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TWO-STAGE OPTIMIZATION METHOD WITH FATIGUE CONSTRAINTS FOR THIN-WALLED STRUCTURES

Mirosław Mrzygłód

Cracow University of Technology, Institute of Rail Vehicles, Kraków, Poland
e-mail: mrzyglod@mech.pk.edu.pl

The paper describes a two-stage approach to optimization of thin-walled structures. The optimization procedure takes into consideration high-cycle fatigue constraints. Hence, Dang Van's criterion for fatigue damage estimation is used. In the first stage of optimization, a parametric model and evolutionary algorithms are used. The topology optimization method is applied in the second stage. The work is illustrated by an example of optimization of a thin-walled mechanical structure subjected to high-cycle load conditions. In both stages of the optimization, the same parametric FE model, boundary conditions and constraints were used. The topology optimization applied in the second stage of optimization significantly improved the final result. The optimization methodology allowed to effectively lower the mass of the structure maintaining its durability on an established level.

Key words: thin-walled structures, fatigues constraints, topology optimization

1. Introduction

In optimization of a thin-walled metal structure, the parametric methods are easy to apply. However, in the case of great number of decision variables in complex optimization models, like in vehicles, the simple stochastic methods are inefficient in terms of the computation time. The evolutionary algorithms are more efficient, but although they give good results they are also time-consuming. The efficiency of the optimization process can be significantly improved by using parallel evolutionary algorithms (Mrzygłód and Osyczka, 2006).

The topology optimization methodology can also be applied to thin-walled metallic structures (Bendsoe and Sigmund, 2003; Mrzygłód and Osyczka, 2006;

Mrzygłód, 2008). Topology optimization in large vehicle design has been so far rarely used. But, for fairly simple vehicle structures some successful tests have been carried out (Fredricson, 2005). For composites, multi-layer objects a very interesting proposition is the joined topology-sizing optimization method (Zhou *et al.*, 2004). But for the one-layer design space, the application of topology optimization is limited.

On the other hand, for structures subjected to high-cycle load conditions the optimization procedure should take into account fatigue constraints (Mrzygłód and Zieliński, 2006; 2007a,b).

In the proposed optimization methodology, the author wants to join the advantages of both parametric and topology optimization algorithms as well as to take into consideration the fatigue constraints. For thin-walled structures, the parametric model and evolutionary algorithms are to be used in the first stage. Then, the application of the topology optimization algorithm is planned.

2. Two-stage optimization approach

Let us assume the optimization problem to be formulated as follows

$$\min f(\boldsymbol{\eta}) \tag{2.1}$$

the constraints are

$$\underline{g_j} < g_j(\boldsymbol{\eta}) \leqslant \overline{g_j} \qquad j = [1, 2, \ldots, K] \tag{2.2}$$

where $\boldsymbol{\eta} = [\eta_1, \eta_2, \ldots, \eta_N]$ is a vector of N design variables; g_j is the j-th constraint (state parameter); $\underline{g_j}$, $\overline{g_j}$ are the lower and upper bound of constraints, respectively; $f(\boldsymbol{\eta})$ is the objective function (the volume of the structure), K is the number of constraints.

The design variables in the first stage of optimization represent the size related N_I parameters, in the second stage of optimization the design variables represent a pseudo-density of each N_{II} FE element that varies between 0 and 1.

The optimization process consists of the following two stages:

Stage I. The optimization investigation of the objective function is performed using evolutionary algorithms and a parametric FE model.

Stage II. After finding a solution, the vector of optimal values of decision variables $\boldsymbol{\eta}^*$ form the design space for the topology optimization algorithm. In this stage the same FE model, under the same boundary conditions, becomes a subject to topology optimization with the aim of further improvement of the value of the objective function.

Both stages of the optimization are executed with the same constraints.

As the main tool of the first stage of optimization, the software package Evolutionary Optimization System (EOS) was chosen. The EOS is designed to solve single and multi-criteria optimization problems for nonlinear programming problems. For the purpose of numerical optimization, the EOS software was combined with the FEM package ANSYS® and the parallel computing procedure (Mrzygłód and Osyczka, 2006).

For the realization of the second stage of optimization, a simple procedure of topology optimization was coded using the ANSYS® APDL language (ANSYS Inc., 2007).

The following topology optimization algorithm was incorporated (see Fig. 1) (Mrzygłód, 2008, 2009):

1) calculation and record of stress values of the M loading steps;
2) checking the constraint limits (e.g. von Mises eqv. stress); if the state parameter crosses its bound limit, a layer of finite elements is added to the structure boundary;
3) selection and removal of the groups of low stressed finite elements ($\sigma < \sigma_{min}$); the bound value σ_{min} is increased at every iteration;
4) checking the value of the objective function; if the value $f(\boldsymbol{\eta})$ is under the total volume bound value V_{ref} the optimization process is finished, otherwise go to 1).

It should be noted that it is also possible to start the optimization procedure from an unfeasible solution and increase the structure to its optimum topology.

3. High-cycle fatigue constraints

The structural optimization with stress constraints is usually suitable for cases where complex load conditions must be investigated. Among contemporary stress criteria in the topology optimization, von Mises hypothesis is commonly used (Bendsoe and Sigmund, 2003). However, this criterion does not take into account fatigue characteristics of materials but only their yield limit. In such cases, a multi-axial high-cycle fatigue (MHCF) hypothesis as well as cumulative damage matrix should be applied. The modern MHCF criteria like

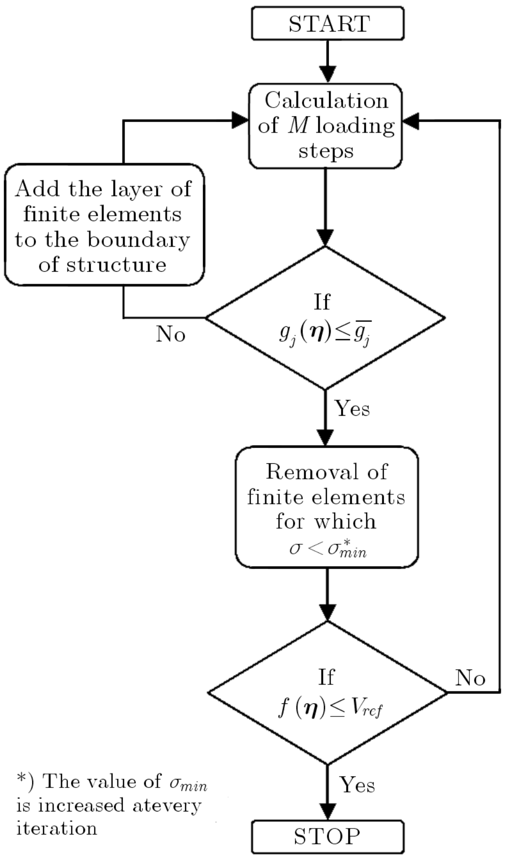

Fig. 1. Topology optimization algorithm

Crossland, Dang Van or Papadopoulos use at least two fatigue limits (e.g. for bending and torsion) to characterize endurance of the material (Ballard *et al.*, 1995; Papadopoulos *et al.* 1997; Dang Van and Papadopoulos, 1999). In this work, the modern MHCF hypothesis of Dang Van was chosen as the damage criterion because of its numerical convenience and high reliability (Ballard *et al.*, 1995; Papadopoulos *et al.* 1997). Moreover, the author takes a few assumptions for the modeling and analysis (Mrzygłód and Zieliński, 2006, 2007a,b):

- real time load history is simplified,
- possible short time violation of high-cycle loads has been taken into consideration by means of a suitable safety factor,
- particular components of loads have the in-phase or reverse character,

- frequency of loads is considerably lower than the first eigen-frequency of studied structures,
- inertial effects are not taken into account.

The Dang Van criterion takes into consideration the average value of shear and normal stresses in an elementary volume V_{el}. According to Dang Van, the fatigue damage appears in a definite time, when the combination of local shear stresses $\tau(t)$ and hydrostatic stress $\sigma_H(t)$ cut the borders of the admissible fatigue area. A numerically convenient form of the Dang Van criterion is presented below (Mrzygłód and Zieliński, 2006, 2007a,b)

$$\max_V[\tau(t) + \kappa\sigma_H(t)] \leqslant \lambda \tag{3.1}$$

where V is volume of the studied object

$$\tau(t) = \frac{\sigma_1(t) - \sigma_3(t)}{2} \qquad \sigma_H(t) = \frac{1}{3}(\sigma_1(t) + \sigma_2(t) + \sigma_3(t))$$

σ_1, σ_2, σ_3 – are principal stresses.

The material parameters can in principle be expressed by data from two high-cycle fatigue tests: reversed bending (fatigue limit f_{-1}) and reversed torsion (fatigue limit t_{-1}). For the Dang Van criterion it results in $\lambda = t_{-1}$ and $\kappa = 3t_{-1}/f_{-1} - 3/2$.

Dang Van criterion (3.1) proposes an analysis with respect to the time variable. For the optimization, the following pattern of reduction of the load time history was proposed:

- real load time history (see Fig. 2a) is simplified to an equivalent sinusoidal form with an equivalent amplitude and mean load values,
- moreover, five load steps: min, $0.5min$, 0, $0.5max$, max are assumed (see Fig. 2b) and checked in every single fatigue analysis.

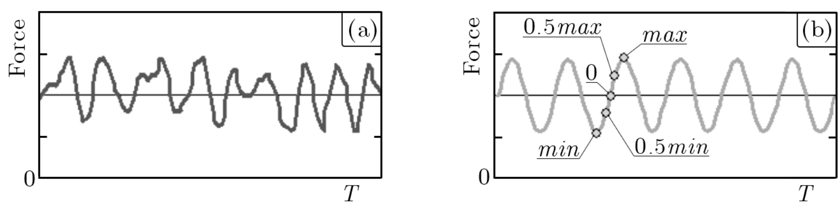

Fig. 2. Example of a real load time history (a), transformed to the sinusoidal form with five load steps: min, $0.5min$, 0, $0.5max$, max (b)

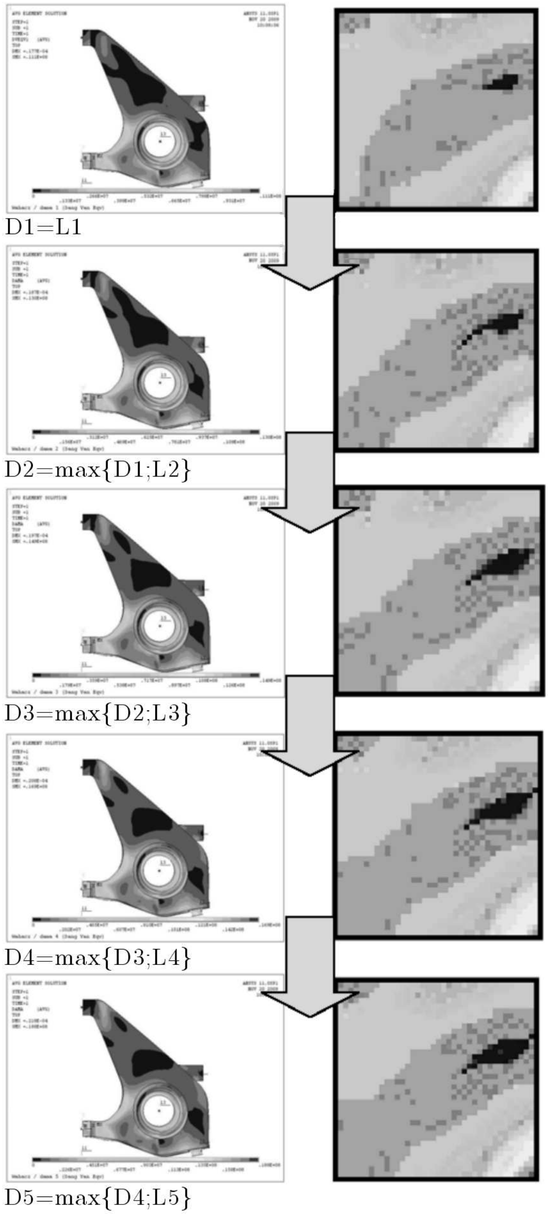

Fig. 3. Graphical representation of the "compare and save maximum" algorithm of assembling the cumulative damage matrix; L1, . . . , L5 – are fatigue equivalent stress matrices for load cases 1 to 5; and D1, . . . , D5 – are damage matrices 1 to 5. As it is shown above, the contour maps of damage fatigue equivalent stresses change their areas from step 1 to 5

For the topology optimization, the damage matrix should be constructed for every iteration consisting of a set of load steps. The stress matrix values for load cases 1 to 5 have been added according to Rainflow Cycle Counting rules (Dowling, 1993). It means that only the maximum values between two compared matrices are transferred to the resulting damage matrix (see Fig. 3). In particular, for the first damage matrix D1 all stresses are transferred from the equivalent stress matrix L1. For the second load step, a "compare and save maximum" procedure is employed. For every finite element its equivalent stress value of matrix L2 is compared to the previous one stored in the damage matrix D1, if the present value of L2 is higher than the former value form D1. The new value is transferred to the new damage matrix D2. Otherwise, the equivalent stresses from D1 are transferred to D2 without changes. The operation is repeated for load cases 3, 4 and 5. The final matrix D5 represents the cumulative damage matrix of the whole loading sequence. In Fig. 3, it is shown how the contour maps of damage matrices of load cases 1 to 5 are gradually changing their areas.

4. Example of optimization

An example of a car rear suspension arm was chosen to demonstrate two-stage methodology (Fig. 4a). The arm is built from two thin sheet drawn pieces assembled by welding. This part is subjected to high-cycle load conditions. The Multi-Body simulation of the real car suspension arm was conducted for determination of the load history. The equivalent loads for fatigue analysis were prepared according to the previously established rules (see Table 1). The parametric FE model of the suspension arm is presented in Fig. 4b.

For the assumed material model $f_{-1} = 260\,\mathrm{MPa}$, $t_{-1} = 160\,\mathrm{MPa}$, equation (3.1) takes the form

$$\sigma_{eqv,DV} = 0.5\sigma_{TG}(t) + 0.346\sigma_H(t) \leqslant 160\,\mathrm{MPa} \tag{4.1}$$

where σ_{TG} is the equivalent normal stress according to the Tresca criterion.

To find the limit value of damage parameter $\overline{g_j}$, a preliminary fatigue analysis for the original shape of the suspension arm (Fig. 4b) was done . The maximum value of the Dang Van equivalent stress was found (Fig. 6a) and the state parameter bound $\overline{g_j}$ was accepted on the level $\sigma_{eqv,DV} = 21\,\mathrm{MPa}$. The low level of admissible stresses can be noticed due to a large safety factor in the real working structure.

For the optimization process, five from the twelve decision variables were selected after the sensitivity investigation. For the evolutionary algorithm,

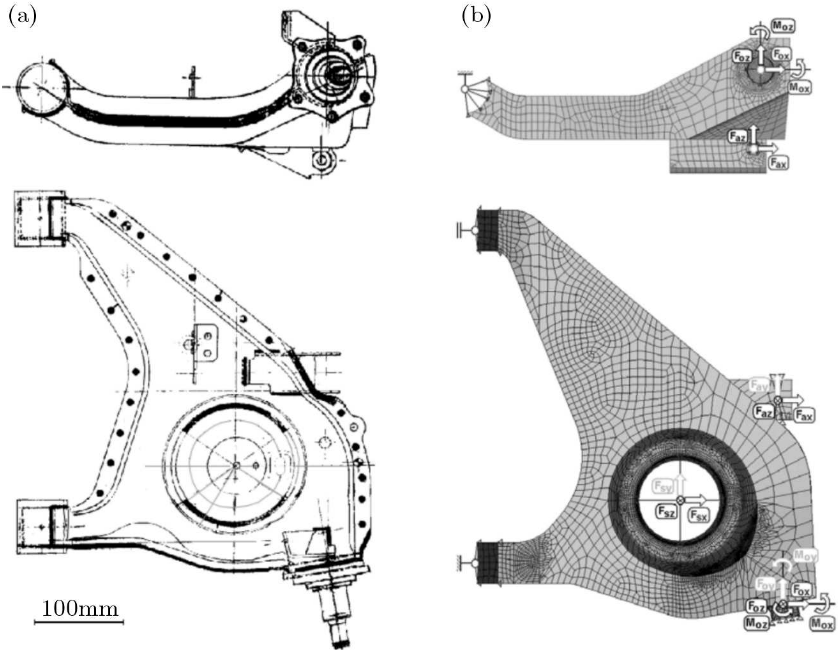

Fig. 4. Suspension arm (a) and its FE model (b)

Table 1. Equivalent loads for fatigue analysis of the real car suspension arm

Loads	Steps				
	I	II	III	IV	V
F_{ox} [N]	−300	−200	−100	0	100
F_{oy} [N]	−1030	−1197.5	−1365	−1532.5	−1700
F_{oz} [N]	2050	2337.5	2625	2912.5	3200
M_{ox} [Nm]	−310	−350	−390	−430	−4700
M_{oy} [Nm]	−50	−60	−70	−80	−90
M_{oz} [Nm]	−40	−30	−20	−10	0
F_{sx} [N]	−950	−1175	−1400	−1625	−1850
F_{sy} [N]	−240	−275	−310	−345	−380
F_{sz} [N]	−2450	−2612.5	−2775	−2937.5	−3100
F_{ax} [N]	250	157.5	65	−27.5	−120
F_{ay} [N]	5	2.5	0	−2.5	−5
F_{az} [N]	390	242.5	95	−52.5	−200

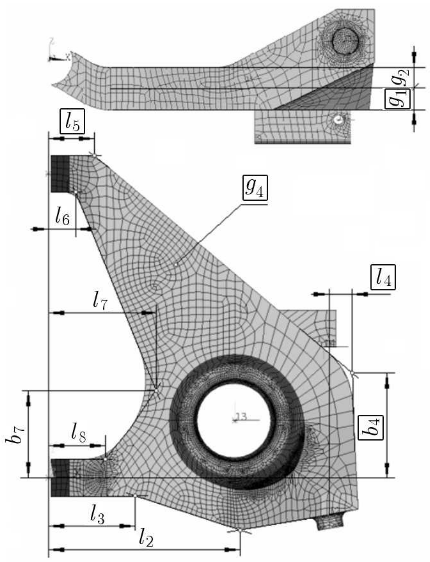

Fig. 5. Decision variables initially accepted for the optimization process; variables selected through sensitivity investigation are marked with box

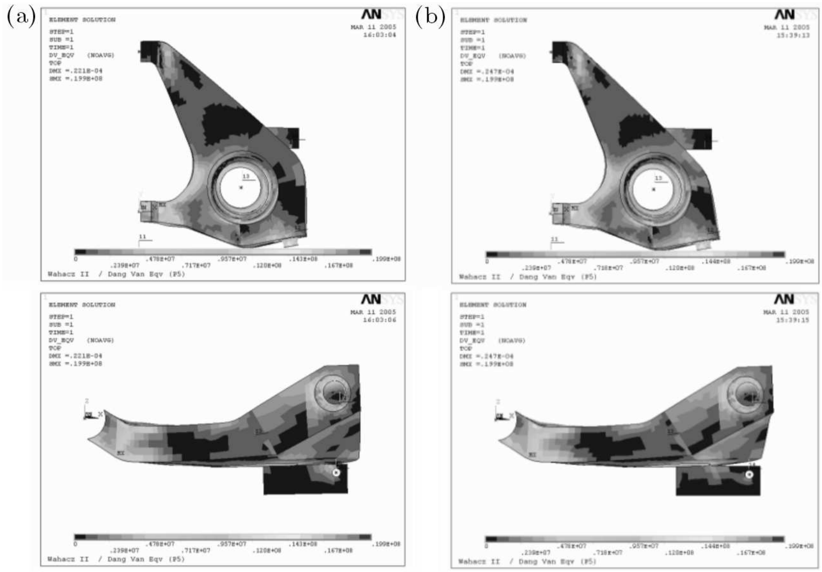

Fig. 6. Initial (a) and optimal shape of structure after the first stage of optimization (b); (a) objective function $f(\eta) = 6438.003\,\mathrm{g}$; (b) objective function $f(\eta) = 5713.026\,\mathrm{g}$

the following starting parameters were accepted: size of population $J = 350$; crossover parameter $p_c = 0.7$; mutation parameter $p_m = 0.4$; number of generations $L_g = 70$.

As a result of the first stage of optimization, $\varepsilon_1 = 11.3\%$ decrease of the structure mass was obtained. The results of evolutionary optimization of the first stage are presented in Fig. 6b.

In the second stage of optimization, $\varepsilon_2 = 21.5\%$ decrease of the mass was achieved. The second stage results of topology optimization are presented in Fig. 7.

Fig. 7. Results of topology optimization of the second stage of optimization; objective function $f(\eta) = 4484.54\,\mathrm{g}$

In the two-stages optimization, about $\varepsilon_\Sigma = 32.8\%$ of mass reduction was done, the Dang Van equivalent stress value was maintained on a level of the original shape of the suspension arm (see Fig. 6a). The contour-map of the Dang Van equivalent stresses represents their maximum cumulative value obtained in the five-step load history.

It should be noted that the obtained optimized structure with sharp edge holes is difficult to manufacture, and in the final design of the suspension arm the openings must be replaced by equivalent circular or oval holes. The dimensions of new holes can be defined by the next-step standard parametric optimization.

5. Conclusions

In the paper, the complex two-stage optimization methodology with fatigue constraints was presented. In the proposed methodology, the author combined the parametric evolutionary method with the topology algorithm and applied them to thin-walled structures.

As observed in the numerical example of the structure subjected to high-cycle loads, the proposed approach allowed one to effectively reduce the mass of the structure, maintaining its fatigue durability on an established level taken from the real object. The second stage of topology optimization improved the result by removing the excessive material which, clearly, could not be removed by the first method. What is interesting, the topology optimization method gives very good results for already optimized objects. The two-stage optimization approach presented in the paper can be easily applied to complex thin-walled structures.

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Dwuetapowa metoda optymalizacji konstrukcji cienkościennych z ograniczeniami zmęczeniowym

Streszczenie

Artykuł opisuje dwuetapowe podejście do optymalizacji konstrukcji cienkościennych uwzględniające wysoko-cyklowe ograniczenia zmęczeniowe. Do oszacowania uszkodzenia zmęczeniowych w procedurze optymalizacyjnej użyto hipotezy Dang Van'a. W pierwszym etapie optymalizacji został zastosowany model parametryczny oraz algorytmy ewolucyjne. W etapie drugim wykorzystano metodę optymalizacji topologicznej. Działanie algorytmu optymalizacyjnej zilustrowana na przykładzie cienkościennej konstrukcji mechanicznej poddanej wysoko-cyklowemu reżimowi obciążeń. Dla obu etapów optymalizacji użyto tego samego model parametrycznego MES oraz takich samych warunków brzegowych i ograniczeń. Zauważono, że zastosowana w drugim etapie optymalizacja topologiczna znacząco poprawiła końcową wartość funkcji celu. Zaprezentowana w pracy metodyka optymalizacyjna pozwoliła skutecznie obniżyć masę konstrukcji, utrzymując jej trwałość zmęczeniową na założonym poziomie.

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