Scientific Journals
Maritime University of Szczecin

Zeszyty Naukowe
Akademia Morska w Szczecinie

2012, 30(102) pp. 37–45

2012, 30(102) s. 37–45

Quantum issues in diagnostics of marine energy machines and devices

Zagadnienia kwantowe w diagnostyce okrętowych urządzeń energetycznych

Jerzy Girtler

Gdańsk University of Technology, Faculty of Ocean Engineering & Ship Technology
Department of Marine and Land Power Plants,
Politechnika Gdańska, Wydział Oceanotechniki i Okrętownictwa
Katedra Siłowni Morskich i Lądowych
80-233 Gdańsk, ul. Narutowicza 11/12, e-mail: jgirtl@pg.gda.pl

Key words: diagnostics, quantum mechanics, marine energy machines / devices

Abstract

The paper rises the issue of a possibility to consider some achievements of the quantum mechanics in the diagnostics of marine energy machines and devices. It has been shown that the observations from the quantum mechanics and technical diagnostics obtained in the knowledge field of the empirical research are convergent. The need has been justified as for empirical research to make not only calculations of errors, but also to determine the uncertainty of test results (measurements) and identify whether they are mainly the result of inaccuracy of energy machines / devices as the research objects – or inaccuracy of the measuring technique (measuring methods and means). Moreover, the possibility of using mathematical statistics has been presented for describing the results of the studies connected with determining the instantaneous values of the measured quantities. It has been shown at the same time the necessity for the empirical research of a clear definition of the instantaneous value characterized by two coordinates: a measured value of a physical quantity and a value of time which the measured value of the physical quantity is assigned to.

Słowa kluczowe: diagnostyka, mechanika kwantowa, okrętowe urządzenie energetyczne

Abstrakt

W artykule podjęto zagadnienie możliwości uwzględniania części dorobku mechaniki kwantowej w diagnostyce okrętowych urządzeń energetycznych. Wykazano, że są zbieżne spostrzeżenia mechaniki kwantowej i diagnostyki technicznej uzyskane w obszarze wiedzy z zakresu badań empirycznych. Uzasadniono konieczność wykonywania podczas badań empirycznych, nie tylko rachunku błędów, lecz także określania niedokładności wyników badań (pomiarów) wraz z ustaleniem, czy są one głównie wynikiem niedokładności urządzeń energetycznych jako obiektów badań, czy też – niedokładności techniki pomiarowej (metod i środków pomiarowych). Przedstawiono także możliwość zastosowania statystyki matematycznej do opracowania wyników badań związanych z określaniem wartości chwilowych wielkości mierzonych. Wykazano przy tym konieczność stosowania w badaniach empirycznych jednoznacznej definicji wartości chwilowej, która jest charakteryzowana przez dwie współrzędne: wartość mierzoną wielkości fizycznej i wartość czasu, której przyporządkowana jest wartość mierzona wielkości fizycznej.

Introduction

In diagnostics of marine energy machines / devices (e.g. internal combustion piston engines and gas turbine engines, displacement and axial compressors, pumps, boilers and others), like in diagnostics of other diagnosed systems (*SDN*), diagnostic tests are performed in the first place, which are followed by different types of diagnostic inferences, like based on: signal, symptom, measurement, structure and operation [1]. Diagnostic investigation consists inter alia in measuring the values of diagnostic parameters of signals generated by the machines / devices, with measuring instruments of a diagnosing system (*SDG*). Diagnostic study enables making sets of measurement results which include measurement errors. Analysis of the errors may be made by using achievements of surveying [2].

The analysis of measurement results for marine energy machines / devices cannot indiscriminately use deterministic measuring methods and make calculation (analysis) of errors, because the measurements are performed generally in the conditions of uncertainty. This requires to apply a statistic approach to determine not only measurement errors, but also their inaccuracy. This results from that the values of the measured physical quantities characterizing energy properties of marine machines / devices (e.g. temperatures of piston crowns or bearing bushing of crank-piston mechanisms in main engines, etc.) undergo changes while being measured. This means that during diagnostic tests one cannot expect a specific measurement outcome, but only a frequency of obtaining the values of the measured quantities. From this reason it is important to define inaccuracies of the measures. One of the main concerns referred to identification of the technical condition of marine energy machines / devices is undoubtedly the quantum nature of many phenomenon which makes that the obtained set of values of the measured quantities is discrete in nature. This follows not only from the features of the applied measuring methods, but most of all from the used measuring instruments which are characterized by a limited angular resolution. That is why in the surveying the definition of measurement has been accepted as an action in which it can be stated that at a given time and under defined conditions and by applying proper methods and measuring instruments, as well as by performing suitable measuring actions, the measured quantity W has had a value falling into the interval of values of [a, b], i.e. the relation [2, 3, 4] has been satisfied:

$$a \le w \le b \qquad (1)$$

Establishing during the measurement that the value w is not lower than a, and at the same time is not greater than b, constitutes the result of measurement and the difference $b - a = 2\varepsilon$ is called the threshold of sensitivity. This threshold of sensitivity different for different measurement instruments is their indispensable attribute associated with precision of their performance, but also with imperfection of the senses of the person performing the measurement, and cannot be reduced to zero [2]. Thus, acceptance of the fact that $2\varepsilon > 0$ is a basic assumption (postulate) of the metrology. This imperfection of the measuring instruments and perception capabilities of the person performing the measurement makes it impossible to distinguish the two neighboring values of indication of the measuring instrument, which differ by less than the value of 2ε and just for this reason this value is called (as already mentioned) the threshold of sensitivity. Which measuring instruments will be used and what therefore the thresholds of sensitivity shall be, depend not only on the physical quantities whose values must be measured, but also on the methods used in such studies. For this reason it is important to analyze the usefulness of different methods and to select this method which will provide the least inaccuracy in the measuring technique applied in the studies, and this is because the adoption of a given research method decides also which measuring instruments are to be used for the diagnostic study. The mentioned uncertainty of the measurement may result from inaccuracy of the standard and imprecision of performance of the measurement procedure [3, 4].

From the presented considerations follows that it must be tendency to scrupulous finding the reasons of inaccuracy of the measurements in diagnostics of marine energy machines / devices in order to be able to recognize the obtained research results as sufficiently reliable and therefore useful not only for utilitarian purposes, but also cognition ones [5, 6]. The fulfillment of this second condition enables development of new knowledge concerning the technical diagnostics of energy systems, which is essential for science. Development of such knowledge requires, however, application in addition to the methods directly connected with performing the diagnostic tests, also the methods of non-deductive (inductive) inference. If necessary, the methods of deductive reasoning can also be used. However, for these studies the inductive methods allowing verification of hypotheses should be the favorites, as the inductive methods [3, 6]:

- enable investigation of empirical systems;
- are more useful than deductive, because empiricism and the associated induction are easier to apply.

Whereas application of deduction requires:

- scientific mind with predisposition to acquire knowledge from the field of mathematical sciences;
- high skills for creative thinking (formulating and justifying sentences in accordance with the direction of logical reasoning);

- high intuition, i.e. skills (ability) to guess the relations existing between the facts identified during research and the causes which generate them;
- deep and extensive knowledge within epistemology, formal logic and the theory of empirical research.

In this sort of studies where new scientific knowledge is important, the method of analogy should also be applied in addition to inductive and deductive methods. This is because the analogy enables searching for a common reason among the various objects of studies, while [6]:

- induction consists in matching a reason to the consequence (because it is a kind of reasoning that leads from a detailed truth considered as firm to an uncertain general truth);
- deduction consists in matching a consequence to the reason (because it is in fact a kind of inference that leads from a general truth to a truth in details).

Using an analogy, however, usually requires employment of achievements from other fields of knowledge.

For the purpose of developing a new knowledge in the field of technical diagnostics the quantum mechanics seems to be interesting, which is regarded as a theory laying at the basis of the modern science and technology. This statement follows from that [3, 7, 8, 9]:

- quantum theory is the basis of: atomic and molecular physics, physics of solids, liquids, gases and plasma, nuclear and molecular physics, molecular biology, genetic engineering, quantum chemistry, cryophysics (quantum liquid), interaction of electromagnetic radiation, etc.;
- quantum theory has enabled development of spectroscopy, holography, electronics and microelectronics (today there are used more and more complex microelectronic systems of measurement), crystal structure dynamics (which considers quantization of energy);
- without the quantum theory there would not exist such systems like a laser, transistor, DNA molecule, it would not be possible to explain such natural phenomena as electrical conductivity, the presence of lines in atomic spectra;
- till now the quantum theory has not been yet extended within physical sciences only for gravitational phenomena (but the works on development of the quantum theory of gravity are in progress) and for the physics of atomic nuclei and elementary particles in the very high energies and small scales of length.

Considering the listed achievements of the quantum mechanics and taking into account the existing knowledge within the technical diagnostics of marine energy machines / devices there can be noticed the reasons leading to take advantage of the achievements of the quantum theory.

Reasons for using achievements of the quantum mechanics in diagnostics

The general reasons for taking advantage of the achievements of the quantum mechanics in the diagnostics of marine energy machines / devices can include as follows [3, 7]:

1) the quantum theory is regarded as one of the most perfect theories applied in various fields of physics;
2) no incompatibility has been recorded between the quantum theory and the results of empirical research in the fields of knowledge where it has been used;
3) the quantum mechanics tends to describe the relationships between macroscopic phenomena, events and processes that have been initiated by microobjects;
4) knowledge of surveying where the quanta of the measured quantities must be used as in the quantum mechanics, is applied in diagnostic investigations;
5) properties of energy machines / devices are expressed with physical quantities which are of random nature.

The listed reasons can be further justified by the research situation existing in the diagnostics of marine energy machines / devices which causes the randomness and unpredictability of the registered events. It is known that [4, 10, 11, 12]:

- in operation of marine energy machines / devices there exists a degree of uncertainty of the research results, which is associated with all the investigated phenomena and events (e.g.: wear, damage, generation of diagnostic signals, etc.);
- it is impossible to predict accurately the changes in the energy technical properties of energy marine machines / devices in time;
- there are changes in condition of marine energy machines / devices (as the objects of studies) during measuring and errors of the applied measuring methods and instruments;
- negligible variation can be determined by applying the calculation of errors;
- significant variation requires application of the statistical estimation;
- true value (correct value) is an abstract concept;

- it is necessary to assume that the arithmetic mean obtained from the measurements is different than the true value of the measured physical quantity;
- one cannot expect the same results, but can expect the same frequency for the given result.

The considerations show that uncertainty of the results obtained during measurements arises in diagnostic studies of energy machines / devices. Therefore, it is necessary to determine the reasons of the uncertainty of the results. The most significant reasons of the uncertainty of the test results may be as follows [2, 3, 4, 12]:

- quantization of changes in properties of each of energy machines / devices (energy, wear):

$$E_{\max}=\underset{\underset{e}{\searrow}}{E_1}\to\underset{\underset{e}{\searrow}}{E_2}\to\ldots\to\underset{\underset{e}{\searrow}}{E_{n-2}}\to\underset{\underset{e}{\searrow}}{E_{n-1}}\to E_n=E_{\min} \tag{2}$$

where:

E_i (i = 1, 2,..., n–1) energies determined in the result of recording (by a diagnosing system) successive drops in energy (E) of the machine / device, in the form of a portion (quantum) e;

$E_{\max}$ – maximum energy that can be generated by a machine / device at the time of proper operation, possible to be registered by the diagnosing system;

$E_{\min}$ – minimum energy that can be generated by a machine / device at the time of its failure (it does not require the possibility to be registered by the diagnosing system);

- quantum symptom of changes of the physical quantities characterizing: the flow of heat or electric current, energy radiation in the form of a stream of particles or electromagnetic waves, radioactive decay, etc.;
- in metrology “quantum of time” has been implemented instead of “time”;
- digital signal is a quantity quantized and sampled, encumbered with errors of: quantization, aperture and error of sampling time;
- set of tensions generated by the analog-digital converter is discrete, successive voltage values differ by a quantum “q”;
- there are used programs for generating the reference voltage (U_w) called “quantum after quantum programs” which generate in successive steps „i” the voltage $U_w = i \cdot q$, q – voltage quantum value;
- frequency is quantized by nature in time;
- the essence of measurement is quantized $a \le w \le b \;\to\; b-a=2\varepsilon$;
- values are measured with random variables;
- conditions that cause the variation of measures are variable;
- there is inaccuracy of the applied methods and measuring instruments;
- measurement duration is different due to the necessity of repeating the measurement;
- experiences of persons performing measurements are different;
- there is inaccuracy of properties of energy machines / devices as the objects of studies;
- recording of instantaneous values is a stochastic process;
- ambiguous causality implies ambiguous determinism, i.e. determinism resulting from the probabilistic laws of the quantum mechanics.

From the considerations results that the changes in both technical and energy conditions of marine energy machines / devices will have a quantum nature. These changes investigated during operation of the machines / devices will constitute the realizations that can be considered as stochastic processes with discrete states and continuous time. An example of realization of such a process demonstrating changes in dissipated energy (diffusion) of any marine energy machine / device is shown in figure 1.

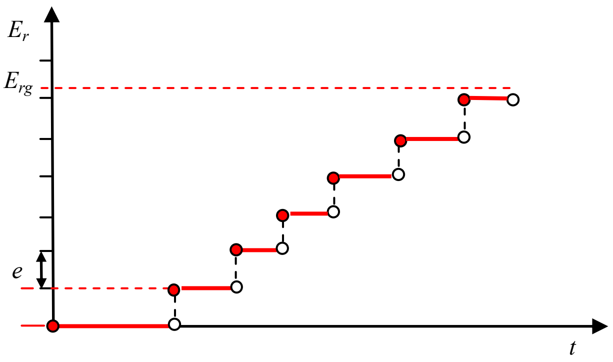

Fig. 1. The interpretation of the process of cumulating dissipated energy E_r for a marine energy machine / device: e – energy portion (quanta) which the energy E_r, is decreased by, E_{rg} – dissipated energy in the limit

Rys. 1. Interpretacja procesu narastania energii rozproszonej E_r okrętowego urządzenia energetycznego: e – porcja energii (kwant), o który ulega zmniejszeniu energia E_r, E_{rg} – energia rozpraszana graniczna

The presented research situation shows that the uncertainty of test results must be taken into account in the technical diagnostics of marine energy machines / devices. It is necessary because diagnostic testing of each energy machine / device is the first link in the diagnosing chain. Results of such study have a significant influence on the reliability of diagnostic inference (based on symptom, measure, structure and operation) [5]. In this relation the consequences of uncertainty of the test results should be identified.

Consequences of uncertainty of test results

The most important consequences of uncertainty of measurement results for such research objects as marine energy machines / devices [3, 4] are as follows:

- measurement indefiniteness arises due to existence of inaccuracy of the research object and measurement technology, which should be explained at least in the scope whether the mentioned inaccuracy is mainly an effect of:
  - changes in energy machines / devices as research objects, resulting mainly from influence of interfering factors, or
  - errors of the applied methods and measuring means (errors in measuring techniques);
- it is important to:
  - estimate the value of inaccuracy of the measuring technique, and thus determine the values of $\pm e_{\mathrm{TP}}$;
  - select the proper proportions of inaccuracy of the measuring technique ($\pm e_{\mathrm{TP}}$) to the existing inaccuracy of the research object ($\pm e_{\mathrm{SDN}}$).

This kind of consequences of the uncertainty in measurement can be simply explained in a form of a graph presented in figure 2.

The figure 2 shows that for the existing inaccuracy of the research object (as SDN1) $2e_{\mathrm{SDN1}}$ the more suitable is SDG with the inaccuracy of the measuring technique $2e_{\mathrm{TP1}}$. While in the case of inaccuracy of the research object (as SDN2) $2e_{\mathrm{SDN2}}$ it must be recognized that SDG with inaccuracy of the measuring technique $2e_{\mathrm{TP}}$ is not proper for it. Additionally, for diagnostic testing of this object as SDN2, SDG should be used with inaccuracy of the measuring technique other than the $2e_{\mathrm{TP}}$. In this case, application of SDG with inaccuracy of the measuring technique $2e_{\mathrm{TP}}$ for testing this object does not provide adequate information about changes in properties of the research object.

The important consequences of measurement uncertainty for such research objects as marine energy machines / devices may also include:

- repeating of diagnostic tests;
  application of the statistics for analyzing the results of empirical research;
- recording the instantaneous values of the measured physical quantity $\langle q_r, t_r \rangle$.

Repeating tests is necessary in order to obtain the relevant statistics describing the particular physical quantities characterizing technical and operational properties of the energy machines / devices. In this case the mentioned physical quantities must be considered as random variables [13, 14]. This requires application of the statistics [14, 15] for ana-

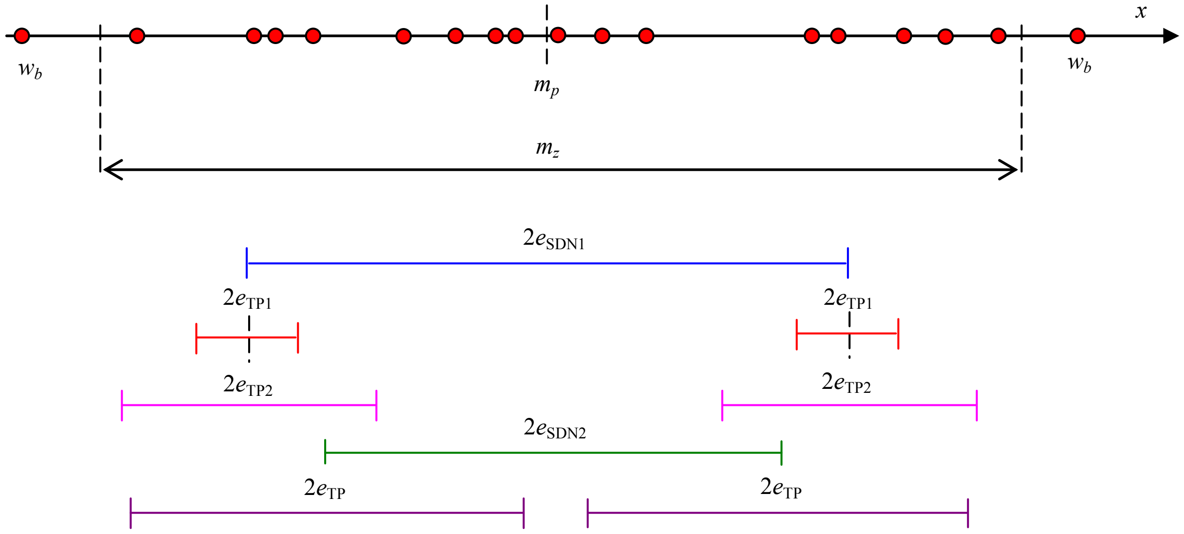

Fig. 2. Graph interpretation of inaccuracy of the research object as a diagnosed system (SDN) and the measuring technique being a part of a diagnosing system (SDG): m_p –measure of position (measure of central tendency), m_z – measure of variability (measure of dispersion), w_b – result obtained due to a large error (random error of excessive value)), $2e_{\mathrm{SDN}}$ – inaccuracy of SDN (object of diagnosis, object of diagnostic studies), $2e_{\mathrm{TP}}$ – inaccuracy of the measuring technique (measuring methods and means) being a part of SDG

Rys. 2. Interpretacja graficzna niedokładności obiektu badań jako systemu diagnozowanego (SDN) i techniki pomiarowej wchodzącej w skład systemu diagnozującego (SDG): m_p – miara położenia (miara tendencji centralnej), m_z – miara zmienności (miara rozproszenia), w_b – wynik pomiaru uzyskany wskutek popełnienia błędu grubego (błędu przypadkowego o nadmiernej wartości), $2e_{\mathrm{SDN}}$ – niedokładność SDN (obiektu diagnozowania, obiektu badań diagnostycznych), $2e_{\mathrm{TP}}$ – niedokładność techniki pomiarowej (metod i środków pomiarowych) wchodzących w skład SDG lyzing the research results (measurements), where the inaccuracy of the mentioned results should be defined, and which for this reason have a statistical sense. Inaccuracy of N measurement results characterizes a random-boundary interval around the indication of the instrument w^* (measurement result), which contains the actual (true) value of the result with a specified probability α, i.e. [4]:

$$P\left(w^* - N \le w \le w^* + N\right) \ge \alpha \tag{3}$$

This inaccuracy is caused by the reasons like inter alia:

- molecular or quantum nature of the phenomena causing momentary discrepancies of the measured characteristic value of an energy machine / device as a SDN from the mean;
- randomly changing environmental conditions shaped by pressure, temperature, humidity, voltage in the electrical network, etc., affecting the measuring instruments;
- aging and linear (surface) wear or volumetric (microcracks) wear of materials which the standards and measuring instruments are made of;
- influence of a measuring instrument on the physical quantity of which the value is measured, e.g. measurement of voltage with a voltmeter having low internal resistance causes receipt of too small value (error of measurement by a voltmeter is the smaller, the larger its resistance is in comparison with the resistance of the electrical circuit), while the current measurement error is the smaller, the lower the resistance of the ammeter is, similarly a larger sized thermocouple discharges more heat to the environment due to which it changes more significantly the temperature field inside the body);
- lack of sufficient knowledge in the scope of extracting a useful component of the measured diagnostic signal, having especially the properties of a stochastic process.

In case of diagnostic tests the quantities Q_r (r = 1, 2, 3…) being a function of time of which the values $q_{r(l)}$ (l = 1, 2, 3…) must be measured. The main reason for variability of the quantities is their molecular (quantum) nature [2, 3]. However, the study of these quantities is necessary because their time course is the most complete source of information about the values of $q_{r(l)}$ (l = 1, 2, 3…). This time course allows to obtain a family of random variables (of a stochastic process) $\{Q(t);\ t \ge 0\}$, whose exemplary realization is presented in figure 3.

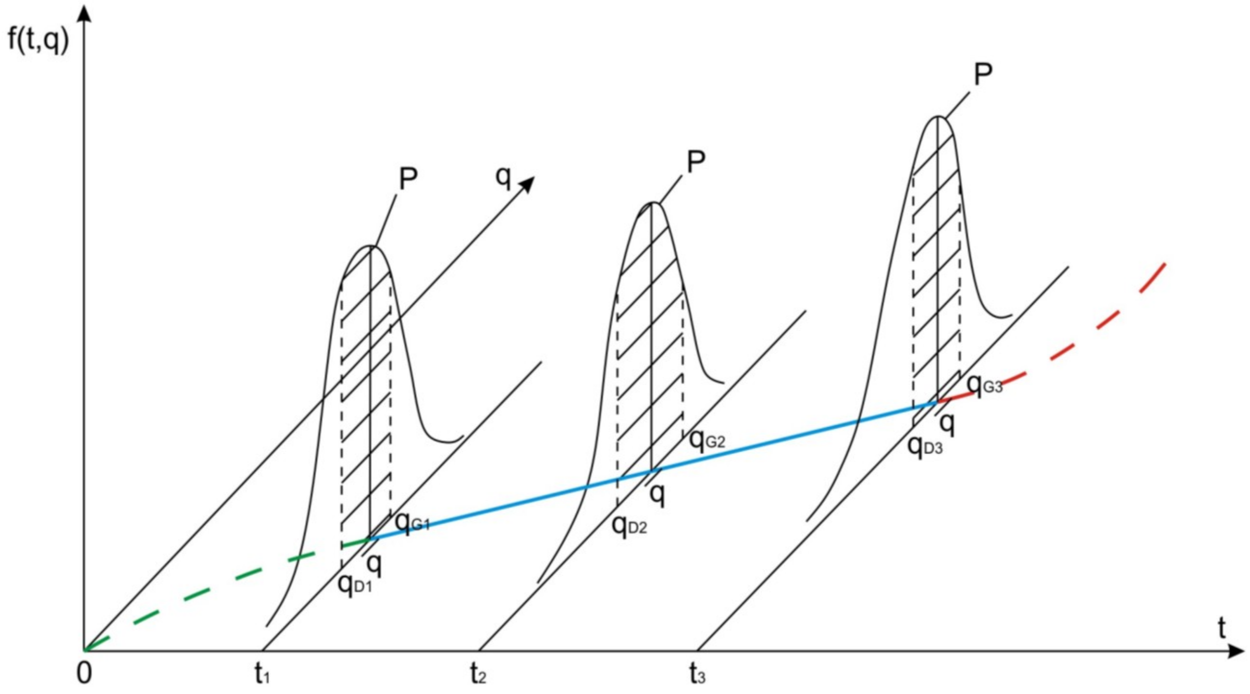

Fig. 3. An example of a family of random variables Q_k (k = 1, 2, 3) obtained in result of realization of the process of measuring a physical quantity Q for an energy machine / device in successive times t_r (r = 1, 2, 3), $\bar{q}$ – mean, q_{D1}, q_{D2} – respectively lower and upper value of the interval comprising the expected value of the random variables Q_k (k = 1, 2, 3)

Rys. 3. Przykładowa rodzina zmiennych losowych Q_k (k = 1, 2, 3) uzyskanych wskutek zrealizowania procesu pomiaru wielkości fizycznej Q urządzenia energetycznego w kolejnych chwilach t_r (r = 1, 2, 3), $\bar{q}$ – wartość średnia, q_{D1}, q_{D2} – odpowiednio wartość dolna i górna przedziału, w którym zawarta jest wartość oczekiwana zmiennych losowych Q_k (k = 1, 2, 3)

The states of the stochastic process $\{Q(t);\ t \geq 0\}$ are random variables Q_k ($k = 1, 2, 3$) assigned to the specific times t_1, t_2, t_3 (Fig. 3).

The usefulness of such approach to the study of energy machines / devices working especially in considerably different operating conditions, can be presented clearly on the example of measurements realization shown in figure 4.

Any time of $t_1, t_2, t_3, \ldots, t_6$ is assigned a random variable $Q_1, Q_2, Q_3, \ldots, Q_6$, to, respectively.

Expected values of particular random variables Q_r ($r = 1, 2, \ldots, 6$) for the respective times t_r ($r = 1, 2, \ldots, 6$) are defined with the formula [14, 15]:

$$\bar{q}_r = \frac{1}{n}\sum_{u-1}^{n} q_{u(r)} \tag{4}$$

where:

q_u – values obtained after performance of successive measurements of the values of the quantities of Q_r, $u = 1, 2, \ldots, n$ at a given time t (for the case as in figure 3, $u = 1, 2, .., 5$ for each: $t_1, t_2, t_3, \ldots . t_6$;

n – number of measurements (for the case shown in figure 3, $n = 5$) for each t.

In mathematical statistics it is found that the mean $\bar{q}$ is a value of statistics $\bar{Q}$, having asymptotically normal distribution $N\left(m_1, \frac{\sigma}{\sqrt{n}}\right)$, regardless of the functional form of a random variable distribution Q [14, 15]. Due to the fact that the convergence of this distribution to a normal distribution $N(m_1, \sigma)$ is very fast, it can be assumed that the random variable $\frac{Q - m_1}{s}\sqrt{n-1}$ has a Student's t-distribution with $z = n - 1$ degrees of freedom. Standard deviation s of this variable can be estimated from the following formula:

$$s = \sqrt{\frac{1}{n-1}\sum_{u=1}^{n}\left(q_u - \bar{q}\right)^2} \tag{5}$$

The value s is an estimate of the unknown value of the standard deviation σ. This estimation is obtained from the formula (5) during statistic research of the variable Q.

In this case, the confidence interval for an unknown expected value of the variable Q can be determined as follows [14, 15]:

$$P\left\{\bar{q} - t_{\alpha,n-1}\frac{s}{\sqrt{n-1}} \leq m_1 \leq \bar{q} + t_{\alpha,n-1}\frac{s}{\sqrt{n-1}}\right\} = \beta \tag{6}$$

The presented approach to the estimation of the value of the physical quantities Q characterizing an energy machine / device requires taking into account the fact that the instantaneous values of the measured physical quantity are identified by two

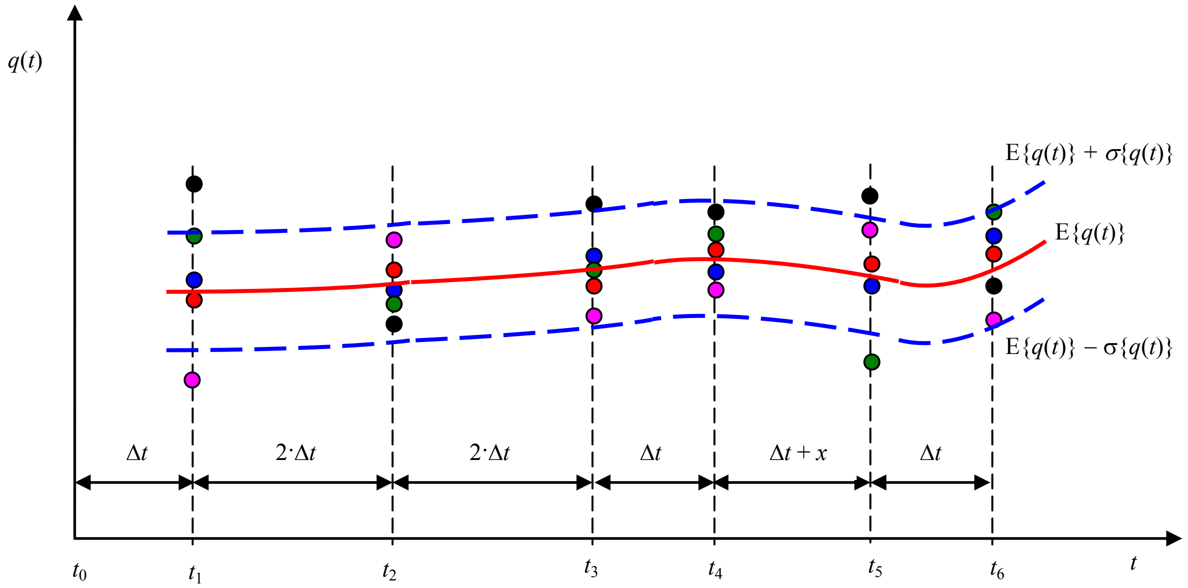

Fig. 4. An example of obtained measurement results in consequence of realization of the process $\{Q(t)\colon t \geq 0\}$ of performing measurements of the physical quantity Q: t – time of experimental studies, Δt – time interval accepted in the given plan of experiment, x – extra time, q – value of the measured quantity Q considered a random variable, which is characterized by the expected value m_1 and standard deviation σ

Rys. 4. Przykład uzyskania wyników pomiarów wskutek realizacji procesu $\{Q(t)\colon t \geq 0\}$ wykonywania pomiarów wielkości fizycznej Q: t – czas badań eksperymentalnych, Δt – przyjęty przedział czasu w danym planie eksperymentu, x – czas dodatkowy, q – wartość wielkości mierzonej Q uznanej za zmienną losową, którą charakteryzuje wartość oczekiwana m_1 oraz odchylenie standardowe σ

coordinates on the number axis of time: value of the measured physical quantity and time of recording this value. In addition, recording the instantaneous values of the measured quantity during operation of energy machines / devices, a random function is obtained with a parameter signifying the time, whose the values are random variables denoting the measurement results of the measured physical quantity.

In case of considering jointly the results obtained in particular times $t_1, t_2, \dots t_r$ of the time t of empirical research, a random variable should be considered [14, 15]:

$$Q = \sum_{i=1}^{r} Q_i \tag{7}$$

The expected value E(Q) of the random variable Q is then expressed as the following formula:

$$\begin{aligned} \mathrm{E}(Q) &= \mathrm{E}(Q_1 + Q_2 + \dots + Q_r) = \\ &= \mathrm{E}(Q_1) + \mathrm{E}(Q_2) + \dots + \mathrm{E}(Q_r) \end{aligned} \tag{8}$$

and the variance as follows:

$$\begin{aligned} \mathrm{D}^2(Q) &= \mathrm{D}^2(Q_1 + Q_2 + \dots + Q_r) = \\ &= \mathrm{D}^2(Q_1) + \mathrm{D}^2(Q_2) + \dots + \mathrm{D}^2(Q_r) \end{aligned} \tag{9}$$

Therefore, the standard deviation $\sigma = \sqrt{\mathrm{D}^2(Q)}$ is defined by the formula:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_r^2} \tag{10}$$

This means that it can be written down as [14]:

$$\mathrm{E}(Q) = \mathrm{E}\left(\sum_{i=1}^{r} Q_i\right) = \sum_{i=1}^{r} \mathrm{E}(Q_i) = \sum_{i=1}^{r} m_{1i} \tag{11}$$

$$\mathrm{D}^2(Q) = \mathrm{D}^2\left(\sum_{i=1}^{r} Q_i\right) = \sum_{i=1}^{r} \mathrm{D}^2(Q_i) = \sum_{i=1}^{r} \sigma_i^2 \tag{12}$$

therefore:

$$\sigma = \sqrt{\sum_{i=1}^{r} \sigma_r^2} \tag{13}$$

Assuming that the random variable $\overline{Q}$ is an arithmetic mean of r independent random variables $Q_1 + Q_2 + \dots + Q_r$, then [14]:

$$\mathrm{E}(\overline{Q}) = \mathrm{E}\left(\frac{1}{r}\sum_{i=1}^{r} Q_i\right) = \frac{1}{r}\sum_{i=1}^{r} \mathrm{E}(Q_i) \tag{14}$$

$$\mathrm{D}^2(\overline{Q}) = \mathrm{D}^2\left(\frac{1}{r}\sum_{i=1}^{r} Q_i\right) = \frac{1}{r^2}\sum_{i=1}^{r} \mathrm{D}^2(Q_i) \tag{15}$$

In case the studied random variables Q_i ($i = 1, 2, \dots, r$) have the same expected values E(Q) = m_1 and the equal variance D^2(Q) = σ^2, then:

$$\mathrm{E}(\overline{Q}) = \mathrm{E}(Q) = m_1 \tag{16}$$

$$\mathrm{D}^2(\overline{Q}) = \frac{1}{r}\mathrm{D}^2(Q) = \frac{\sigma^2}{r} \tag{17}$$

and therefore: $\sigma_{\overline{Q}} = \dfrac{\sigma}{\sqrt{r}}$

Such approach can be justified by the fact that according to the Chebyshev theorem, the sequence of random variables $\overline{Q}$ is stochastically convergent to the value of m_1, i.e.:

$$\lim_{r \to \infty} P\left\{\left|\overline{Q} - m_1\right| > \varepsilon\right\} = 0 \tag{18}$$

The possibility of using the asymptotically normal distribution for cumulative description of measurements results from Lindeberg-Levy theorem and Lyapunov's theorem [14].

The Lindeberg-Levy central limit theorem shows that the sum of r independent random variables Q_k with the same (arbitrary) distribution of the common expected value E(Q_k) = m_1 and variance D^2(Q_k) = $\sigma^2 \neq 0$ has an asymptotically normal distribution $N(r \cdot m;\ \sigma\sqrt{r})$.

In accordance with Lindeberg-Levy central limit theorem the random variable $\overline{Q}_r$ is a variable with asymptotically normal distribution $N\left(m_1, \dfrac{\sigma}{r}\right)$. This means that the arithmetic mean of r independent random variables Q_k having arbitrary but equal distribution with a common expected value E(Q_k) = m_1 and variance $\mathrm{D}^2(Q_k) = \sigma^2$ has asymptotically normal distribution $N\left(m_1, \dfrac{\sigma}{r}\right)$.

Lindeberg-Levy theorem was extended by Lyapunov. From this theorem, called the Lyapunov's theorem (or the central limit theorem), results that the sum of independent random variables will have asymptotically normally distribution, even when its components are random variables with different distributions [14].

According to this theorem, the random variable $\overline{Q}_r$ of r independent random variables Q_k is a random variable with asymptotically normal distribution regardless of the distributions of the individual random variables Q_k.

Summary – remarks and conclusions

It should be accepted that the basic assumption of diagnostics and metrology at the same time is that the arithmetic mean obtained from measurements of any measured physical quantity characterizing each energy machine / device is different than the real (actual) value of this physical quantity.

It must be recognized that there is no explicit representation of the reason in its result (effect) as a measurement result, but there is representation resulting from the probabilistic laws of the quantum mechanics.

Due to the randomly changing conditions under which measurements are made, a statistical approach should be applied for describing the results of measurements of individual physical quantities. The statistical approach takes into account simultaneously both the change of the research object proceeding during measurement, as well as errors of the applied measuring methods and instruments. Thus, the measurement uncertainty occurs which in diagnostic studies (as in all scientific studies) should be explained at least as for its main reason, i.e. factors whether the uncertainty is mainly a result of:

- change of the research object, resulting mainly from the influence of interfering factors, or
- errors of the measuring methods and means (errors of the measuring technique).

The main reasons resulting in inaccuracy of the measurements include first of all the following:

- variable conditions under which the measurements are made, that cause variability of the measures of: position (central tendency), diffusion (dispersion);
- inaccuracy of the applied measuring methods and instruments;
- variable time of measurement duration, which results from the necessity of repeating the measurement;
- experience of the person performing measurements;
- uncertainty in the research object which the measurements relate to;
- difficulty in testing the measuring instruments and checking in this way the accuracy of regaining the standard, to make sure that the characteristic properties of these devices (essential for accurate measurements) are consistent with the required ones.

Measuring instruments that are in state of full ability should have the required static and dynamic properties. This is particularly important in case of measuring transducers. This follows from the fact that they operate in the toughest conditions.

References

1. Będkowski L.: Elementy diagnostyki technicznej. WAT, Warszawa 1992.
2. Piotrowski J.: Podstawy miernictwa. WNT, Warszawa 2002.
3. Girtler J.: Aspekty kwantowej diagnostyki maszyn. XXXIX Sympozjum Diagnostyki Maszyn. Wisła, 04–10. 03.2012 r., Wyd. Wydział Transportu Politechniki Śląskiej, Katowice 2012.
4. Polański Z.: Planowanie doświadczeń w technice. PWN, Warszawa 1984.
5. Girtler J.: Probabilistic measures of a diagnosis' likelihood about the technical state of transport means. Archives of Transport. Polish Academy of Sciences, Committee of Transport, Quartelly, Vol. 11, iss. 3–4, Warszawa 1999, 33–42.
6. Pabis S.: Metodologia i metody nauk empirycznych. PWN, Warszawa 1985.
7. Białynicki-Birula I., Cieplak M., Kamiński J.: Teoria kwantów. Mechanika falowa. Wydawnictwo Nukowe PWN, Warszawa 2001.
8. Gribbin J.: W poszukiwaniu kota Schrödingera. Zysk i S-ka Wydawnictwo s.c., Poznań 1997. Tytuł oryginału: In Search of Schrödinger's Cat. Quantum Physics Reality.
9. Nalewajski R.F.: Podstawy i metody chemii kwantowej. Wydawnictwo Naukowe PWN, Warszawa 2001.
10. Girtler J.: Sterowanie procesem eksploatacji okrętowych silników spalinowych na podstawie diagnostycznego modelu decyzyjnego. Monografia. Zeszyty Naukowe AMW, Gdynia 1989, nr 100A.
11. Girtler J.: Diagnostyka jako warunek sterowania eksploatacją okrętowych silników spalinowych. Studia Nr 28, WSM, Szczecin 1997.
12. Inżynieria diagnostyki Maszyn. Praca zbiorowa pod redakcją B. Żółtowskiego i C. Cempla. PTDT, Wyd. ITE, Warszawa, Bydgoszcz, Radom 2004.
13. Benjamin J.R., Cornell C.A.: Probability, Statistics, and Decision for Civil Engineers. Copyright 1970 by McGraw-Hill, Inc. Wyd. polskie: Rachunek prawdopodobieństwa, statystyka matematyczna i teoria decyzji dla inżynierów. WNT, Warszawa 1977.
14. Firkowicz S.: Statystyczna ocena jakości i niezawodności lamp elektronowych. WNT, Warszawa 1963.
15. Pawłowski Z.: Statystyka matematyczna. PWN, Warszawa 1976.