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INFLUENCE OF COLLISIONS WITH A MATERIAL FEED ON COPHASAL MUTUAL SYNCHRONISATION OF DRIVING VIBRATORS OF VIBRATORY MACHINES

Jerzy Michalczyk
Piotr Czubak
AGH University of Science and Technology, Cracow, Poland
e-mail: michalcz@agh.edu.pl

The relation between angular oscillations of vibratory machine bodies – disturbing the vibratory transport – and the loss of cophasal of driving vibrators was indicated in this paper. It was shown that the loss of cophasal running could be caused by periodical collisions of the body with a material feed. The mathematical model of this phenomenon was developed. The obtained analytical dependencies, allowing one to assess disphasing of vibrators and to estimate amplitudes of angular oscillations of the machine, were verified by comparison with the results obtained by digital simulation of the system behaviour.

Key words: vibratory machines, cophasal run, synchronisation

1. Problem formulation

Several essential processing and transportation processes are realised in the industry by means of vibratory machines and devices, such as vibrating screens and conveyers, foundry shake-out grids, vibrating tables for production of concrete prefabricates as well as vibratory devices for synchronous eliminations of vibrations.

Correct performance of this type of machines depends on obtaining synchronous, cophasal, angular motion of unbalanced masses constituting the source of the needed dynamic forces (Lavendel, 1981; Michalczyk and Cieplok, 1999).

As an example, let us discuss the scheme of a vibratory machine of a linear trajectory of vibrations – Fig. 1, in which the drive constitutes two independent inertial vibrators set in motion by means of induction motors.

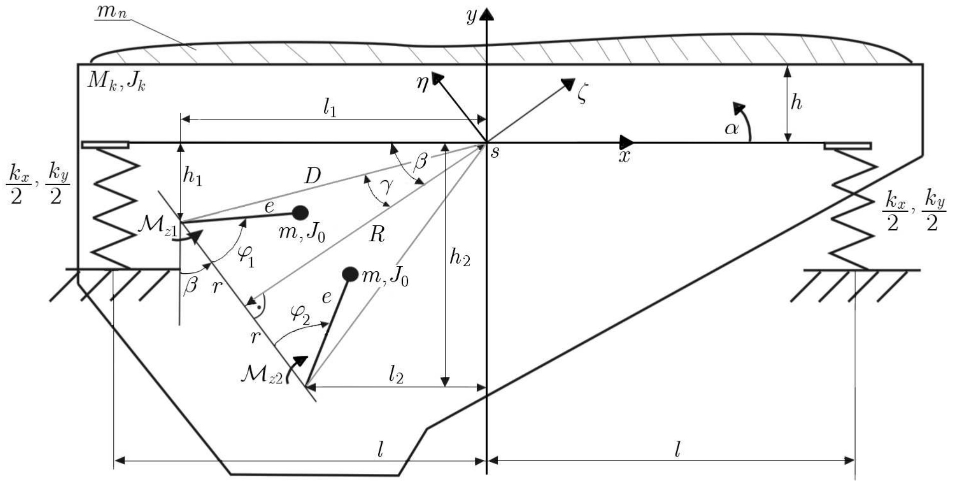

Fig. 1. Calculation diagram of the two-vibrator vibratory machine

In Figure 1: M_k, J_k – mass and central moment of the body inertia, m, e – mass and eccentricity of the single vibrator, m_n – mass of a material feed, k_x, k_y – coefficients of elasticity of the body suspension in directions x and y.

The desirable situation is that both vibrators are counter and cophasal running generating the resulting force in the direction of working vibrations ζ. The direction of this force should pass through the machine mass centre, which ensures lack of excitations (when the system of elastic supports is symmetrical) for angular oscillations.

Conditions for occurrence of tendency for the desirable synchronous and cophasal vibrator running can be determined on the basis of the integral criterion formulated by Blekhman (1994), Blekhman and Yaroshevich(2004)

$$D(\varphi_1-\varphi_2, \varphi_1-\varphi_3, \dots, \varphi_1-\varphi_n) = \frac{1}{T}\Big[\int\limits_0^T (E-V)\, dt - \int\limits_0^T (E_w - V_w)\, dt\Big] = \min \tag{1.1}$$

According to this criterion, the set of phase angles is stable around values $\varDelta\varphi_{12}, \varDelta\varphi_{13}, \dots, \varDelta\varphi_{1n}$, if the function D, determined by Equation (1.1) for these values, acquires the local minimum, where:

$\varphi_1, \varphi_2, \dots, \varphi_n$	–	angles of rotation of individual vibrators versus their initial positions,
T	–	period of forced vibrations, $T = 2\pi/\omega$,

E	–	kinetic energy of the machine body with rotor masses concentrated in the pivoting point,
V	–	potential energy of the support system of the machine body,
E_w, V_w	–	kinetic and potential energy of constrains between vibrators, respectively.

For machines operating in a far-over-resonant mode, for which influence of elastic forces in the suspension can be neglected and which corresponds to the scheme presented in Fig. 1, the above given condition leads to (Lavendel, 1981)

$$D > 0 \tag{1.2}$$

However, the above criterion does not determine occurrence and precision of synchronisation in the case when counter-acting factors exist. The range of allowable disphasing angles of vibrators – for various types of vibratory machines given in Lavendel (1981) paper – indicates the importance of vibrator dissynchronisation for the working process:

$$\Delta\varphi \leqslant \begin{cases} 3^\circ - 5^\circ & \text{for vibrating screens} \\ 5^\circ - 12^\circ & \text{for feeders} \\ 12^\circ - 16^\circ & \text{for vibratory conveyers.} \end{cases}$$

The thesis that one of the factors disturbing synchronisation of drives is the influence of instantaneous forces originated from colliding of the material feed with the machine will be stated and verified in the paper. The dependences between the material feed mass and the work cycle character and the disphasing angle of vibrators causing angular oscillations of the body being responsible for irregular transportation of materials along the machine body will be also determined.

2. Analysis of influence of collisions with the material feed on cophasal running of vibrators

Analysis of undisturbed running of vibrators will be performed by means of an averaging method. It allows one to write equations of motion of the machine body, shown in Fig. 1, separating the "quickly" and "slowly" variable phenomena. Thus, assuming for synchronous running the equality of angular velocities of both vibrators $\dot{\varphi}_1 = \dot{\varphi}_2$ and their "slow" variation ($\omega \approx \text{const}$), we can write approximate equations of motion of the body in between the collisions with the material feed in the absolute system ξ, η in the form

$$\begin{aligned} &M\ddot{\xi} + k_\xi \xi = me\omega^2(\sin\varphi_1 + \sin\varphi_2) \\ &M\ddot{\eta} + k_\eta \eta = me\omega^2(\cos\varphi_2 - \cos\varphi_1) \\ &J\ddot{\alpha} + k_y l^2 \alpha = me\omega^2 r(\sin\varphi_2 - \sin\varphi_1) + me\omega^2 R(\cos\varphi_1 - \cos\varphi_2) \end{aligned} \tag{2.1}$$

where

ξ, η	–	absolute coordinates determining the position of the body mass centre,
α	–	angle of rotation of the machine body,
M	–	mass of a vibrating part of the machine, $M = M_k + 2m$,
J	–	central moment of inertia of the machine with unbalanced masses brought to the axis of rotation of the vibrators,
k_ξ, k_η	–	coefficient of elasticity in direction ξ and η, respectively, and

$$k_\xi = k_x \cos^2\beta + k_y \sin^2\beta \qquad\qquad k_\eta = k_y \cos^2\beta + k_x \sin^2\beta$$

the remaining markings are given in Fig. 1.

Denoting $\varphi_1 - \varphi_2 = \Delta\varphi =$ const and assuming $\Delta\varphi \ll 1$ and $\varphi_2 = \omega t$, $\omega =$ const, Equations (2.1) can be presented in the approximated form

$$\begin{aligned} &M\ddot{\xi} + k_\xi \xi = 2me\omega^2 \sin\Big(\omega t + \frac{\Delta\varphi}{2}\Big) \qquad\qquad M\ddot{\eta} + k_\eta \eta = me\omega^2 \Delta\varphi \sin(\omega t) \\ &J\ddot{\alpha} + k_y l^2 \alpha = -me\omega^2 D \Delta\varphi \sin(\omega t + \gamma) \end{aligned} \tag{2.2}$$

where $\tan\gamma = r/R$ (Fig. 1).

Particular integrals of the above equations, determining the steady state, are of the following form

$$\begin{aligned} &\xi(t) = \frac{2me\omega^2}{k_\xi - M\omega^2} \sin\Big(\omega t + \frac{\Delta\varphi}{2}\Big) \qquad\qquad \eta(t) = \frac{-me\omega^2 \Delta\varphi}{M\omega^2 - k_\eta} \sin(\omega t) \\ &\alpha(t) = \frac{me\omega^2 D \Delta\varphi}{J\omega^2 - k_y l^2} \sin(\omega t + \gamma) \end{aligned} \tag{2.3}$$

Let us also determine the second time derivatives in stationary motion – for those coordinates

$$\begin{aligned} &\ddot{\xi}(t) = \frac{2me\omega^4}{M\omega^2 - k_\xi} \sin\Big(\omega t + \frac{\Delta\varphi}{2}\Big) \qquad\qquad \ddot{\eta}(t) = \frac{me\omega^4 \Delta\varphi}{M\omega^2 - k_\eta} \sin(\omega t) \\ &\ddot{\alpha}(t) = \frac{-me\omega^4 D \Delta\varphi}{J\omega^2 - k_y l^2} \sin(\omega t + \gamma) \end{aligned} \tag{2.4}$$

Dynamic equations analysed up to the present were describing body vibrations on the assumption that the angular motion of vibrators can be considered as uniform for the steady state. This assumption is equivalent to disregarding the influence of body vibrations on vibrators running. Presently, we will develop equations of motion of vibrators taking into consideration those couplings, it means in a non-inertial coordinate system related to the machine body performing vibrations described above.

Applying moments from the inertial forces resulting from vibration of their axis to vibrators, we obtain equations of angular motion in the form

$$\begin{aligned} J_0\ddot{\varphi}_1 &= \mathcal{M}_{z1} - me\ddot{\xi}_1\cos\varphi_1 - me\ddot{\eta}_1\sin\varphi_1 \\ J_0\ddot{\varphi}_2 &= \mathcal{M}_{z2} - me\ddot{\xi}_2\cos\varphi_2 + me\ddot{\eta}_2\sin\varphi_2 \end{aligned} \tag{2.5}$$

where

$\mathcal{M}_{z1}, \mathcal{M}_{z2}$	–	external moments (difference of the driving moment and the moment of friction),
J_0	–	inertial moment of the vibrator versus its axis of rotation.

Let us mark the vibratory moments $\mathcal{M}_{wi}$, $i = 1, 2$, as expressions

$$\begin{aligned} \mathcal{M}_{w1} &= -me(\ddot{\xi}_1\cos\varphi_1 + \ddot{\eta}_1\sin\varphi_1) \\ \mathcal{M}_{w2} &= -me(\ddot{\xi}_2\cos\varphi_2 - \ddot{\eta}_2\sin\varphi_2) \end{aligned} \tag{2.6}$$

On the basis of the previously determined solutions of the body motion (not taking into account any influences of vibratory moments on the vibrators running) we will determine components of accelerations of axes of both vibrators, disregarding centripetal accelerations as being small as compared with the remaining ones

$$\begin{aligned} \ddot{\xi}_1 &= \ddot{\xi} - \ddot{\alpha}D\sin\gamma \qquad & \ddot{\eta}_1 &= \ddot{\eta} - \ddot{\alpha}D\cos\gamma \\ \ddot{\xi}_2 &= \ddot{\xi} + \ddot{\alpha}D\sin\gamma \qquad & \ddot{\eta}_2 &= \ddot{\eta} - \ddot{\alpha}D\cos\gamma \end{aligned} \tag{2.7}$$

Taking into consideration in the above presented expresions Equations (2.4) and substituting them into (2.6) we will obtain the following equations to the vibratory moments

$$\begin{aligned} \mathcal{M}_{w1} = -m^2e^2\omega^4\Big[&\frac{2}{M\omega^2 - k_\xi}\sin\Big(\omega t + \frac{\Delta\varphi}{2}\Big)\cos(\omega t + \Delta\varphi) + \\ &+\frac{D^2\Delta\varphi\sin\gamma}{J\omega^2 - k_yl^2}\sin(\omega t + \gamma)\cos(\omega t + \Delta\varphi) + \\ &+\frac{\Delta\varphi}{M\omega^2 - k_\eta}\sin(\omega t)\sin(\omega t + \Delta\varphi) + \frac{D^2\Delta\varphi\cos\gamma}{J\omega^2 - k_yl^2}\sin(\omega t + \gamma)\sin(\omega t + \Delta\varphi)\Big] \end{aligned}$$

$$
\begin{aligned}
\mathcal{M}_{w2} = -m^2e^2\omega^4\Big[&\frac{2}{M\omega^2-k_\xi}\sin\Big(\omega t+\frac{\Delta\varphi}{2}\Big)\cos(\omega t)+\\
&-\frac{D^2\Delta\varphi\sin\gamma}{J\omega^2-k_yl^2}\sin(\omega t+\gamma)\cos(\omega t)+\\
&-\frac{\Delta\varphi}{M\omega^2-k_\eta}\sin^2(\omega t)-\frac{D^2\Delta\varphi\cos\gamma}{J\omega^2-k_yl^2}\sin(\omega t+\gamma)\sin(\omega t)\Big]
\end{aligned}
\tag{2.8}
$$

We will calculate now the value, averaged for the period $T = 2\pi/\omega$, of the vibratory moment acting on vibrator No. 1, applying the assumption $\Delta\varphi \ll 1$

$$
\begin{aligned}
\mathcal{M}_{w1av} &= \frac{1}{T}\int\limits_0^T \mathcal{M}_{w1}(t)\;dt =\\
&= -m^2e^2\omega^4\frac{\omega}{2\pi}\Big[\frac{2}{M\omega^2-k_\xi}\int\limits_0^{2\pi/\omega}\sin\Big(\omega t+\frac{\Delta\varphi}{2}\Big)\cos(\omega t+\Delta\varphi)\;dt+\\
&+\frac{D^2\Delta\varphi\sin\gamma}{J\omega^2-k_yl^2}\int\limits_0^{2\pi/\omega}\sin(\omega t+\gamma)\cos(\omega t+\Delta\varphi)\;dt+\\
&+\frac{\Delta\varphi}{M\omega^2-k_\eta}\int\limits_0^{2\pi/\omega}\sin(\omega t)\sin(\omega t+\Delta\varphi)\;dt+\\
&+\frac{D^2\Delta\varphi\cos\gamma}{J\omega^2-k_yl^2}\int\limits_0^{2\pi/\omega}\sin(\omega t+\gamma)\sin(\omega t+\Delta\varphi)\;dt\Big]=\\
&=\frac{-m^2e^2\omega^4}{2}\Big[\frac{2}{M\omega^2-k_\xi}\sin\Big(-\frac{\Delta\varphi}{2}\Big)+\frac{D^2\Delta\varphi\sin\gamma}{J\omega^2-k_yl^2}\sin(\gamma-\Delta\varphi)+\\
&+\frac{\Delta\varphi}{M\omega^2-k_\eta}\cos(\Delta\varphi)+\frac{D^2\Delta\varphi\cos\gamma}{J\omega^2-k_yl^2}\cos(\gamma-\Delta\varphi)\Big]=\\
&=\frac{-m^2e^2\omega^4}{2}\Big[\frac{-\Delta\varphi}{M\omega^2-k_\xi}+\frac{D^2\Delta\varphi\sin\gamma}{J\omega^2-k_yl^2}(\sin\gamma-\Delta\varphi\cos\gamma)+\\
&+\frac{\Delta\varphi}{M\omega^2-k_\eta}+\frac{D^2\Delta\varphi\cos\gamma}{J\omega^2-k_yl^2}(\cos\gamma+\Delta\varphi\sin\gamma)\Big]
\end{aligned}
\tag{2.9}
$$

Disregarding terms containing $(\Delta\varphi)^2$, we will finally obtain

$$
\mathcal{M}_{w1av} = \frac{-m^2e^2\omega^4}{2}\Big(\frac{D^2}{J\omega^2-k_yl^2}+\frac{1}{M\omega^2-k_\eta}-\frac{1}{M\omega^2-k_\xi}\Big)\Delta\varphi
\tag{2.10}
$$

In a similar fashion, calculating the averaged within the period $T = 2\pi/\omega$ value of the vibratory moment for vibrator No. 2, we will obtain

$$
\begin{aligned}
\mathcal{M}_{w2av} &= \frac{1}{T}\int\limits_0^T \mathcal{M}_{w2}(t)\,dt = \\
&= -m^2e^2\omega^4\frac{\omega}{2\pi}\Big[\frac{2}{M\omega^2-k_\xi}\int\limits_0^{2\pi/\omega}\sin\Big(\omega t+\frac{\Delta\varphi}{2}\Big)\cos(\omega t)\,dt + \\
&\quad -\frac{D^2\Delta\varphi\sin\gamma}{J\omega^2-k_yl^2}\int\limits_0^{2\pi/\omega}\sin(\omega t+\gamma)\cos(\omega t)\,dt + \\
&\quad -\frac{\Delta\varphi}{M\omega^2-k_\eta}\int\limits_0^{2\pi/\omega}\sin^2(\omega t)\,dt - \frac{D^2\Delta\varphi\cos\gamma}{J\omega^2-k_yl^2}\int\limits_0^{2\pi/\omega}\sin(\omega t+\gamma)\sin(\omega t)\,dt\Big] = \\
&= \frac{-m^2e^2\omega^4}{2}\Big[\frac{2}{M\omega^2-k_\xi}\sin\Big(\frac{\Delta\varphi}{2}\Big) - \frac{D^2\Delta\varphi\sin\gamma}{J\omega^2-k_yl^2}\sin\gamma - \frac{\Delta\varphi}{M\omega^2-k_\eta} + \\
&\quad -\frac{D^2\Delta\varphi\cos\gamma}{J\omega^2-k_yl^2}\cos\gamma\Big] = \frac{m^2e^2\omega^4}{2}\Big(\frac{D^2}{J\omega^2-k_yl^2}+\frac{1}{M\omega^2-k_\eta}-\frac{1}{M\omega^2-k_\xi}\Big)\Delta\varphi
\end{aligned}
\tag{2.11}
$$

As it can be seen, the values of both moments are equal while their directions reverse. Thus, their difference equals

$$
\begin{aligned}
\Delta\mathcal{M}_w &= \mathcal{M}_{w2av} - \mathcal{M}_{w1av} = \\
&= m^2e^2\omega^4\Big(\frac{D^2}{J\omega^2-k_yl^2}+\frac{1}{M\omega^2-k_\eta}-\frac{1}{M\omega^2-k_\xi}\Big)\Delta\varphi
\end{aligned}
\tag{2.12}
$$

The above expression constitutes the measure of the ability of the system to generate the synchronising moment, when due to a certain reason the system with natural tendency for synchronous cophasal running operates dissynchronised by an angle $\Delta\varphi$.

Let us now consider the influence of periodical collisions with the feed material on vibrators running.

After satisfying certain, given below, limitations for the machine motion, the feed material performs periodical motion. The period of this motion equals the vibration period of the machine. Typical motion of the system is shown in Fig. 2.

It is being proven, in the theory of motion of a material point on a plate vibrating with a harmonic translatory motion, that the time instant of the

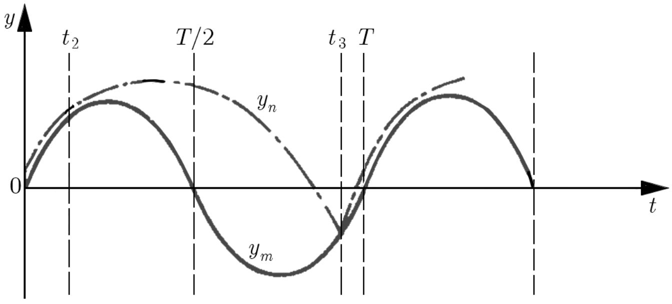

Fig. 2. Feed and machine body motion; y_n, y_m are vertical displacements of the feed and the body, respectively

feed material falling on the machine body t_3 is a function of a dimensionless parameter k_p called the coefficient of throw (Czubak and Michalczyk, 2001; Michalczyk, 1995).

This parameter, determining the ratio of the perpendicular component of the machine body vibration accelerations to the acceleration of gravity is expressed as follows

$$k_p = \frac{A\omega^2 \sin\beta}{g} \tag{2.13}$$

where

A	–	vibration amplitude along the ξ axis,
g	–	acceleration of gravity,
β	–	inclination angle of body vibrations versus the horizontal line, Fig. 1.

Since for $m \ll M$ the disturbances of machine motion caused by collisions with the feed are quite small, it is allowed to use the equation for the time of falling t_3

$$t_3 = \frac{1}{\omega}\arcsin\Big(\frac{1}{k_p}\Big) + \frac{2\pi}{\omega}n \tag{2.14}$$

where the first component determines the time t_2 of the feed material detachment from the body, the second component – the time of a free flight, while n is the root of the equation developed by A. Czubak (Czubak and Michalczyk, 2001)

$$k_p = \sqrt{\Big[\frac{\cos(2\pi n) + 2\pi n^2 - 1}{2\pi n - \sin(2\pi n)}\Big]^2 + 1} \tag{2.15}$$

The time t_3 is counted versus the initial moment $t = 0$ assumed in the instant when the machine body achieves its maximum velocity $\dot{\xi}_{max}$. This description is correct for one-stroke motion (which means that the flight is not longer than for 1 vibration period of the machine), which occurs for $1 < k_p \leqslant 3.3$.

The initial instant t=0, for accurately synchronised vibrators, occurs for $\varphi_1 = \varphi_2 = \pi$, which means that the collision takes place when those angles are: $\varphi_1(t_3) = \varphi_2(t_3) = \omega t_3 + \pi = \varphi_0$.

Thus, the angle φ_0 determining the position of vibrators at the moment of collision with the material feed is related to the basic motion parameter of the machine: k_p.

Presently, we will determine the value of the force impulse of the collision. We can take advantage of the fact that the reverse impulse maintains periodicity of material feed motion of the mass m_n remaining in the gravity field.

Thus

$$\int_{t_3}^{t_3+\Delta t} P(t)\, dt = -m_n g T \tag{2.16}$$

Let us divide both terms of the above equation by the body mass M

$$\int_{t_3}^{t_3+\Delta t} \frac{P(t)}{M}\, dt = \frac{-m_n g T}{M} \tag{2.17}$$

On this basis, we will determine the integral from the body vertical acceleration during the collision time within the interval $t_3 \leqslant t \leqslant t_3 + \Delta t$ as follows

$$\int_{t_3}^{t_3+\Delta t} \ddot{y}(t)\, dt = \frac{-m_n}{M} g T \tag{2.18}$$

The applied hereby assumptions require some comments:

- The impulse of the horizontal force was disregarded. Such a procedure is allowed for the stationary motion at the horizontal positioning of the trough, since in that case the material feed does not change its horizontal velocity from period to period, which proves that this impulse is (approximatelly) equal to zero.
- An increased pressure of supporting springs (due to carrying material feed) on the body was also omitted. For over-resonance machines, the so-called "softly" supported ones, this pressure is not significantly changing due to the body working vibrations. This allows one to consider the acceleration in the upward direction as a constant one. This type of the body acceleration is the source of a constant transportation force, in

equations of vibrators motion in the non-inertial system. Such a force is neither the source of any constant driving moment nor a moment of resistance.

- In the case $k_p < 3.3$, the collision impulse in the moment of time t_3 does not counterbalance the force of gravity acting on the material feed for the period T, since its free flight is shorter. However, directly after the collision a short-lasting phase of a common flight occurs, during which the contact force impulse complements the collision impulse to a value of $m_n g T$.
- It was assumed that the restitution coefficient at the material feed collision with the body $R \cong 0$, which corresponds to the case of a loose material feed.

In order to be able to use Equations (2.6) for determining the vibratory moment originated as a result of the body collision with the material feed, the components along the axis ξ and η as well as accelerations $\ddot{y}(t)$, common for both vibrators, should be determined first

$$\ddot{\xi}(t) = \ddot{y}(t)\sin\beta \qquad\qquad \ddot{\eta}(t) = \ddot{y}(t)\cos\beta \tag{2.19}$$

Then

$$\begin{aligned}\mathcal{M}_{w1} &= -me[\ddot{y}(t)\sin\beta\cos\varphi_1 + \ddot{y}(t)\cos\beta\sin\varphi_1] = \\ &= -me[\sin\beta\cos\varphi_1 + \cos\beta\sin\varphi_1]\ddot{y}(t) \\ \mathcal{M}_{w2} &= -me[\ddot{y}(t)\sin\beta\cos\varphi_2 - \ddot{y}(t)\cos\beta\sin\varphi_2] = \\ &= -me[\sin\beta\cos\varphi_2 - \cos\beta\sin\varphi_2]\ddot{y}(t)\end{aligned} \tag{2.20}$$

We can calculate the value of the vibratory moment averaged for the vibration period originated from the collision occurring at $\varphi_1 = \varphi_2 = \varphi_0$ and of duration $\varDelta t \to 0$

$$\begin{aligned}\mathcal{M}_{w1av} &= \frac{1}{T}\int\limits_0^T \mathcal{M}_{w1}(t)\;dt = \frac{1}{T}\Big[-me(\sin\beta\cos\varphi_0 + \cos\beta\sin\varphi_0)\int\limits_0^T \ddot{y}(t)\;dt\Big] = \\ &= -\frac{me}{T}(\sin\beta\cos\varphi_0 + \cos\beta\sin\varphi_0)\frac{-m_n}{M}gT = \\ &= \frac{mem_ng}{M}(\sin\beta\cos\varphi_0 + \cos\beta\sin\varphi_0)\end{aligned} \tag{2.21}$$

Analogously, for vibrator No. 2 we can obtain

$$\mathcal{M}_{w2av} = \frac{1}{T}\int\limits_0^T \mathcal{M}_{w2}(t)\;dt = \frac{mem_ng}{M}(\sin\beta\cos\varphi_0 - \cos\beta\sin\varphi_0) \tag{2.22}$$

The fact that the integral of the collision force for the vibration period T is equivalent to the integral for the period from t_3 to $t_3 + \varDelta t$, since behind this range the collision force equals zero, was utilised. We will calculate the difference of these moments

$$\varDelta\mathcal{M}_w = \mathcal{M}_{w2av} - \mathcal{M}_{w1av} = -\frac{2mem_n g}{M}\cos\beta\sin\varphi_0 \tag{2.23}$$

As can be seen from Equation (2.23) depending on angle φ_0, the vibratory moment load originated from collisions with the material feed can be higher for one of the vibrators. For example, for $\sin\varphi_0 > 0$, when the feed falls on the body being below the state of static equilibrium, the collision constitutes a higher load for vibrator No. 2. In order to maintain equality of the average angular velocity, the dissynchronisation of vibrators (of the type analysed previously) must occur. Vibrator No. 1 has to run with a lead $\varDelta\varphi$ as compared to vibrator No. 2, which causes diversification of vibratory moments originated by vibrations of axes in the opposite direction than those originated from collisions with the material feed. Thus, equating modules of Equations (2.12) and (2.23), it is possible to determine the disphasing angle $\varDelta\varphi$ of vibrators and then – on the basis of $(2.3)_3$ – the time history of body oscillations resulting from collisions with a material feed. It should be emphasised, that in consideration of the equality of the average angular velocity of both vibrators, the static characteristic inclination of the driving motors (which shapes the cumulative value of the power input) does not participate in transmission of an increased power into the more loaded vibrator.

The form of Equation (2.23) indicates the possibility of equalisation of loads of vibrators originated from collisions with the material feed and thus avoiding dissynchronisations of vibrators and angular oscillations of the body. To this end, it is enough to assure that $\sin\varphi_0 = 0$ at the moment when the material feed falls on the body. For a single-stroke motion this happens when $t_3 = \pi/\omega$ or $t_3 = 2\pi/\omega$, i.e. when the collision occurs at the moment when the body passes through the balance point.

It can be stated, on the basis of Equations (2.14) and (2.15), that such a case occurs for the coefficient of throw: $k_p = 1.14$ and $k_p = 2.97$.

Since the value of $k_p = 1.14$ is most often not sufficient for an effective technological process, the assumption of $k_p = 2.97$ is recommended for avoiding body oscillations which cause irregular distribution of vibration amplitudes along the body.

3. Simulation investigations

Simplifying assumptions adopted in the given above analysis indicate usefulness of the verification of the obtained equations by means of computer simulation of the system motion.

The model of the system presented in Fig. 3 was used for the numerical simulation.

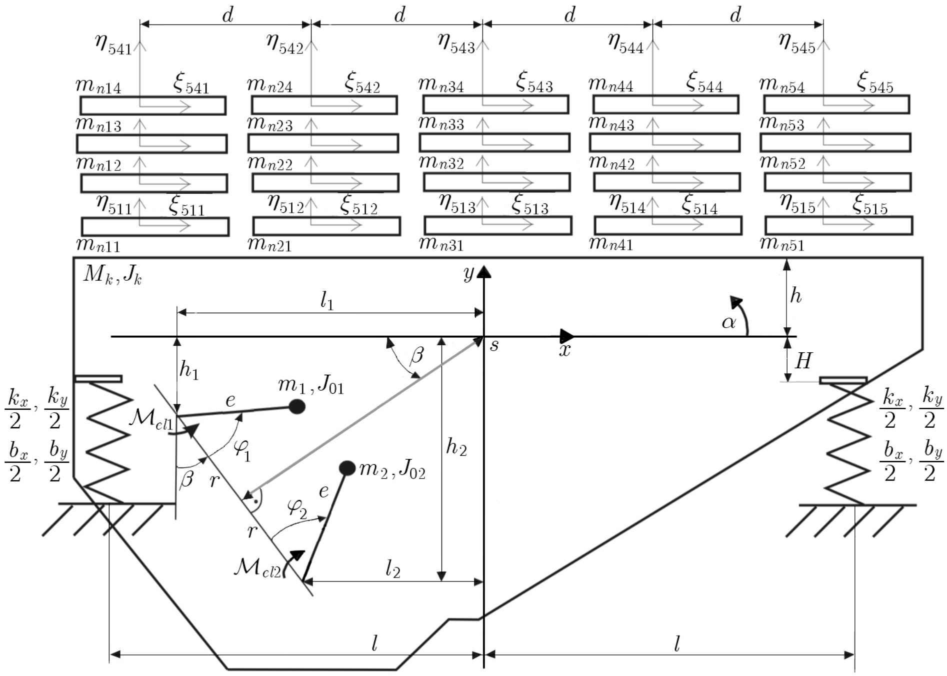

Fig. 3. Model of the feeder together with the material feed

The model consists of: two inertial vibrators of an independent induction drive (described by static characteristics), the machine body performing plane motion and supported by a system of vertical coil springs and five four-layer models of the loose material feed (Czubak and Michalczyk, 2001; Michalczyk and Cieplok, 2006) arranged in different points of the machine working surface. The effect of the gravity force on angular motion of the vibrators is taken into account in this model.

The mathematical model of such a system consists of matrix equation (3.1) describing the machine motion, equations (3.6) concerning electromagnetic moments of driving motors, equations (3.5) determining motions of successive layers of the material feed as well as Equations (3.3) and (3.4) describing normal and tangent interactions in between the material feed layers and between the material feed and the machine body

$$\mathbf{M}\ddot{\boldsymbol{q}} = \boldsymbol{Q} \tag{3.1}$$

where

$$\begin{aligned}
&\mathbf{M} = \begin{bmatrix} \mathcal{M}_k + m_1 + m_2 & 0 & m_1h_1 + m_2h_2 & m_{14} & m_{15} \\ 0 & \mathcal{M}_k + m_1 + m_2 & -m_1a_1 - m_2a_2 & m_{24} & m_{25} \\ m_1h_1 + m_2h_2 & -m_1l_1 - m_2l_2 & m_{33} & m_{34} & m_{35} \\ m_{41} & m_{42} & m_{43} & J_{01} & 0 \\ m_{51} & m_{52} & m_{53} & 0 & J_{02} \end{bmatrix}\\
&\ddot{\boldsymbol{q}} = [\ddot{x}, \ddot{y}, \ddot{\alpha}, \ddot{\varphi}_1, \ddot{\varphi}_2]^\top \\
&\boldsymbol{Q} = [Q_1, Q_2, Q_3, Q_4, Q_5]^\top
\end{aligned} \tag{3.2}$$

where

$$\begin{aligned}
m_{14} &= m_{41} = m_1e_1\cos(\beta + \varphi_1)\\
m_{15} &= m_{51} = m_2e_2\cos(\varphi_2 - \beta)\\
m_{24} &= m_{42} = m_1e_1\sin(\beta + \varphi_1)\\
m_{25} &= m_{52} = -m_2e_2\sin(\varphi_2 - \beta)\\
m_{33} &= m_2h_2^2 + m_2l_2^2 + m_1h_1^2 + m_1l_1^2 + J_k\\
m_{34} &= m_{43} = m_1h_1e_1\cos(\beta + \varphi_1) - m_1l_1e_1\sin(\beta + \varphi_1)\\
m_{35} &= m_{53} = m_2h_2e_2\cos(\varphi_2 - \beta) + m_2l_2e_2\sin(\varphi_2 - \beta)\\
Q_1 &= -m_2e_2\dot{\varphi}_2^2\sin(\varphi_2 - \beta) - m_1e_1\dot{\varphi}_1^2\sin(\beta + \varphi_1) - k_x(x + H\alpha) +\\
&\quad -b_x(\dot{x} + H\dot{\alpha}) - T_{101} - T_{102} - T_{103} - T_{104} - T_{105}\\
Q_2 &= m_2e_2\dot{\varphi}_2^2\sin(\varphi_2 - \beta) + m_1e_1\dot{\varphi}_1^2\cos(\beta + \varphi_1) - \frac{1}{2}k_y(y + l_1\alpha) +\\
&\quad -\frac{1}{2}k_y(y - l_2\alpha) - \frac{1}{2}b_y(\dot{y} + l_1\dot{\alpha}) - \frac{1}{2}b_y(\dot{y} - l_2\dot{\alpha}) +\\
&\quad -F_{101} - F_{102} - F_{103} - F_{104} - F_{105}\\
Q_3 &= -m_1h_1e_1\dot{\varphi}_1^2\sin(\beta + \varphi_1) - m_1l_1e_1\varphi_1^2\cos(\beta + \varphi_1) +\\
&\quad -m_2h_2e_2\dot{\varphi}_2^2\sin(\varphi_2 - \beta) + m_2l_2e_2\dot{\varphi}_2^2\cos(\varphi_2 - \beta) - k_xH^2\alpha - k_xHx +\\
&\quad -b_xH\dot{x} - b_xH^2\dot{\alpha} - \frac{1}{2}k_y(y + l\alpha)l + \frac{1}{2}k_y(y - l\alpha)l - \frac{1}{2}b_y(\dot{y} + l\dot{\alpha})l +\\
&\quad +\frac{1}{2}b_y(\dot{y} - l\dot{\alpha})l + (T_{101} + T_{102} + T_{103} + T_{104} + T_{105})H_n + F_{101}2d +\\
&\quad +F_{102}d - F_{104}d - F_{105}2d\\
Q_4 &= \mathcal{M}_{el1} - b_{s1}\dot{\varphi}_1^2\operatorname{sgn}(\dot{\varphi}_1) - m_1ge_1\sin(\beta + \varphi_1)\\
&\mathcal{M}_{el2} - b_{s2}\dot{\varphi}_2^2\operatorname{sgn}(\dot{\varphi}_2) - m_2ge_2\cos(\varphi_2 - \beta)
\end{aligned}$$

and

$F_{j,j-1,k}$	–	normal component of the j-th layer pressure on the $j-1$ layer in the k-th column,
$T_{j,j-j,k}$	–	tangent component of the j-th layer pressure on the $j-1$ layer in the k-th column,
j	–	material feed index ($j = 0$ concerns the machine body),
k	–	material feed column index
J_{0ic}	–	central moment of inertia of m_i, $i = 1, 2$

$$m_i e_i^2 + J_{0ic} = J_{0i} \qquad i = 1, 2$$

It was further assumed that

$$J_{01} = J_{02} = J_0$$

If successive layers of the material feed j and $j-1$ (in the given column) are not in contact, the contact force in the normal direction $F_{j,j-1,k}$ and in the tangent direction $T_{j,j-1,k}$ between these layers equals zero

$$F_{j,j-1,k} = 0 \qquad T_{j,j-1,k} = 0 \qquad \text{for} \quad \eta_{j,k} \geqslant \eta_{j-1,k}$$

Otherwise, the contact force in the normal direction between the layers j,k and $j-1,k$ of the material feed occurs (or in the case of the first layer: between the layer and the body), the model of which (Michalczyk, 2008) is of the form

$$F_{j,j-1,k} = (\eta_{j-1,k} - \eta_{j,k})^p k_H \Big\{ 1 - \frac{1-R^2}{2} [1 - \operatorname{sgn}(\eta_{j-1,k} - \eta_{j,k}) \operatorname{sgn}(\dot{\eta}_{j-1,k} - \dot{\eta}_{j,k})] \Big\} \tag{3.3}$$

and the force originated from friction in the tangent direction

$$T_{j,j-1,k} = -\mu F_{j,j-1,k} \operatorname{sgn}(\dot{\xi}_{j,k} - \dot{\xi}_{j-1,k}) \tag{3.4}$$

where R is the restitution coefficient of normal impulses at collision, k_H, p – Hertz-Stajerman constants.

The form of dependence (3.3) was developed in Michalczyk (2008) on the basis of the Hertz-Stajerman contact forces model modified by taking into account material damping.

Parameters of the hysteresis loop were assumed in such a way as to have the ratio of the bodies relative velocity after the collision to their velocity before the collision equal to R. It means that formula (3.3) ensures that this ratio is equal to the assumed restitution coefficient.

Equations of motion of individual layers in the directions ξ and η, with taking into consideration the influence of the conveyer on the lower layers of the material feed are in the following form

$$
\begin{aligned}
m_{nj,k}\ddot{\xi} &= T_{j,j-1,k} - T_{j+1,j,k} \\
m_{nj,k}\ddot{\eta} &= -m_{nj,k}g + F_{j,j-1,k} - F_{j+1,j,k}
\end{aligned}
\tag{3.5}
$$

M_{eli} – electromagnetic moment generated by the i-th motor assumed in the form corresponding to the static characteristic of the motor

$$
\mathcal{M}_{eli} = \frac{2\mathcal{M}_{ut}(\omega_{ss} - \dot{\varphi}_{i1})(\omega_{ss} - \omega_{ut})}{(\omega_{ss} - \omega_{ut})^2 + (\omega_{ss} - \dot{\varphi}_i)^2} \qquad i = 1,2 \tag{3.6}
$$

where: M_{ut} – stalling torque of the driving motors, ω_{ss} – their synchronous frequency and ω_{ut} – stalling frequency.

The simulation was performed for the following parameters: $l = 0.5\,\mathrm{m}$, $l_1 = 1\,\mathrm{m}$, $l_2 = 0.5\,\mathrm{m}$, $H = 0.0\,\mathrm{m}$, $h_1 = 0.5\,\mathrm{m}$, $h_2 = 1\,\mathrm{m}$, $b_x = b_y = 400\,\mathrm{Ns/m}$, $k_x = k_y = 150000\,\mathrm{N/m}$, $m_1 = m_2 = 5\,\mathrm{kg}$, $M_k = 120\,\mathrm{kg}$, $J_{01} = J_{02} = J_0 =$ variable, $J_k = 25\,\mathrm{kgm}^2$, $e_1 = e_2 =$ variable $e(k_p)\,\mathrm{m}$, $D = 1.118\,\mathrm{m}$, $m_{ut} = 50\,\mathrm{Nm}$, $\omega_{ss} = 50\pi\,\mathrm{rad/s}$, $\omega_{ut} = 15.9 \cdot 2\pi\,\mathrm{rad/s}$, $b_{s1} = b_{s2} = 0.00009\,\mathrm{Nms}^2$.

The simulation model developed for the verification of analytical solutions takes into consideration not only factors included in the analytical solutions but also other phenomena of essential meaning for the process of vibrators synchronisation, such as e.g. force of gravity. In addition, no limitations for disphasing angles were introduced and the vibratory moments were treated as variables (not averaged) within the period of machine vibrations.

4. Conclusions

In order to verify the analytical solution, the amplitudes of angular oscillations of the body A_α, being the result of vibrators disphasing due to collisions with the material feed were determined on the basis of equations (2.12), (2.23) and $(2.3)_3$. The obtained values were compared with the simulation results. The calculations and simulations were performed for various values of the coefficient of throw k_p from the range $[1, \sqrt{\pi^2+1}]$ and for two masses of the feed: $m_n = 20\,\mathrm{kg}$ and $60\,\mathrm{kg}$. The results obtained by analytical and simulation methods are presented in Figs. 4, 5, 6 and 7.

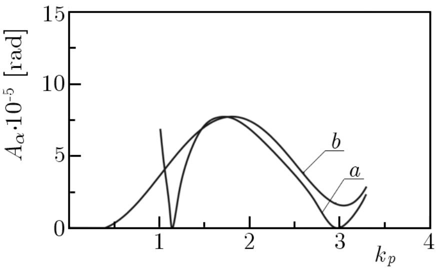

Fig. 4. Angular amplitudes of the body A_α versus coefficient of throw k_p for the material feed in a lump form of mass of 20 kg, (a) theoretical curve, (b) digital simulation

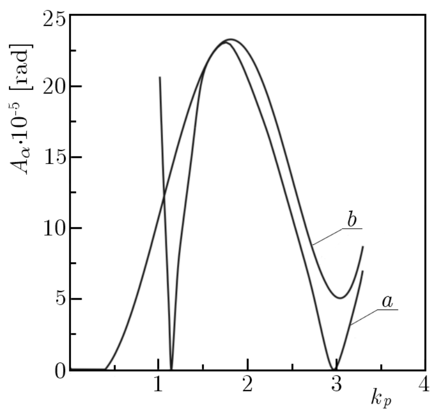

Fig. 5. Angular amplitudes of the body A_α versus coefficient of throw k_p for the material feed in a lump form of mass of 60 kg, (a) theoretical curve, (b) digital simulation

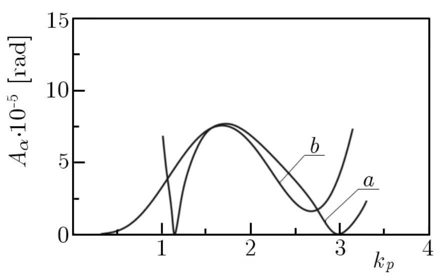

Fig. 6. Angular amplitudes of the body A_α versus coefficient of throw k_p for the loose material feed of mass of 20 kg, (a) theoretical curve, (b) digital simulation

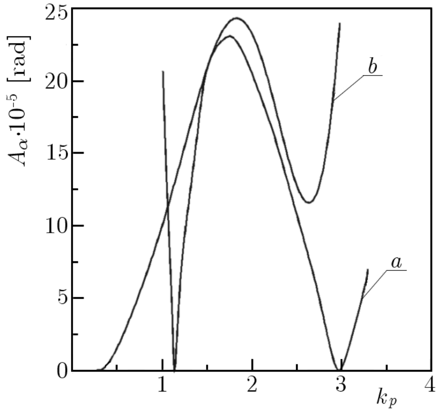

Fig. 7. Angular amplitudes of the body A_α versus coefficient of throw k_p for the loose material feed of mass of 60 kg, (a) theoretical curve, (b) digital simulation

The comparison of results obtained analytically and by means of digital simulation leads to the following conclusions:

- The mathematical model, developed in this paper, properly describes the influence of a lumped material feed on diversification of phase angles of the vibrators and the resulting angular oscillations of the body for the coefficient of throw within the range $k_p = 1.5$ to 3.3, corresponding to variability of this parameter in industrial conditions. It allows one to predict, with high accuracy, the maximum value of body angular oscillations and its position ($k_p \approx 1.75$). It also indicates the existence of the amplitude minimum for $k_p \approx 3.0$.

  However, the value of this minimum is not zero, as indicates the theory, but approximately 20% of the peak value.

- In the case of a loose material feed, the analytical dependencies allow prediction, satisfactory for the practice, of the maximum body angular oscillation and its position.

  However, in this case, the minimum of body oscillations does not occur for $k_p \approx 3.0$ as predicts the theory, but near $k_p = 2.7$, and its value varies by 20% (for a material feed constituting app. 15% of the body mass) up to 47% (for a material feed constituting app. 46% of the body mass) of the peak value.

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Wpływ zderzeń z nadawą na współfazowość synchronizacji wzajemnej wibratorów napędowych maszyn wibracyjnych

Streszczenie

W pracy wskazano na istnienie związku poomiędzy zakłócającymi przebieg transportu wibracyjnego wahaniami korpusów maszyn wibracyjnych a utratą współfazowości wibratorów napędowych. Wykazano, że utrata współfazowości spowodowana być może przez okresowe zderzenia korpusu z nadawą i zbudowano model matematyczny tego zjawiska. Uzyskane zależności analityczne, pozwalające na oszacowanie rozfazowania wibratorów i ocenę amplitudy wahań kątowych maszyny, zweryfikowano przez porównanie z rezultatami otrzymanymi na drodze symulacji cyfrowej zachowania układu.

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