Scientific Journals
Maritime University of Szczecin

Zeszyty Naukowe
Akademia Morska w Szczecinie

2012, 30(102) pp. 25–29

2012, 30(102) s. 25–29

Design of robust, nonlinear control system of the ship course angle, in a model following control (MFC) structure based on an input-output linearization

Projektowanie odpornego, nieliniowego układu regulacji kąta kursu statku w strukturze MFC z linearyzacją typu wejście-wyjście

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Key words: ship course control, model following control, input-output linearization

Abstract
The paper presents a designing procedure of controllers in the structure of tracking model (*Model Following Control*, *MFC*) for nonlinear model of a ship as an object of the course angle control. In the article feedback linearization method for known nonlinearity in the input-output channel of plant has been used. Ideal linearization in the classical control system occurs only when the design nonlinearity of the model and the object are identical. Therefore, the article proposes the use of MFC structure, which is able to compensate for differences of non-linear characteristics of the process and model.

Słowa kluczowe: regulacja kursu statku, sterowanie z modelem, linearyzacja wejście-wyjście

Abstrakt
Artykuł przedstawia procedurę projektowania regulatorów w strukturze ze śledzeniem modelu (*Model Follo-wig Control*, *MFC*) dla nieliniowego modelu statku, jako obiektu regulacji kąta kursu. W artykule została wykorzystana metoda linearyzacji z ujemnym sprzężeniem zwrotnym dla znanej nieliniowości modelu obiektu w torze wejście-wyjście. Idealna linearyzacja w klasycznym układzie regulacji zachodzi jedynie wtedy, gdy projektowe nieliniowości modelu i obiektu są identyczne. Dlatego też w artykule zaproponowano zastosowanie układu MFC, który jest w stanie skompensować różnice nieliniowych charakterystyk procesu i jego modelu.

Introduction

Real plants and processes are strongly nonlinear, for example, a ship as an object of the course angle control, or a main ship's engine from a speed, temperature or viscosity control point of view, and the like. Solutions of the control problems for such plants are complex and difficult. The main difficulty is a choice of transformation method of nonlinear plant into linear. Till now two popular methods of linearization have been applied: a linearization of plant at the operating point (*Jacobian linearization*) or harmonic linearization (*describing functions*).

In the case of Jacobian linearization, the changes of the operating point generate a need to modify the description of the dynamics of the plant: the nonlinear plant is replaced with set of linear plants at different operating points.

In the case of harmonic linearization, the describing function takes into account only the first harmonic, which presupposes such an inaccurate description of a nonlinear plant.

Another solution is the use of nonlinear controllers, which cancel the nonlinearity of the plant in the input-output channel [1].

The feedback linearization cancels the nonlinearities, directly and establishes a linear input--output linearization map.

One difficulty is that input-output linearization algorithm requires the control with stable inversion of nonlinear function of plant and state feedback.

Another linearization method is the *backstepping*; this method is sensitive to changes in parameters and difficult in practical applications [2] and cannot be applied to all plants [3].

A new modern alternative way to compensate of completely or partially unknown nonlinear characteristics (Eq. 1) of the plant is the using adaptive control [4]:

$$\dot{\mathbf{x}} = f(\mathbf{x}, u) \\ y = h(\mathbf{x}) \qquad (1)$$

In MFC structure (Fig. 1) – presented in this article – the differences between the signal outputs of the model and real plant (due to for example, structural or parametric uncertainties, static and dynamic inaccuracies of model, faults or failures of the actuators and/or measurement sensors, etc.) are compensated by an additional control signal of corrective (auxiliary) controller.

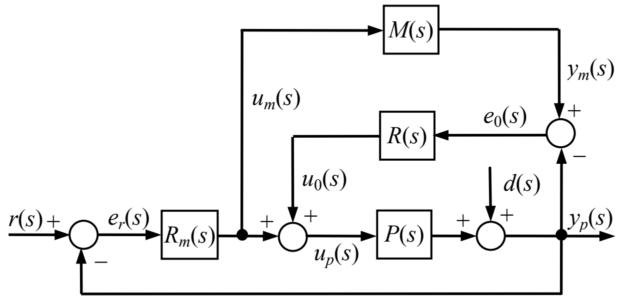

Fig. 1. Block diagram of classical MFC structure, where transfer functions mean: $M(s)$ – model of plant at nominal conditions, $P(s)$ – plant, $R_m(s)$ – main controller, $R(s)$ – corrective (auxiliary) controller and $r(s)$ – set point value (reference signal), $y_p(s)$ – process output, $y_m(s)$ – model output, $d(s)$ – plant disturbances reduced to output

Rys. 1. Schemat blokowy klasycznej struktury ze śledzeniem modelu MFC

Review of structures with Model Following Control

The basic MFC structure, described by means the transfer functions was shown in figure 1 [5].

In the linear or linearized MFC system as the main controller $R_m(s)$ and auxiliary controller $R(s)$ are used PID or PI controllers [5]. But, the proportional controllers may be used also as auxiliary controller with an algorithm $u_o = k_p e_0$ (Fig. 1), or as the state controller with the algorithm $u_o = K_m x_m - K_p x_p$ (Fig. 2), where K_m and K_p are the vectors of the gain coefficients of state vector x_m of model and state vector x_p of process [6].

The proportional controller responds quickly to appearing differences of output signals between process and model. The source of these differences may be, for example, in structural or parametric uncertainties of the model of process, faults or malfunctions appearing in the process, plant's parameter changes (for example, different values in the ballast states of the ship during the cruise, as a result of consumption of fuel oil, lubricating oil, food supplies, etc.).

Some similarity with the described situation take place in cascade control system.

As it follows from the figure 1, the corrective controller R generates correction signal u_o based on

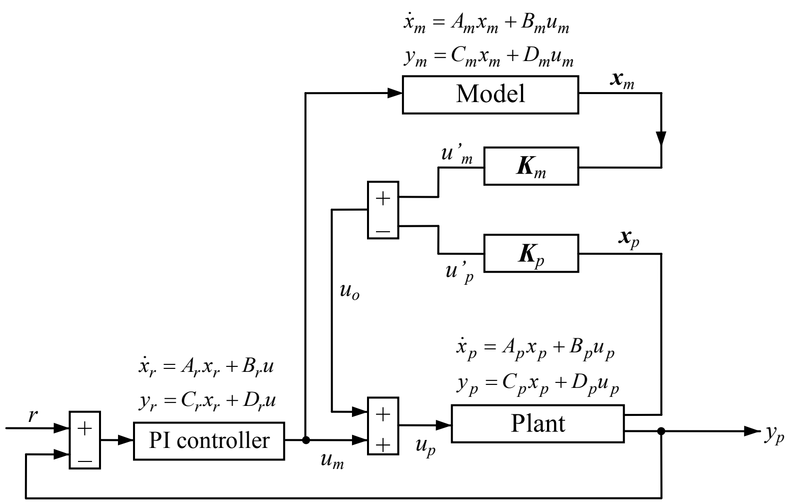

Fig. 2. Block diagram of MFC structure with state feedback controllers K_m and K_p

Rys. 2. Schemat blokowy struktury MFC z korekcyjnymi regulatorami od stanu K_m i K_p

the information contained in the error signal e_o, i.e. of the difference between the output signals: y_m of model and y_p of plant. In this situation, the plant state variables are "beyond control" and they can not follow the state variables of the model in a transparent manner. Character of the changes of state variables of the process is very important, for example in the drives' control systems. These systems require of desired waveform of acceleration, velocity and position. Measurements of these physical quantities are possible for those plants, because these state variables are mostly directly available. In the cases, when the state variables are unavailable, the various state observers are used.

The linear system (Fig. 2) describe the following matrices (Eq. 2) in the state space:

$$A = \begin{bmatrix} A_p - B_p D_r C_p - B_p K_p & B_p K_m & B_p C_r \\ -B_m D_r C_p & A_m & B_m C_r \\ -B_r C_p & 0 & A_r \end{bmatrix}$$

$$B = \begin{bmatrix} B_p D_r \\ B_m D_r \\ B_r \end{bmatrix}; \; C = \begin{bmatrix} C_p & 0 & 0 \end{bmatrix} \tag{2}$$

$$x(t) = \begin{bmatrix} x_p(t) & x_m(t) & x_r(t) \end{bmatrix}^T; \; u(t) = r(t)$$

where: $x_p \in R^n$, $x_m \in R^n$, $x_r \in R^1$ are state vectors of plant, model and PI controller, A_p, B_p, C_p; A_m, B_m, C_m; A_r, B_r, C_r, D_r are constant matrices of plant, model and PI controller of appropriate dimensions. The pairs of matrices (A_p, B_p) and (A_m, B_m) are stabilizable and A_m is stable matrix. K_m and K_p are the vectors of the gain coefficients of the state variables of model and process, respectively.

K_m and K_p coefficients, i.e. parameters of corrective controller (Fig. 2), have been calculated on the base of LQR procedure (for classical feedback control loop with model and process, separately) with the same performance criteria in generic form as (Eq. 3):

$$J = \int_0^\infty (z^T Q z + u^T R u)\,\mathrm{d}t \tag{3}$$

where: symmetric matrix $Q \geq 0$ has dimension $n \times n$, symmetric matrix $R > 0$ has dimension $p \times p$ and $z = x_m$ or $z = x_p$ and $u = u_m$ or $u = u_p$.

Of course, the system in figure 2 has better properties than a system with an auxiliary proportional controller with an input signal e_o (Fig. 1).

Good results gets also the using of the state-feedback auxiliary proportional controller with the algorithm $u_o = K_e(x_m - x_p)$, which has been verified by the author. In this case, the following matrices (4) describe linear system in the considered state space:

$$A_e = \begin{bmatrix} A_p - B_p D_r C_p - B_p K_e & B_p K_e & B_p C_r \\ -B_m D_r C_p & A_m & B_m C_r \\ -B_r C_p & 0 & A_r \end{bmatrix}$$

$$B_e = \begin{bmatrix} B_p D_r \\ B_m D_r \\ B_r \end{bmatrix}; \; C_e = \begin{bmatrix} C_p & 0 & 0 \end{bmatrix} \tag{4}$$

As can be seen the state variables of the process which are used in the auxiliary controller can be used simultaneously in the main controller R.

Design of nonlinear controller

The nonlinear controller was designed using the method of input-output linearization for the classical feedback control loop.

The main idea of the input-output linearization is to algebraically transform a nonlinear plant dynamics into a linear, so that the linear control theory can be applied. This method uses a negative feedback loop for linearization and can be viewed as ways of transforming of the original, nonlinear system into the equivalent model in a simpler form.

If the SISO (single-input single-output) plant has an affine-in-control form (Eq. 5):

$$\begin{aligned} \dot{x} &= f(\boldsymbol{x}) + g(\boldsymbol{x})u \\ y &= h(\boldsymbol{x}) \end{aligned} \tag{5}$$

where y, $\boldsymbol{x}$, u denote output, state and control vectors respectively, f and g are the functions of state vector $\boldsymbol{x}$, then it is input-output linearizable [4], if control signal u is (Eq. 6):

$$u = \frac{-L_f^p h(\boldsymbol{x}) + v}{L_g L_f^{p-1} h(\boldsymbol{x})} \tag{6}$$

The control law (Eq. 6) reduces the input-output map to:

$$y^{(p)} = v$$

which is a chain of integrators where p is the relative degree of the nonlinear system.

In equation (Eq. 6) $L_f^p h(x)$ denotes p-th the Lie derivative of h along f, $L_g L_f^{p-1} h(x)$ denotes ($p - 1$)-th the Lie derivative of h along g.

Thus, the control law in this control system is:

$$v = r^{(p)} - b_0 e - b_1 \dot{e} - b_2 \ddot{e} - \ldots - b_{p-1} e^{(p-1)}$$

(where $e(t) = y(t) - r(t)$) and the factors b_i should be chosen, so that the following polynomial

$$s^p + b_{p-1} s^{p-1} + b_{p-2} s^{p-2} + \ldots + b_1 s + b_0$$

has its roots strictly in the left-half complex plane, leads to exponentially stable dynamics

$$e^{(p)} + b_{p-1}e^{(p-1)} + b_{p-2}e^{(p-2)} + ... + b_1\dot{e} + b_0 e = 0$$

which implies exponentially convergent tracking $e(t) \to 0$, i.e. $y(t) \to r$.

Nonlinear MFC example

The use of the presented methodology in the model following control structure is demonstrated through illustrative example – a course angle control of a ship.

This example presents a design of the ship course tracking controller in MFC structure, based on the nonlinear mathematical model of a ship as an object of the course angle control.

The known nonlinear Norrbin model of ship as an object of the course angle control [7] can be described by the equation (7):

$$T\dot{r} + a_3 r^3 + a_2 r^2 + a_1 r + a_0 = k\delta \qquad (7)$$

where: δ – rudder deflection-control variable, r – angular velocity of hull ($\dot{\psi} = r$), ψ – controlled variable-course of ship), and T, k, a_i, are parameters of ship dynamics. Generally, $H(r)$ is nonlinear function of $\dot{\psi}$ and can be estimated by third-order polynomial. The 3rd-order polynomial of $H(r)$

$$H(r) = a_3 r^3 + a_2 r^2 + a_1 r + a_0 \qquad (8)$$

describes the nonlinear steering curves of ship (*Dieudonne curves*). The coefficients in (Eq. 7) or (Eq. 8) are constant for fixed external conditions. This function can be found from direct spiral test made for course-stable ships and "reversed" spiral test for course-unstable ships [8].

The exemplary parameters of the nonlinear dynamics of ship at nominal conditions are: $k = 0.1256$; $T = 48$ sec; $a_3 = 1.2322$; $a_2 = 0.0665$; $a_1 = 1$; $a_0 = 0.075$ [9].

For the modeling purposes, the nonlinear plant-ship in the state space (for the case $H_p(r) = H_m(r)$, which means that the MFC system is a classical control system) has been described by equations (9):

$$\begin{aligned} \dot{x}_1 &= x_2 = r \\ \dot{x}_2 &= -\frac{1}{T}H_m(r) + \frac{k}{T}\delta \\ y &= x_1 = h(x_1) \end{aligned} \qquad (9)$$

According to the procedure described above, we have:

$$\delta = \frac{T_m}{k_m}\ddot{y} + \frac{1}{k_m}H_m(r) \qquad (10)$$

where: $\ddot{y} = \ddot{\psi}$, $H_m(r) = a_3 r^3 + a_2 r^2 + a_1 r + a_0$ is nonlinear steering curve of ship's model at nominal conditions, T_m and k_m are parameters of ship's model.

Because the transient component of the course control error describes the following differential equation (Eq. 11):

$$\ddot{e}(t) = -b_1\dot{e}(t) - b_0 e(t) \qquad (11)$$

where the signal error is (Eq. 12):

$$e(t) = \psi - \psi_R \qquad (12)$$

(ψ_R – denotes desired ship course angle) then placing (Eq. 12) into (Eq. 11) and assuming that $\ddot{\psi}_R = 0, \dot{\psi}_R = 0$, we obtain equation (Eq. 13):

$$\ddot{\psi} = -b_1\dot{\psi} + b_0(\psi_R - \psi) \qquad (13)$$

Substituting, in turn (Eq. 13) into (Eq. 10), and taking into account that $\dot{\psi} = r,\ u = \delta$, we obtain nonlinear control law (Eq. 14) [9]:

$$\delta = \frac{T_m}{k_m}\left[b_0(\psi_R - \psi) - b_1 r\right] + \frac{1}{k_m}H_m(r) \qquad (14)$$

b_1 coefficient affects the rate of decay of the oscillations of the control system (a high value of b_1 causes intense oscillation suppression); b_0 coefficient affects the frequency of oscillation (the increasing b_0 increases the frequency of oscillation).

The coefficients b_1 and b_0 can be determined, for example, by poles placement using the closed system natural frequency ω_o.

The equation (Eq. 14) represents the main (R_m) nonlinear PD controller in MFC structure. For set point value (ψ_R) changes, this controller acts as a proportional and for $\dot{\psi} = r$ changes it is nonlinear PD controller – the derivative action is adjusted according to nonlinear characteristics of the ship's model (main controller R_m contains function $H_m(r)$ of the model at nominal conditions).

We also assume that proportional controller with a gain coefficient k_R will be auxiliary controller in MFC structure.

As it is shown in [10] the nonlinear MFC system ($H_p(\dot{\psi}) \neq H_m(\dot{\psi})$) describes the nonlinear differential equation (Eq. 15). This equation contains nonlinear characteristics of the real ship $H_p(\dot{\psi})$, its model $H_m(\dot{\psi})$, parameters b_0 and b_1 of the main nonlinear controller R_m and gain coefficient k_R

(included in b) of auxiliary controller R (see previous sections):

$$\ddot{\psi} + (d + b_1)\dot{\psi} + (b - c + b_0)\psi + \left(\frac{a}{k_m} - \frac{1}{T_m}\right)H_m(\dot{\psi}) + \frac{H_p(\dot{\psi})}{T_p} = b\psi_m - (c - b_0)\psi_R \qquad (15)$$

where: k_p, T_p and $H_p(\dot{\psi})$ are parameters of the real ship, and $a = \frac{k_m}{T_m} - \frac{k_p}{T_p}$; $b = k_p \frac{k_R}{T_p}$; $c = \frac{a T_m b_0}{k_m}$; $d = -\frac{a T_m b_1}{k_m}$.

If $H_p(\dot{\psi}) = H_m(\dot{\psi})$, $k_p = k_m$, $T_p = T_m$ then equation (Eq. 15) takes the form (Eq. 16):

$$\ddot{\psi} + b_1\dot{\psi} + (b + b_0)\psi = b\psi_m + b_0\psi_R \qquad (16)$$

On the basis of the linear differential equation (Eq. 16) one can be written the transfer function $G_R(s)$ (Eq. 17) of linearized MFC system (if $\psi_m = 0$),

$$G_R(s) = \frac{\psi(s)}{\psi_R(s)} = \frac{b_0}{s^2 + b_1 s + (b + b_0)} \qquad (17)$$

and transfer function $G_m(s)$ (Eq. 18) of linearized MFC system if $\psi_R = 0$

$$G_m(s) = \frac{\psi(s)}{\psi_m(s)} = \frac{b}{s^2 + b_1 s + (b + b_0)} \qquad (18)$$

Gain coefficient of auxiliary controller will be determined by the pole placement method.

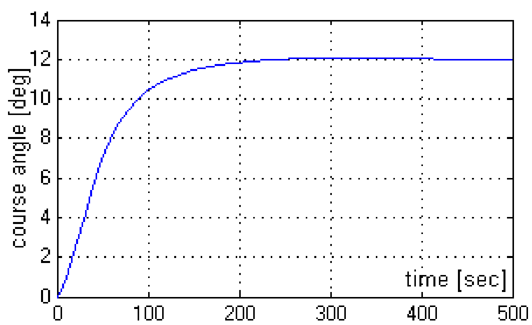

Fig. 4. Step response in designed MFC structure with main controller (Eq. 14) and auxiliary proportional controller with gain $k_R = 0.1$ for the case if $H_m(r) \neq H_p(r)$ and set-point value $\psi_R = 12$ [deg]

Rys. 4. Odpowiedź skokowa kąta kursu statku w zaprojektowanym układzie regulacji o strukturze MFC z korekcyjnym regulatorem proporcjonalnym o wzmocnieniu $k_R = 0.1$ dla przypadku, gdy $H_m(r) \neq H_p(r)$ przy wartości zadanej $\psi_R = 12$ [deg]

As it is easy to verify, a closed control system of the ship's course angle control (but without controller) has two poles with the following values: $s_1 \approx -0.01 + 0.05i$; $s_2 \approx -0.01 - 0.05i$.

We want to move these poles, so that the poles will have the following values: $s_1 = -0.04$; $s_2 = -0.07$.

For an exemplary ship, we assume that $b_0 = \omega_0^2 = 0.0028$ and determine b_1 ($b_1 = 0.11$). Now, on the basis of transfer function (Eq. 17) one can determine the condition of an overdamped step response, that is, $k_R \leq 0.1$. We assume $k_R = 0.1$.

Step response in designed MFC structure is shown in figure 4.

Conclusions

The article presents the use of model following control structure for nonlinear process control. Changes of nonlinear characteristics of plant-ship are compensated by a signal from the auxiliary controller. This controller reacts on the difference between the outputs signal of real ship and its model. The nonlinear plant, linearized by the input-output method has good properties (the short time control, acceptable control signals). In many cases, MFC system can replace the complex and expensive adaptive control system.

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