m_{d_3}^2 & 0 & \vdots & 0 \\ 0 & 0 & 0 & 0 & m_{d_{j+1}}^2 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \vdots & \mathbf{Q}_{Z_i} \end{bmatrix}$$

The below given system of equations makes solution of this (9) adjustment problem:

$$\begin{cases} \hat{\mathbf{d}}x_{i+1} = \mathbf{\Theta}_{i+1} \mathbf{I}_{i+1} \\ \widetilde{\mathbf{Q}}_{i+1} = \mathbf{\Theta}_{i+1} \end{cases} \tag{10}$$

where:

$$\mathbf{\Theta}_{i+1} = \left(\mathbf{A}_{i+1}^T \widetilde{\mathbf{P}}_{i+1} \mathbf{A}_{i+1}\right)^{-1}, \quad \mathbf{I}_{i+1} = -\mathbf{A}_{i+1} \widetilde{\mathbf{P}}_{i+1} \widetilde{\mathbf{L}}_{i+1},$$

$$\hat{\mathbf{d}}x_{i+1} = \begin{bmatrix} \hat{\mathbf{d}}x_{Z_{i+1}} \\ \hat{\mathbf{d}}x_{Z_{i(II)}} \end{bmatrix}$$

Finally, the adjusted coordinates of the Z vessel in $i + 1$ position are to be obtained from the following dependence [3, 9]:

$$\hat{\mathbf{X}}_{i+1} = \mathbf{X}_{i+1}^0 + \hat{\mathbf{d}}x_{i+1} \tag{11}$$

where:

$$\hat{\mathbf{X}} = \begin{bmatrix} \hat{X}_{Z_{i+1}} \\ \hat{Y}_{Z_{i+1}} \end{bmatrix}, \quad \mathbf{X}_{i+1}^0 = \begin{bmatrix} X_{R_{i+1}}^0 \\ Y_{R_{i+1}}^0 \\ X_{P_{i+1}}^0 \\ Y_{P_{i+1}}^0 \end{bmatrix}, \quad \hat{\mathbf{d}}x_{i+1} = \begin{bmatrix} \hat{\mathrm{d}}\,x_{Z_{i+1}} \\ \hat{\mathrm{d}}\,y_{Z_{i+1}} \end{bmatrix}$$

The (5)–(11) calculations are to be repeated in the next stages of the adjustment (moments of determination of the vessel's positions under observations).

Numerical tests

Let's assume that the vessel goes along $KR_{i=1\ldots n} = 140.0°$ course, as it is given on figure 1. Navigator present on board of the "*Z*" vessel, at the time of the entire example duration, is observing four inland beacons of rectangular coordinates, in "UTM" imaging, given in table 1. The system of rectangular coordinates is accepted due to simplification of calculation procedures, targeted on verification of the theoretical assumptions connected with determination of the vessel's position.

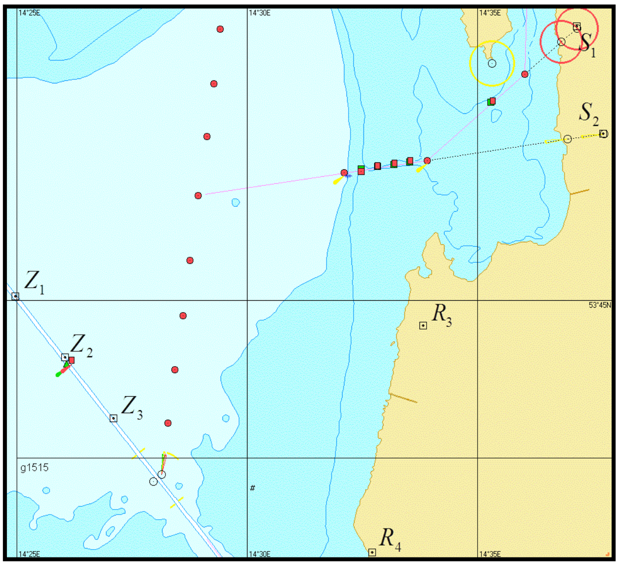

Fig. 1. Graphical interpretation of vessel's route [own study]

Table 1. Rectangular coordinates of navigational marks and vessels under operation in presented example

Navigational mark, vessel	Geographical coordinates (WGS 84 ellipsoid)		„UTM" coordinates	
	φ	λ	X [m]	Y [m]
S1 – mark 1	53°48.5' N	014°37.2' E	5962223.5	474928.4
S2 – mark 2	53°47.1' N	014°37.7' E	5959694.6	475557.3
R3 – mark 3	53°44.7' N	014°33.8' E	5955200.2	471227.2
R4 – mark 4	53°41.8' N	014°32.7' E	5949862.1	469983.9
Z1 – vessel Z_i	53°45.1' N	014°24.8' E	5956034.9	461392.8
Z2 – vessel Z_{i+1}	53°44.3' N	014°26.0' E	5954494.9	462658.6
Z3 – vessel Z_{i+2}	53°43.5' N	014°27.1' E	5953059.7	463827.3

For the needs of this article, entire route of the vessel has been divided into three stages.

Stage I – the vessel reaches position Z1;
Stage II – the vessel reaches position Z2;
Stage III – the vessel reaches position Z3.

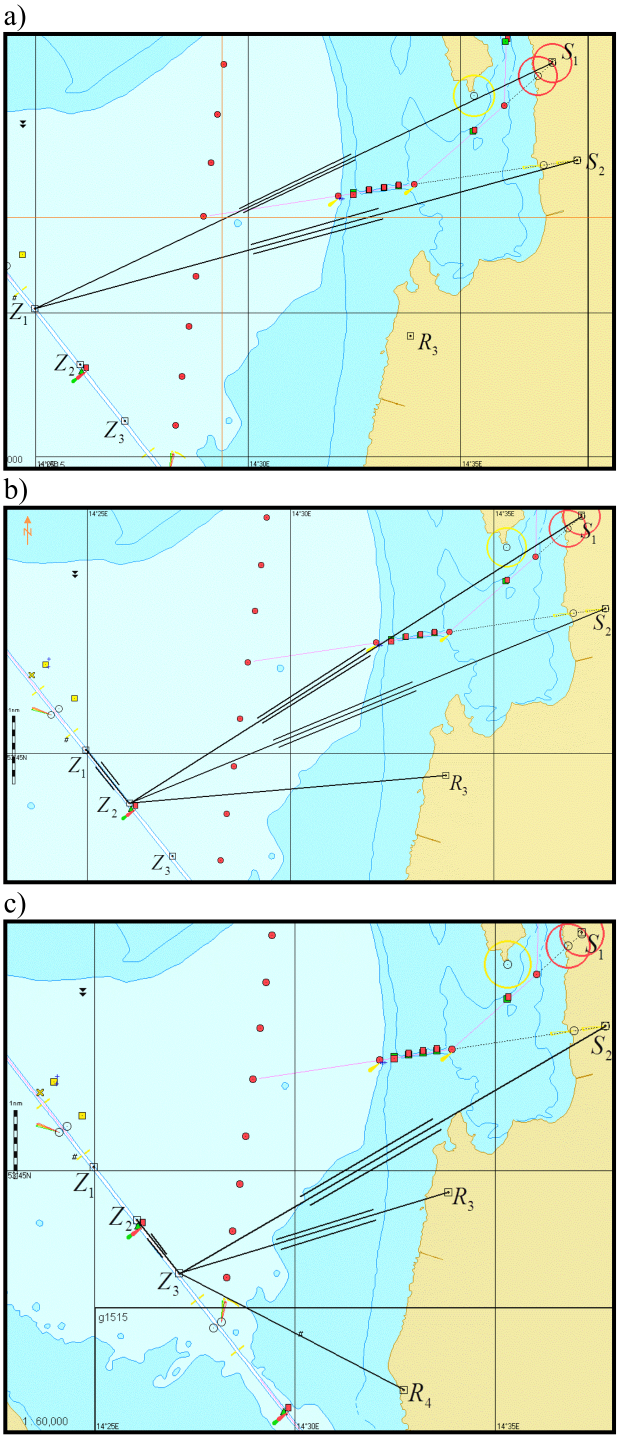

Fig. 2. Graphical interpretation of a) stage I, b) stage II, c) stage III [own study]

Let's assume, at the beginning, that the navigator present on board of the ship at the "*i*" moment, executed bearings and distances in respect to the first (S1, S2) navigational marks. The measured bearings and distances have the following values: $NR_1 = 065.0°$; $NR_2 = 075.0°$; $d_{R1} = 14\,874.9$ m;

d_{R2} = 14 620.8 m. Increments in respect to the Z_i, vessel's coordinates were determined, in accordance with the (1)–(4) dependences, and they are, respectively, as the following:

$$\hat{\mathbf{d}}x_{Z_i} = \begin{bmatrix} \hat{\mathrm{d}}\, x_{Z_i} \\ \hat{\mathrm{d}}\, y_{Z_i} \end{bmatrix} = \begin{bmatrix} -48.76 \\ 26.53 \end{bmatrix}$$

resulting in the corrected position of the vessel at the "i" moment (the Z_i vessel):

$$\hat{\mathbf{X}}_i = \begin{bmatrix} \hat{X}_{Z_i} \\ \hat{Y}_{Z_i} \end{bmatrix} = \begin{bmatrix} 5955986.2 \\ 4614193 \end{bmatrix}$$

Application of the sequence adjustments obliges to determine only matrix of cofactors in the intermediate stages. This matrix obtains the following form in the first stage of the calculations:

$$\mathbf{Q}_{Z_i} = \begin{bmatrix} 3245.5 & -1133.3 \\ -1133.3 & 452.3 \end{bmatrix}$$

In the second stage, new bearings and distances from the vessel to the navigational marks of highly accurate (S1, S2) coordinates were determined, as well as an additional bearing onto a visible (R3) mark, hence enhancing our navigational structure. The said navigational parameters obtain the following values: NR_3 = 057.5°; NR_4 = 068.0°; NR_5 = 085.0°; d_{R3} = 14 490.5 m; d_{R4} = 13 896.0 m. In the determination process of the stage II also $KR_{i=1...n}$ = 140.0° course of the going vessel, the covered distance ($d_{i=1...n}$ = 1985.0 m) and coordinates of the previous Z_i position were taken into consideration. According to the (5)–(11) dependences, increments in respect to the coordinates of the vessel operating at sea at the „i+1" moment were determined, and they, respectively, are as the following:

$$\hat{\mathbf{d}}x_{Z_{i+1}} = \begin{bmatrix} \hat{\mathrm{d}}\, x_{Z_{i+1}} \\ \hat{\mathrm{d}}\, y_{Z_i+1} \end{bmatrix} = \begin{bmatrix} -35.48 \\ 29.05 \end{bmatrix}$$

resulting in appropriately corrected position of the vessel at the „i+1" moment (the Z_{i+1} vessel):

$$\hat{\mathbf{X}}_{i+1} = \begin{bmatrix} \hat{X}_{Z_{i+1}} \\ \hat{Y}_{Z_{i+1}} \end{bmatrix} = \begin{bmatrix} 5954459.4 \\ 4626877 \end{bmatrix}$$

Matrix of cofactors in the stage II of the calculations is, adequately, as the following:

$$\mathbf{Q}_{Z_{i+1}} = \begin{bmatrix} 92.3 & -19.6 \\ -19.6 & 50.0 \end{bmatrix}$$

Third stage consists in measurements of next three bearings onto (S2, R3, R4) navigational marks, with simultaneous adoption of a new R4 mark into the existing navigational structure and two distances to the (S2 and R3) marks. The said parameters have, respectively, the following values: NR_6 = 060.0°; NR_7 = 074.0°; NR_8 = 117.0°; d_{R5} = 13 470.5 m, d_{R6} = 7695.0 m. Moreover, similar to the stage II, in the third stage, also the $KR_{i=1...n}$ = 140.0° course of the going vessel, the covered route ($d_{i=1...n}$ = 1860.0 m) and coordinates of the previous position – Z_{i+1} in this case – were taken into consideration. Increments in respect to the operating vessel's coordinates were determined, in accordance with the (1)–(4) and (5)–(11) dependences, and they are, respectively, as the following:

$$\hat{\mathbf{d}}x_{Z_{i+2}} = \begin{bmatrix} \hat{\mathrm{d}}\, x_{Z_{i+2}} \\ \hat{\mathrm{d}}\, y_{Z_{i+2}} \end{bmatrix} = \begin{bmatrix} -44.0 \\ 29.48 \end{bmatrix}$$

resulting in appropriately corrected position of the vessel at the „i+2" moment (the Z_{i+2} vessel):

$$\hat{\mathbf{X}}_{i+2} = \begin{bmatrix} \hat{X}_{Z_{i+2}} \\ \hat{Y}_{Z_{i+2}} \end{bmatrix} = \begin{bmatrix} 5953015.7 \\ 4638567 \end{bmatrix}$$

Analysis of covariance matrix allowed setting a mean error of the vessel coordinates' determination in particular stages of navigation [10, 11]. Quality of obtained determinations has been presented in a form of confidence ellipse. The above said accuracy parameters of the position determination are given in table 2.

Table 2. Accuracy parameters of determined position's coordinates

Vessel's positions	Mean error of position	Elements of confidence ellipse (for γ = 95)			Notes
		„a" – big semi-axis	„b" – small semi-axis	φ – torsional angle of ellipse	
	[m]	[m]	[m]	[°]	
Vessel in Z1 position ("i" moment)	42.42	259.7	30.5	–19.5	Stage I (without sequence)
Vessel in Z2 position ("i+1" moment)	6.06	40.9	26.7	–21.4	Stage II (with sequence)
Vessel in Z3 position ("i+2" moment)	4.19	27.1	18.4	–37.4	Stage III (with sequence)

Graphical interpretation of the above said confidence ellipses in particular stages for the vessel has been presented on figure 3.

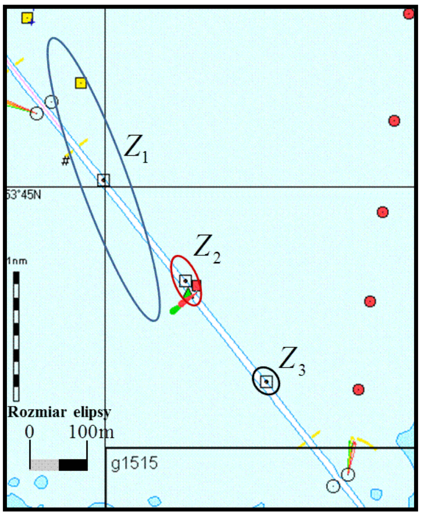

Fig. 3. Graphical interpretation of confidence ellipses of towed vessel [own study]

Accuracy of the proposed method is based, first of all, on accuracy of the measured navigational parameters. In maritime navigation, it is accepted that [1, 2]:

– error of course and bearing determination coming from device most commonly used for the reason, i.e. gyro-compass, is contained in the $\sigma_{KŻ} = \sigma_{NŻ} = 0.5°\div1.5°$ range. The value of $\sigma_{KŻ} = \sigma_{NŻ} = 0.5°$ has been accepted in the presented example;
– mean error of distance determination depends on used device. In case of sextant, for instance, the error of angle measurement is accepted in the $\sigma_S = 1.0'\div2.0'$ range. Thus, the mean error of determination of distance to an inland object, as well as to a towed vessel may be accepted in the $\sigma_d = 5.0\div10.0$ m range. In the presented example, the value of $\sigma_d = 10.0$ m has been accepted for both, the distance measurement and the covered route between particular stages.

Conclusions

1. With a use of the sequence adjustments, one has an influence on accuracy level of a vessel position determination – by way of considering, in the navigation process, newly executed navigational parameters with parameters executed earlier, with the navigational marks used for determination of those parameters simultaneously taken into consideration.
2. Application of mathematical dependences in the sequence adjustments enables to shorten, to an indispensable minimum, a process of calculations connected with determination of accuracy level of the position determination.
3. The sequence adjustments enable quick verification of knowledge about accuracy of a vessel position (kind of arrangement of confidence ellipses' semi-axes in respect to axis of a fairway), smaller deviation of the vessel centerline's position from the set voyage route.
4. The presented method may be applied under the following conditions:
   – highly accurate counted coordinates of the vessel should be taken for calculations (then, one may obtain estimated position from iteration in one step – so as it has been presented in this article);
   – navigational parameters should be determined with the highest possible accuracy, by means of the most accurate equipment available on board of the ship.

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