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ADJUSTMENT CALCULUS AND TREFFTZ FUNCTIONS APPLIED TO LOCAL HEAT TRANSFER COEFFICIENT DETERMINATION IN A MINICHANNEL

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The paper presents results of numerical calculations conducted in order to define the heat transfer coefficient in flow boiling in a vertical minichannel with one side made of a heating foil with liquid crystals. During the experiment, we measured the local temperature of the foil, inlet and outlet liquid temperature and pressure, current and voltage drop of the electric power supplied to the heater. Local measurements of foil temperature were approximated with a linear combination of the Trefftz functions. The known temperature measurement errors allowed application of the adjustment calculus. The foil temperature distribution was determined by the FEM combined with the Trefftz functions. Local heat transfer coefficients between the foil and the boiling fluid were calculated from the third-kind condition.

Key words: flow boiling, liquid crystals, heat transfer coefficient

Nomenclature

A, B, c	–	linear combination coefficient
$\mathbf{C}, \mathbf{D}, \mathbf{S}, \mathbf{V}, \mathbf{v}$	–	matrix
G	–	mass flux [kg/(m^{2}s)]
I	–	current supplied to heating foil [A]
J	–	error functional
L	–	minichannel length [m]
l_w, M, N, P	–	number of nodes in element, number of Trefftz functions used for approximation and number of measurement points, respectively
p, T	–	pressure [Pa] and temperature [K]
Re	–	Reynolds number
q_v	–	volumetric heat flux (capacity of internal heat source) [W/m^3]
U, W	–	voltage drop across the foil [V] and foil width [m]
u	–	particular solution of non-homogeneous equation
v_n	–	n-th Trefftz function
x, y	–	spatial coordinates

Greek

α	–	heat transfer coefficient [W/(m^{2}K)]
Δ	–	error
δ	–	thickness [m]
ε, σ	–	temperature measurement correction [K] and measurement error [K]
Φ, φ	–	Lagrange function and basis function
λ	–	thermal conductivity [W/(mK)]
Ω, ω	–	flat domain and Lagrange multiplier

Subscripts

approx	–	measurement data approximation
F, f, G	–	foil, fluid and glass, respectively
i, j, k, m, n	–	numbers
in, *out*	–	at inlet and at outlet
p	–	measurement point

Superscripts

corr	–	referenced to smoothed measurements
j, k	–	numbers
$\widetilde{(\cdot)}$	–	approximate solution

1. Adjustment calculus and Trefftz functions

The paper presents results of numerical calculations conducted in order to define the heat transfer coefficient in flow boiling in a vertical minichannel. The conducted experiment is presented here only in brief, a more detailed description can be found in Hozejowska *et al.* (2009) and Piasecka (2002). The major part of the test stand is a minichannel measurement module. R-123 refrigerant flows through the minichannel which is 1 mm deep, 40 mm wide and 300 mm long. One of the minichannel walls is made of a heating foil supplied with DC of adjustable strength. A layer of liquid crystals is applied to the foil. Liquid crystals hue allows defining the temperature distribution of the foil external surface (the so called liquid crystal thermography). The channels in the back wall of the measurement module make possible to maintain constant temperature on the wall, so that it can be regarded as quasiadiabatic.

In each series of experiments, the heat flux on the heating foil is increased gradually to induce boiling incipience and allow observations of the so called "boiling front" (after the foil temperature increase at the set constant heat flux, a rapid decrease in the foil temperature follows after exceeding the boiling front value). A detailed description of the test stand is provided in Hozejowska *et al.* (2009) and Piasecka (2002).

The paper focuses on determining local heat transfer coefficients between the foil and the fluid by means of the finite element method combined with the Trefftz functions (further called FEMT). The Trefftz functions are functions which satisfy exactly the governing differential equation. The method discussed here uses harmonic polynomials as the Trefftz functions which satisfy Laplace's equation. Such polynomials are defined as a real part and an imaginary part of the complex number $(x + \mathrm{i}y)^n$ (i is the imaginary unit), $n = 0, 1, 2, \ldots$.

Additional information on the Trefftz functions can be found in Ciałkowski and Frąckowiak (2002), Herrera (2000), Kita (1995), Zieliński (1995).

Temperatures of the foil T_k are measured using liquid crystal thermography at the points with coordinates (x_k, δ_G), where δ_G denotes the thickness of glass. The continuous form of the measured temperature may be obtained in the form of a linear combination of the Trefftz functions (T-functions)

$$T_{approx}(x) = \sum_{i=0}^{R} c_i v_i(x, \delta_G) \tag{1.1}$$

where $v_i(x, y)$ is the i-th T-function and c_i denote coefficients of linear combination. The coefficients are computed based on measurement data T_k from the dependence

$$T_{approx}(x, k) = T_k \tag{1.2}$$

Temperature measurements T_k, approximated by the polynomial $T_{approx}(x)$, are corrected by the adjustment calculus (Brandt, 1999; Szargut, 1984). We look for corrections ε_k to measurements T_k, and for new coefficients ci for approximate $T^{corr}_{approx}(x)$ so that the following dependencies can be satisfied

$$a)\ \ T_k^{corr} = T_k + \varepsilon_k \qquad\qquad b)\ \ T^{corr}_{approx}(x_k) - T_k^{corr} = 0 \tag{1.3}$$

The corrections ε_k are determined so as to minimize Lagrange'a function

$$\varPhi = \sum_{k=1}^{K}\Big(\frac{\varepsilon_k}{\sigma_k}\Big)^2 + 2\sum_{k=0}^{K}\omega_k(T^{corr}_{data}(x,k) - T_k^{corr}) \ \rightarrow\ \min \tag{1.4}$$

where ω_k – Lagrange multipliers, σ_k – measurement errors at the k-th measurement point. The error σ_k is obtained from the calibration curve, which defines the relationship between the foil temperature and the liquid crystals hue (Hożejowska *et al.*, 2009; Piasecka, 2002).

For the corrected temperature T_k^{corr}, we recalculate the measurement errors $\sigma_k^{corr} = \sqrt{C_{kk}}$ according to the error propagation law. By introducing $S_{ki} = v_i(x_k, \delta_G)$, we can calculate matrix $\mathbf{C}$ from the formula

$$\mathbf{C} = \mathbf{S}(\mathbf{S}^{\mathrm{T}}\mathbf{D}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{T}} \tag{1.5}$$

As the measurements are independent from each other, the weight matrix is diagonal, $\mathbf{D} = [1/\sigma_k^2]$.

2. Mathematical model and approximate solution

In the minichannel, a steady state is assumed. The unknown glass and foil temperatures, T_G and T_F, satisfy the following equations

$$\nabla^2 T_G = 0 \qquad \text{for} \qquad (x,y) \in \varOmega_G = \{(x,y) \in R^2:\ \ 0 < x < L,\ 0 < y < \delta_G\} \tag{2.1}$$

and

$$\begin{aligned}&\nabla^2 T_F = -\frac{q_v}{\lambda_F} = -\frac{UI}{\delta_F W L \lambda_F}\\ &\qquad \text{for} \qquad (x,y) \in \varOmega_F = \{(x,y) \in R^2:\ \ x_1 < x < x_P,\ \delta_G < y < \delta_G + \delta_F\}\end{aligned} \tag{2.2}$$

where x_1 is the coordinate of the first temperature measurement, x_P is the coordinate of the final temperature measurement, U – voltage drop $\mathbf{V}$, I – current $\mathbf{A}$, δ_F – foil thickness [m], W – foil width [m], L – foil length [m], λ_F – foil thermal conductivity [Wm^{-1}K^{-1}], q_v – volumetric heat flux [Wm^{-3}]. At the glass-foil contact, we assume

$$\lambda_F \frac{\partial T_F}{\partial y} = \lambda_G \frac{\partial T_G}{\partial y} \qquad \text{for} \qquad y = \delta_G \quad \wedge \quad x_1 < x < x_P \tag{2.3}$$

and

$$\begin{aligned}&T_F(x,\delta_G) = T_G(x,\delta_G) \qquad \text{for} \qquad x_1 < x < x_P\\ &T_F(x_p,\delta_G) = T_G(x_p,\delta_G) = T_p \qquad \text{for} \qquad p = 1,2,\dots,P\end{aligned} \tag{2.4}$$

Two mathematical models are considered, in one model $T_p = T_{approx}(x_p)$, while in the other $T_p = T^{corr}_{approx}(x_p)$, for $p = 1,2,\dots,P$. Conditions on other boundaries, see Fig. 1, are

$$\begin{aligned}&\frac{\partial T_G}{\partial y} = 0 \qquad \text{for} \qquad y = 0 \quad \wedge \quad 0 < x < L\\ &\frac{\partial T_G}{\partial x} = 0 \qquad \text{for} \qquad x = 0 \qquad \text{as well as} \qquad x = L \quad \wedge \quad 0 < y < \delta_G\end{aligned} \tag{2.5}$$

and

$$
\begin{aligned}
T_F(x_1, y) &= T_1 \qquad \text{for} \qquad \delta_G < y < \delta_G + \delta_F \\
T_F(x_P, y) &= T_P \qquad \text{for} \qquad \delta_G < y < \delta_G + \delta_F
\end{aligned}
\tag{2.6}
$$

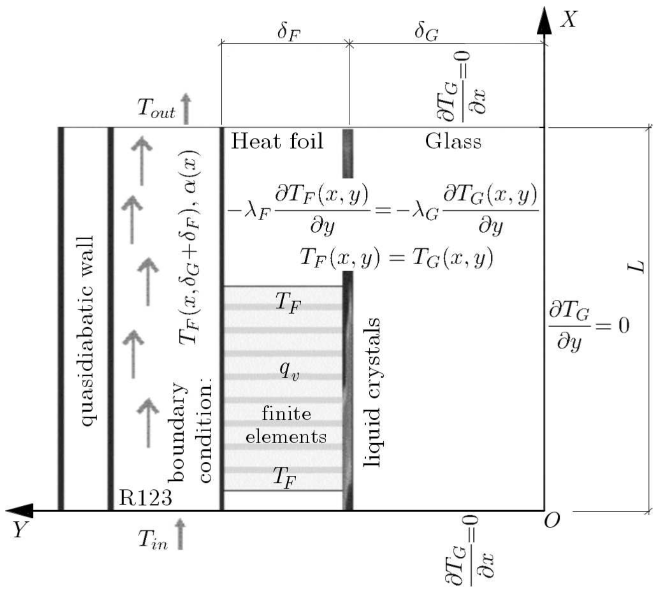

Fig. 1. Scheme of the minichannel and boundary conditions for the flow boiling process

The method of solving equations (2.1) to $(2.6)_2$ is a generalization of the method presented by Ciałkowski and Frąckowiak (2000, 2002). To solve the given problem, the domains Ω_G, Ω_F are divided into elements Ω_G^j, Ω_F^j. The approximate solution to equation (2.1) in each element Ω_G^j is a linear combination of the Trefftz polynomials

$$
\widetilde{T}_G^j(x,y) = \sum_{n=1}^{N} A_{jn} v_n(x,y) \tag{2.7}
$$

Assuming that temperatures $\widetilde{T}_G^{jk}$ in the nodes (x_k, y_k) of the element Ω_G^j are known, the coefficients A_{jn} are calculated from the simultaneous equations

$$
\widetilde{T}_G^j(x_k, y_k) = \widetilde{T}_G^{jk} = \sum_{n=1}^{N} A_{jn} v_n(x_k, y_k) \qquad k = 1, 2, \dots, N \tag{2.8}
$$

In a matrix notation, system (2.8) has a form $\mathbf{vA} = \mathbf{T}$ where after the inversion of the matrix $\mathbf{v}$ we obtain $\mathbf{A} = \mathbf{v}^{-1}\mathbf{T} = \mathbf{VT}$. Therefore

$$
A_{jn} = \sum_{k=1}^{N} V_{nk} \widetilde{T}_G^{jk} \tag{2.9}
$$

Substitution (2.9) into (2.7) leads to the basis functions suitable for the element Ω_G^j in the form

$$
\varphi_{jk}(x,y) = \sum_{n=1}^{N} V_{nk} v_n(x,y) \tag{2.10}
$$

These functions satisfy exactly equation (2.1). The glass temperature in each element Ω_G^j is represented as a linear combination of the basis functions φ_{jk}

$$
\widetilde{T}_G^j(x,y) = \sum_{k=1}^{l_w} \varphi_{jk}(x,y)\, \widetilde{T}_G^{(n)} \tag{2.11}
$$

where j is the element number, l_w – number of nodes in the element, k – node number in the j-th element, n – node number in the entire domain Ω_G.

The unknown coefficients $\widetilde{T}_G^{(n)}$ of linear combination (2.11) are determined by minimizing the functional J_G, which expresses the error of fulfilling the boundary conditions and the discrepancy between the heat flux flowing out from the given element and the heat flux flowing into the neighboring element, but only in the direction of OX axis, because in the direction of OY axis, the domain is not divided into elements (Ciałkowski and Frąckowiak, 2002; Grysa and Leśniewska, 2010)

$$\begin{aligned} J_G &= \int\limits_0^{\delta_G} \Big(\frac{\partial \widetilde{T}_G^1}{\partial x}(0,y)\Big)^2\,dy + \int\limits_0^{\delta_G} \Big(\frac{\partial \widetilde{T}_G^{L1}}{\partial x}(L,y)\Big)^2\,dy + \sum_{p_i=1}^{P} [\widetilde{T}_G^i(x_{p_i},\delta_G) - T_{p_i}]^2 \\ &+ \sum_{i=1}^{L1} \int\limits_{x_{i-1}}^{x_i} \Big(\frac{\partial \widetilde{T}_G^i}{\partial y}(x,0)\Big)^2\,dx + \sum_{i=1}^{L1-1} \int\limits_0^{\delta_G} [\widetilde{T}_G^i(x_i,y) \\ &- \widetilde{T}_G^{i+1}(x_i,y)]^2\,dy + \sum_{i=1}^{L1-1} \int\limits_0^{\delta_G} \Big(\frac{\partial \widetilde{T}_G^i}{\partial x}(x_i,y) - \frac{\partial \widetilde{T}_G^{i+1}}{\partial x}(x_i,y)\Big)^2\,dy \end{aligned} \tag{2.12}$$

where $L1$ is the number of elements in the OX axis direction.

The foil temperature is determined in the same way. The approximate solution to equation (2.2) in each element Ω_F^j has the following form

$$\widetilde{T}_F^j(x,y) = u(x,y) + \sum_{n=1}^{N} B_{jn} v_n(x,y) \tag{2.13}$$

where $u(x,y)$ is the particular solution to equation (2.2).

From the set of equations

$$\widetilde{T}_F^j(x_k,y_k) = \widetilde{T}_F^{jk} = u(x_k,y_k) + \sum_{n=1}^{N} B_{jn} v_n(x_k,y_k) \qquad k=1,2,\ldots,N \tag{2.14}$$

cofficients B_{jn} are computed, as in the case of glass temperature determination. Foil temperature in each element Ω_F^j is represented in the form of a linear combination of the basis functions φ_{jk}, see formula (2.10)

$$\widetilde{T}_F^j(x,y) = u(x,y) + \sum_{k=1}^{l_w} \varphi_{jk}(x,y)[\widetilde{T}_F^{(n)} - u(x_n,y_n)] \tag{2.15}$$

where $u(x_n,y_n)$ stands for the particular solution value in the n-th node of area Ω_F (the subscripts bear the same meaning as in formula (2.11)). The unknown coefficients of linear combination (2.15) are determined by the minimizing functional J_F

$$\begin{aligned} J_F &= \int\limits_{\delta_G}^{\delta_G+\delta_F} [\widetilde{T}_F^1(x_1,y) - T_1]^2\,dy + \int\limits_{\delta_G}^{\delta_G+\delta_F} [\widetilde{T}_F^{L1}(x_P,y) - T_P]^2\,dy + \sum_{p_i=1}^{P} [\widetilde{T}_F^i(x_{p_i},\delta_G) - T_{p_i}]^2 \\ &+ \sum_{i=1}^{L1} \int\limits_{x_1}^{x_{i+1}} [\widetilde{T}_F^i(x,\delta_G) - \widetilde{T}_G^i(x,\delta_G)]^2\,dx + \sum_{i=1}^{L1} \int\limits_{x_i}^{x_{i+1}} \Big(\lambda_F \frac{\partial \widetilde{T}_F^i}{\partial y}(x,\delta_G) - \lambda_G \frac{\partial \widetilde{T}_G^i}{\partial y}(x,\delta_G)\Big)^2\,dx \\ &+ \sum_{i=1}^{L1-1} \int\limits_{\delta_G}^{\delta_G+\delta_F} [\widetilde{T}_F^i(x_i,y) - \widetilde{T}_F^{i+1}(x_i,y)]^2\,dy + \sum_{i=1}^{L1-1} \int\limits_{\delta_G}^{\delta_G+\delta_F} \Big(\frac{\partial \widetilde{T}_F^i}{\partial x}(x_i,y) - \frac{\partial \widetilde{T}_F^{i+1}}{\partial x}(x_i,y)\Big)^2\,dy \end{aligned} \tag{2.16}$$

where $L1$ is the number of elements in the OX axis direction. In the OY axis direction, the domain is not divided into elements.

The known foil temperature distribution allows determination of the heat transfer coefficient from the formula

$$-\lambda_F \frac{\partial \widetilde{T}_F(x, \delta_G + \delta_F)}{\partial y} = \alpha(x)[\widetilde{T}_F(x, \delta_G + \delta_F) - T_f(x)] \tag{2.17}$$

where the fluid temperature $T_f(x)$ is approximated linearly along the entire channel length from the temperature of liquid T_{in} at the channel inlet to the liquid temperature T_{out} at the channel outlet.

Application of the FEMT to solve 2D inverse heat conduction problems was reported and discussed by Grysa and Leśniewska (2010). Non-stationary inverse heat conduction problems are more complicated than stationary ones, and conclusions concerning the FEMT hold also for stationary problems. In Grysa and Leśniewska (2010), a problem of solving the system of algebraic equations resulting from the FEMT is considered and results for different numbers of the Trefftz functions are discussed (to compare accuracy of results for different numbers of the Trefftz functions, approximations with 12 and 15 functions were considered). Generally, when increasing the number of the Trefftz functions one arrives at better results (Grysa and Leśniewska, 2010). However, when the number of the functions is too large, the system of algebraic equations becomes ill-conditioned. In such a case, an approximate solution can be found with the use of the regularization method (Tikhonov and Arsenin, 1977). It is worth to mention that the use of the adjustment calculus radically improves conditioning of the algebraic system of equations resulting from the classical Trefftz method (Piasecka *et al.*, 2004). The study on the impact of the adjustment calculus on results obtained with the use of the FEMT will be presented in our further papers.

In the FEMT, the number of the Trefftz functions can be small, because they satisfy the governing equation and therefore good accuracy of the approximate solution is ensured even with a small number of the functions and with bigger finite elements than applied usually. Here, in the considered problem, only 4 Trefftz functions are used as basic functions in the FEMT, because four-node elements are applied.

The Trefftz functions can be also used to smooth the inaccurate input data. Also such a case was considered in Grysa and Leśniewska (2010) and the conclusion was that smoothing the noisy input data leads to results comparable with those obtained for the exact ones. In this paper, 12 Trefftz functions are used to smooth the results of measured values of temperature.

3. Results

The numerical results of heat transfer identification were derived from experimental data presented in Fig. 2, where we can observe hue distributions on the foil external surface (obtained through liquid crystal thermography). Twelve T-functions (nine-degree polynomials) were used to smooth the measurement data.

Four T-functions were used in the FEMT: 1, $x + \frac{x^2 y}{2} - \frac{y^3}{6}$, $y + \frac{x^3}{6} - \frac{xy^2}{2}$, $xy + \frac{x^2}{2} - \frac{y^2}{2}$. In the domains Ω_G, Ω_F a rectangular grid was brought in, parallel to the coordinate system axes. The domain Ω_G was divided into 407 elements. Depending on the number of measurements, the domain Ω_F was divided into 92-166 elements. The domain Ω_G included 816 nodes, and the domain Ω_F had from 186 to 334 nodes. The element width was equal to the width of foil or glass, respectively. In each element Ω_G^j, Ω_F^j, a set of four nodes was chosen, located in the vertices of the rectangular element, Fig. 5. The function $u(x, y) = -q_v y^2/(2\lambda_F)$ was adopted as the particular solution to equation (2.2).

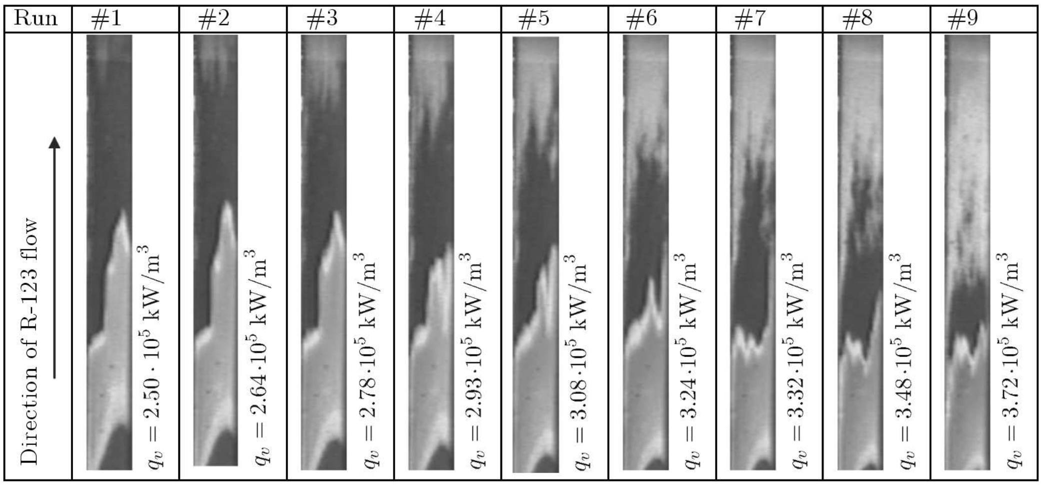

Fig. 2. Hue distribution on the minichannel external surface while increasing the heat flux supplied to the heating foil. Parameters of the runs: $G = 219\,\mathrm{kg/(m^2s)}$, $\mathrm{Re} = 946$, $p_{inlet} = 330\,\mathrm{kPa}$; foil parameters: $\delta_F = 1.02 \cdot 10^{-4}\,\mathrm{m}$, $W = 0.04\,\mathrm{m}$, $L = 0.3\,\mathrm{m}$, $\lambda_F = 8.3\,\mathrm{W/(mK)}$; glass parameters: $\delta_G = 0.005\,\mathrm{m}$, $\lambda_G = 0.71\,\mathrm{W/(mK)}$

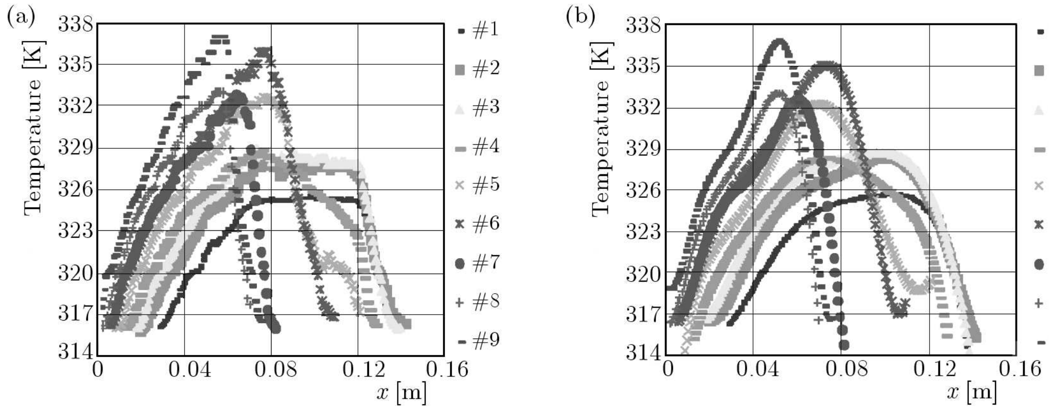

Fig. 3. Temperature measurement at the foil-glass contact area: (a) T_k – obtained through liquid crystals thermography, (b) T_k^{corr} – smoothed with the Trefftz functions and adjustment calculus

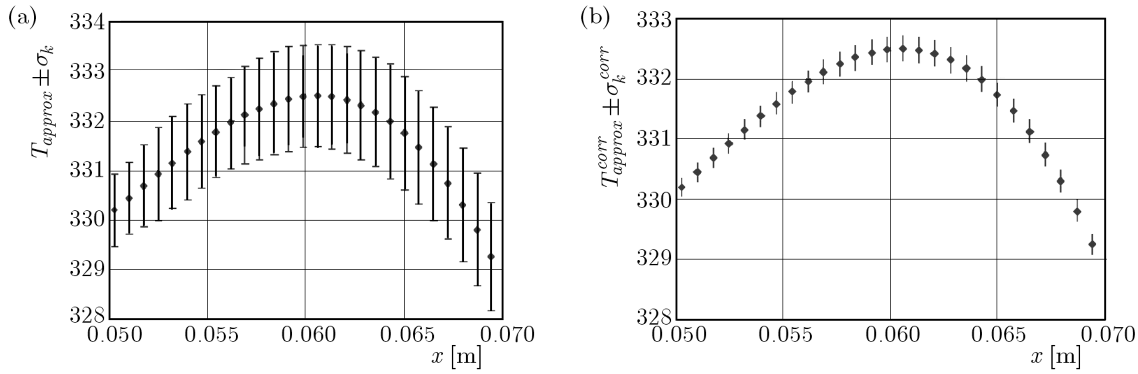

Fig. 4. Distributions near the boiling front (both for run #7): (a) $T_{approx} \pm \sigma_k$ denotes the temperature and mesurement errors, (b) $T_{approx}^{corr} \pm \sigma_k^{corr}$ denotes the temperature and measurement errors for data after applying the adjustment calculus

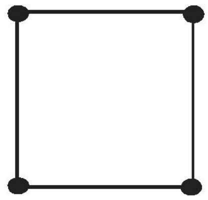

Fig. 5. Node distribution in FEMT element

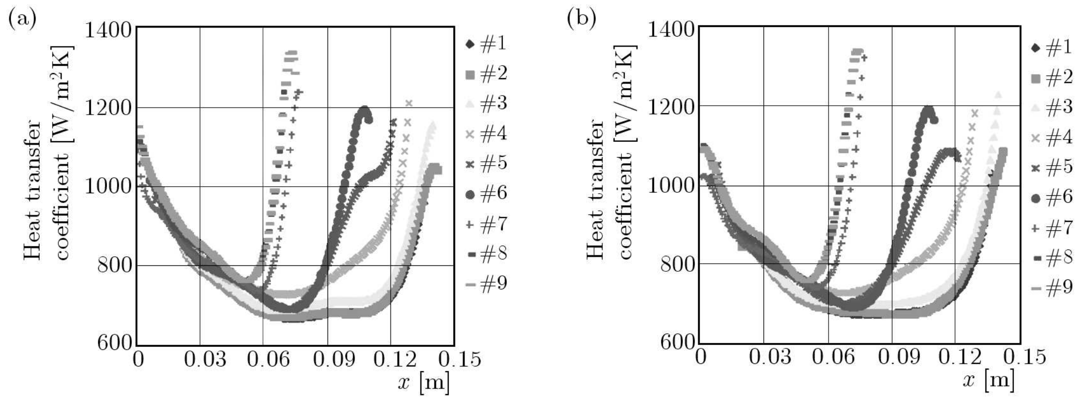

Fig. 6. Heat transfer coefficients calculated with FEMT on the basis of (a) measurement data T_{approx}, (b) adjustment calculus-smoothed data T^{corr}_{approx}

The heat transfer coefficient is calculated from the formula

$$\alpha(x) = \frac{-\lambda_F \frac{\partial \widetilde{T}_F(x,\delta_G+\delta_F)}{\partial y}}{\widetilde{T}_F(x,\delta_G+\delta_F) - T_f(x)} \tag{3.1}$$

thus the heat transfer coefficient error reads

$$\Delta\alpha = \sqrt{\Big(\frac{\partial\alpha}{\partial\widetilde{T}_F}\Delta\widetilde{T}_F\Big)^2 + \Big(\frac{\partial\alpha}{\partial T_f}\Delta T_f\Big)^2 + \Big(\frac{\partial\alpha}{\partial\lambda_F}\Delta\lambda_F\Big)^2 + \Big(\frac{\partial\alpha}{\partial\frac{\partial\widetilde{T}_F}{\partial y}}\Delta\frac{\partial\widetilde{T}_F}{\partial y}\Big)^2} \tag{3.2}$$

where $\Delta\lambda_F$ is the accuracy of the heat conductivity determination, $\Delta\lambda_F = 0.1\,\mathrm{W/(mK)}$ (Piasecka, 2002); $\Delta\widetilde{T}_F$ – accuracy of the foil temperature approximation (since the foil is very thin, we can assume that $\Delta\widetilde{T}_F$ is equal to the error of the temperature measurement, that is, $\Delta\widetilde{T}_F(x_k,\delta_G+\delta_F) = \sigma_k$ or $\Delta\widetilde{T}_F(x_k,\delta_G+\delta_F) = \sigma_k^{corr}$); ΔT_f – the saturation temperature measurement error (Piasecka, 2002), $\Delta T_f = 0.39\,\mathrm{K}$; $\Delta\partial\widetilde{T}_F/\partial y = |\partial^2\widetilde{T}_F/(\partial y\partial x)\Delta x|$, where Δx equals four times dimension of the pixel: $\Delta x = 7.4\cdot 10^{-4}\,\mathrm{m}$ as the measured temperature T_k is an average over the temperature at the point and its four surrounding neighbors (Hożejowska *et al.*, 2009; Piasecka *et al.*, 2004).

The mean relative heat transfer coefficient errors, calculated before and after the adjustment calculus has been applied, are presented in Table 1.

The adjustment calculus reduces the mean relative heat transfer coefficient errors due to the fact that after application of the adjustment calculus, the component $\partial\alpha/(\partial\widetilde{T}_F)\Delta\widetilde{T}_F$ decreases substantially (in the way the measurement error decreases). Moreover, for the FEMT, the mean error $\Delta\alpha/\alpha$ is considerably smaller (even seven times smaller) when compared with the method where the foil and glass temperatures are approximated with the Trefftz functions and the domain is not divided into elements (Hożejowska *et al.*, 2009; Piasecka *et al.*, 2004).

Table 1. Mean relative heat transfer coefficient errors $\Delta\alpha/\alpha$ [%]

Run	Mean relative errors	Mean relative errors after the use of adjustment calculus
#1	2.26	1.69
#2	2.31	1.70
#3	2.35	1.66
#4	2.20	0.26
#5	2.52	1.76
#6	2.54	1.65
#7	2.92	1.76
#8	2.95	1.71
#9	9.70	1.84

4. Conclusions

- Measurement data approximation with the Trefftz functions helps one to "smooth" the measurements and decreases measurement errors.
- The Trefftz functions applied to the FEM as basis functions give a solution which satisfies exactly the governing differential equation.
- The heat transfer coefficient calculated on the basis of measurement data smoothed with the Trefftz functions and adjustment calculus has mean relative errors significantly smaller than the coefficient calculated on the basis of data smoothed with the Trefftz functions alone.

Acknowledgement

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References

1. Brandt S., 1999, *Data Analysis, Statistical and Computational Methods for Scientists and Engineers*, Springer Verlag, New York
2. Ciałkowski M.J., Frąckowiak A., 2000, *Heat Functions and their Application to Solving Heat Conduction in Mechanical Problems*, Publishing House of Poznan Univ. of Technology [in Polish]
3. Ciałkowski M.J., Frąckowiak A., 2002, Solution of a stationary 2D inverse heat conduction problem by Trefftz method, *Journal of Thermal Science*, **11**, 148-162
4. Grysa K., Leśniewska R., 2010, Different finite element approaches for inverse heat conduction problems, *Inverse Problems in Science and Engineering*, **18**, 3-17
5. Hożejowska S., Piasecka M., Poniewski M.E., 2009, Boiling heat transfer in vertical minichannels. Liquid crystal experiments and numerical investigations, *International Journal of Thermal Sciences*, **48**, 1049-1059
6. Herrera I., 2000, Trefftz method: A general theory, *Numerical Methods for Partial Differential Equations*, **16**, 561-580
7. Kita E., 1995, Trefftz method: an overview, *Advanced Software Engineering*, **24**, 3-12
8. Piasecka M., Hożejowska S., Poniewski M.E., 2004, Experimental evaluation of flow boiling incipience of subcooled fluid in a narrow channel, *International Journal of Heat and Fluid Flow*, **25**, 159-172

9. Piasecka M., Maciejewska B., 2010, FEM method using Trefftz functions for determination of heat transfer coefficient in a minichannel, *International Conference Experimental Fluid Mechanics*, Liberec, T. Vit, P. Dancova (Edit.)
10. Piasecka M., 2002, Theoretical and experimental investigations into flow boiling heat transfer in a narrow channel, Ph.D. Thesis, Kielce Univ. of Technology, Dept. Mech. Eng., Kielce [in Polish]
11. Szargut J., 1984, *Equalizing Calculus in Heat Engineering*, Ossolineum, Katowice [in Polish]
12. Tikhonov A.N., Arsenin V.Y., 1977, *Solution of Ill-posed Problems*, Wiley & Sons, Washington, DC
13. Zieliński A.P., On trial functions applied in the generalized Trefftz method, *Advances in Engineering Software*, **24**, 147-155

Zastosowanie rachunku wyrównawczego i funkcji Trefftza do wyznaczenia lokalnego współczynnika przejmowania ciepła w minikanale

Streszczenie

Przedstawiono wyniki obliczeń numerycznych przeprowadzonych w celu określenia współczynnika przejmowania ciepła przy przepływie wrzącej cieczy w pionowym minikanale, którego jedna ściana wykonana jest z folii grzewczej pokrytej ciekłymi kryształami. Podczas eksperymentu mierzono: lokalną temperaturę folii, temperaturę cieczy na wlocie i wylocie z minikanału, ciśnienie cieczy w minikanale oraz spadek natężenia i napięcia prądu stałego dostarczanej do folii grzewczej. Lokalne pomiary temperatury folii aproksymowano kombinacją liniową funkcji Trefftza. Znane błędy pomiaru temperatury pozwoliły na stosowanie rachunku wyrównawczego. Rozkład temperatury folii został określony przy zastosowaniu MES w powiązaniu z funkcjami Trefftza. Lokalne współczynniki wymiany ciepła pomiędzy folią i wrzącym płynem zostały wyliczone z warunku trzeciego rodzaju.

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