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UNIVERSAL WEIGHTING FUNCTION IN MODELING TRANSIENT CAVITATING PIPE FLOW

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This work concerns modeling of time-varying resistance during transient cavitating liquid pipe flow. The wall shear stress is presented in the way introduced by Zielke as the integral convolution of liquid local velocity changes and a weighting function.
A new procedure for determination, so-called, a universal laminar-turbulent weighting function, which combines functions of Zielke and Vardy and Brown or Zielke and Zarzycki, is presented. Based on these weighting functions, the method of simulation transients in the pressure lines in a wide range of Reynolds numbers is presented. It eliminates numerical problems associated with the change of laminar flow into turbulent and vice versa. An example application of this method for simulation of water hammer is presented. The calculation results are compared against experiments presented by Bergant and Simpson, and good agreement is found.

Key words: transient pipe flow, unsteady wall shear stress, weighting function, cavitation

1. Introduction

One of the most common methods used for unsteady pipe flow simulation is the method of characteristics (MOC). To use this method, a model of the wall shear stress (and thus the hydraulic resistance) has to be assumed. The most popular and easiest, in terms of numerical calculations, is the assumption of a quasi steady model. Its determination is based on the formula of Darcy-Weisbach. This assumption, however, is correct only for slow changes in the liquid velocity field in the pipe cross-section, i.e. for low frequencies in the case of pulsating flow, and in the case of waterhammer for the first-wave cycle. This model does not take into account a time-varying velocity gradient in the radial direction, and thus the variable energy dissipation.

The assumption of quasi-steady flow significantly deviates from the reality, especially for fast transients. In this case, it is appropriate to present the instantaneous wall shear stress τ as a sum of the quasi-steady τ_q and unsteady τ_u components, i.e.

$$\tau = \tau_q + \tau_u \tag{1.1}$$

There are two main groups of the unsteady wall shear stress τ_u models. In the first group, the wall shear stress is proportional to the acceleration of the liquid, both the local acceleration (Daily *et al.*, 1956; Cartens and Roller, 1959; Sawfat and Polder, 1973) and convective acceleration (Brunone *et al.*, 1991). Vitkovsky *et al.* (2000) improved the model of Brunone *et al.* by introducing a "$\mathrm{sgn}\,|v|$" to the component of the convective acceleration.

The second group of models of the unsteady wall shear stress τ_u is based on the history of the instantaneous local velocity changes of flow. These models, unlike the first group, better reflect the experiment not only in the degree of attenuation of pressure waves, but also its shape. They are based on the two-dimensional (2D) equation of motion, so they take a time-dependent distribution of the liquid velocity field in cross-section of the pipe.

Zielke (1968) first introduced the unsteady wall shear stress τ_u as a integral convolution of a weighting function and mean local acceleration of the liquid. His model concerned the unsteady laminar flow. Using this model to simulate transients requires a large computer memory. The computation time grows exponentially in this case as the number of time steps increases. Therefore, the model underwent further improvements by Trikha (1975), Kagawa *et al.* (1983), Suzuki *et al.* (1991), Schohl (1993) making it much more efficient.

In the case of unsteady turbulent flow, the models of unsteady shear stress τ_u are based mainly on the distribution of eddy viscosity coefficient in the pipe cross-section which was determined for a steady flow. The authors of these solutions use different numbers of flow layers in which the distribution of eddy viscosity is described. There are two-layer models (Vardy *et al.*, 1991, 1993; Vardy and Brown, 1995, 2003), three-layer (Brown *et al.*, 1969) and four-layer (Ohmi, 1976, Zarzycki 1994, 2000; Zarzyski and Kudźma, 2004).

It is worth to mention that the above cited models are related to smooth pipes, however, Vardy and Brown in 2004 applied the idealized viscosity distribution model for the flow in a rough pipe. Rahl and Barlamond (1996) and then Ghidaoui and Mansour (2002) reported that the use of Vardy and Brown two-layer model to simulate waterhammer gives good consistency with the experiment.

The main problem of this work is focused on simulations of unsteady cavitating pipe flows taking into account the time-dependent hydraulic resistance. The majority of authors in their publications assume that unsteady friction, in case of turbulent flow, is dependent on a fixed Reynolds number (representing the flow before the transient) which influences the shape of the weighting function. A novelty of this work is an attempt to take into account, during the numerical calculation of transient, a variable Reynolds number which constantly modifies the shape of the weighting function.

2. Mathematical models of transient cavitating pipe flow

In order to simulate transient cavitating pipe, flow two common models of cavitation – CSM (column separation model) and BCM (bubbly cavitation model) are used. The first model, CSM, is a discrete model. It assumes that cavitation takes place at a given place of liquid flow and that leads to a disturbance of this flow continuity. In this case, the liquid flow in all flow areas is described by equations [30]:

— continuity equation

$$\frac{\partial p}{\partial t} + \rho_l c^2 \frac{\partial v}{\partial x} = 0 \tag{2.1}$$

— momentum equation

$$\frac{\partial p}{\partial x} + \rho_l \frac{\partial v}{\partial t} + \rho_l g \sin\gamma + \frac{2}{R}\tau = 0 \tag{2.2}$$

where: $v = v(x,t)$ is the mean velocity of the liquid in the pipe cross-section, $p = p(x,t)$ – mean pressure in the pipe cross-section, R – inner radius of the pipe, τ – wall shear stress, ρ_l – liquid density, g – acceleration due to gravity, γ – the angle of inclination of the hydraulic line, c – velocity of pressure wave propagation, t – time, x – distance along pipe.

The next model, BCM, is a continuous, homogeneous cavitation model, where cavitation is assumed to take place along the axis of a pipeline. The fundamental flow equations are given below (Shu, 2003):

— continuity equation

$$\frac{1}{c^2}\frac{\partial p}{\partial t} + (\rho_l - \rho_v)\frac{\partial \alpha}{\partial t} + \rho_m \frac{\partial}{\partial x}\Big(\frac{v}{\alpha}\Big) = 0 \tag{2.3}$$

— momentum equation

$$\rho_m \frac{\partial}{\partial t}\Big(\frac{v}{\alpha}\Big) + \frac{\partial p}{\partial x} + \frac{2}{R}\tau_w + \rho_m g \sin\gamma = 0 \tag{2.4}$$

where: ρ_m is the density of mixture, α – coefficient of liquid phase concentration, ρ_v – vapour density.

The mixture density ρ_m can be expressed

$$\rho_m = \alpha\rho_l + (1-\alpha)\rho_v \tag{2.5}$$

It should be noted that the system of equations (2.1), (2.2) and (2.3), (2.4) is not closed, because it contains three variables: v, p and τ. Thus the main emphasis is placed on the work on modeling the unsteady wall shear stress τ.

Among all methods enabling the solution of equations of this type, particularly noteworthy is the method of characteristics (MOC), which perfectly interprets the essence of the natural phenomenon of unsteady flow, and at the same time, characterized by rapid convergence as well as ease of taking into account different boundary conditions and high accuracy of calculation results.

Method of characteristics enables one to solve partial differential equations of quasi-linear hyperbolic type (2.1), (2.2) and (2.3), (2.4) in a simple way (Shu, 2003; Wylie and Streeter, 1993). The solution is to find the equivalent to the four ordinary differential equations which is then solved using finite differences schemes. An approximation of the finite difference method of the first order gives satisfactory results (Streeter and Lai, 1962).

Zielke in his work (Zielke, 1966, 1968) presented the inverse Laplace transform of a hydraulic line impedance, shown by Brown (Brown, 1962; Brown and Nelson, 1965), and received the following relation describing the instantaneous wall shear stress

$$\tau(t) = \underbrace{\frac{\lambda\rho v|v|}{8}}_{\tau_q} + \underbrace{\frac{2\mu}{R}\int\limits_0^t w(t-u)\frac{\partial v}{\partial t}(u)\,du}_{\tau_u} \tag{2.6}$$

where λ is the coefficient of friction.

The first component τ_q in equation (2.6) represents the quasi-steady wall shear stress while the second τ_u (which is the integral convolution of local fluid acceleration and a weighting function) takes into account the impact of unsteady flow on the wall shear stress. It can be easily seen that for a fixed flow, in which there is no local acceleration of the liquid, $\tau_u = 0$.

Zielke (1968) presented also the proper weighting function for laminar flow. For turbulent flow there are two well known weighting functions: Vardy's and Brown's (Vardy and Brown, 2003, 2004) and Zarzycki's (Zarzycki, 2000; Zarzycki and Kudźma, 2004).

For effective solution of the second component of the wall shear stress τ_u (2.6), the weighting function must be a finite expression of exponential terms (preferably no more than 30 phrases). In our latest article (Urbanowicz and Zarzycki, 2012) was presented the new weighting functions of effective type, which are characterized by an increased range of applicability, and a very good degree of fit to their prototypes, i.e. the classical weighting function according to Zielke for laminar flow (Zielke, 1968) and according to the Vardy and Brown (2003, 2004) (alternatively according to Zarzycki (2000) and Zarzycki and Kudźma (2004)) for turbulent flow.

In this work, a novelty is the inclusion in the model describing unsteady flow in pipes a time-varying hydraulic resistance which is calculated in an effective manner for both laminar and turbulent flow.

3. The universal weighting function

For some time, one can note two types of approaches for modeling unsteady hydraulic resistance:

- The older approaches (Bergant *et al.*, 2006; Shu, 2003; Vitkovsky *et al.*, 2004) assume the shape of weighting function selection before the simulation on the basis of known Reynolds number $\mathrm{Re} = \mathrm{Re}_o$ (constant) – for the steady flow shortly before the onset of the transient. When the number Re_o is less than or equal to the critical value = 2320, the shape of the weighting function is consistent with the shape of the classic weighting function for the laminar flow by Zielke (1968) but when there is a turbulent flow, the shape is determined using a well-known classical turbulent weighting function by Vardy and Brown (2003) (alternatively by Zarzycki (2000)). This approach assumes constancy of the chosen shape for the entire course of the transient simulation.
- In later approaches (Kudźma, 2005; Kudźma and Zarzycki, 2005; Zarzycki and Kudźma, 2004), changes in the shape of a weighting function due to Reynolds number variation are taken into account. There is no assumption of a single weighting function based on initial conditions, but to date, together with any change in the current local Reynolds number Re_c (in the range of turbulent flow) its new shape is determined.

To the best knowledge of authors, there are no papers on the simulation and experimental verification of the transient turbulent cavitating flow in a long liquid line associated with the starting of turbo-machinery with focus on unsteady friction problems. Hence, it can only be considered as authors using the first approach described above could determine a weighting function – probably from the known Reynolds number after reaching a steady state Re_s. Such an approach is rather simplistic to the problem of modeling unsteady hydraulic resistance.

New efficient weighting functions described in detail in our preview article (Urbanowicz and Zarzycki, 2012) are suitable for simulation of transient flows in the case of the first of those approaches.

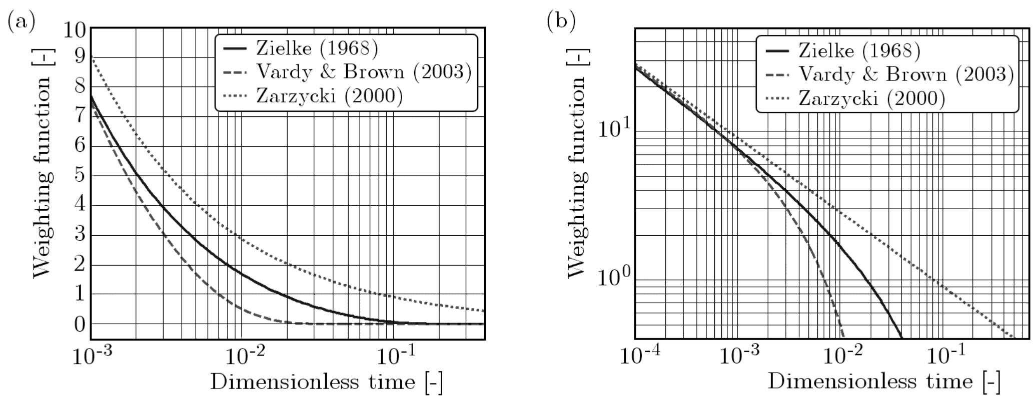

Fig. 1. Courses of weighting functions (determined at $\mathrm{Re}_{cr} = 2320$): (a) log-linear scale, (b) log-log scale

Detailed analysis of the second approach shows that transition from the laminar weighting function to the turbulent one and vice versa raises the problem of numerical nature. This issue is related to inconsistency of the laminar and turbulent weighting functions for a critical number of $\mathrm{Re}_{cr} = 2320$ (see Fig. 1). It is reflected in the variation of pressure and velocity flow simulated runs in the form of abrupt faults. In connection with a different number of components that make up the effective weighting function for laminar flow (26 terms) and the effective weighting function for turbulent flow (16 terms of Vardy-Brown function – alternatively 24 terms of

Zarzycki's function) (Urbanowicz and Zarzycki, 2012) the current numerical procedure for the transition from one type of flow into another assumes parameters from the equation:

$$\tau_u = \frac{2\mu}{R} \sum_{i=1}^{k} \Big(\underbrace{y_i(t)\mathrm{e}^{-n_i \Delta\hat{t}} + m_i \mathrm{e}^{-n_i \frac{\Delta\hat{t}}{2}} [v(t+\Delta t) - v(t)]}_{y_i(t+\Delta t)} \Big) \tag{3.1}$$

which takes into account the history in changes of velocity to be equal to zero $y_i(t_t) = 0$ (where t_t – time of transition). This assumption is valid as an initial condition (for a steady flow), but it certainly is a source of error when there is transition from laminar to turbulent unsteady flow.

To avoid the above mentioned problems by using the second approach, in the present work an effective universal laminar-turbulent weighting function model is proposed. Its shape in the case of laminar flow will coincide with Zielke's function (unchanging when $\mathrm{Re}_c \leqslant 2320$) while for turbulent flow with Vardy's and Brown's turbulent weighting function (alternatively with Zarzycki's turbulent weighting function).

Detailed analysis of the courses of classical weighting functions according to Zielke (1968), Vardy and Brown (2003) and Zarzycki (2000) shows that for the critical Reynolds number ($\mathrm{Re}_{cr} = 2320$) these functions have different shapes (Fig. 1). The function according to Zarzycki in the range of dimensionless time ($10^{-10} < \hat{t} < \infty$) takes values greater than the weighting function according to Zielke. In practice, it means that the simulation results with its usage will be characterized by increased damping. The function according to Vardy and Brown for the range of dimensionless time $7.244 \cdot 10^{-4} < \hat{t} < \infty$) takes instead values smaller than Zielke's function. That means that simulations with its use will be characterized by lower attenuation than when Zielke's or Zarzycki's functions are used. It was also noted that the shape of this function is greatly changed with the increase of Re. The larger the Reynolds number characterizing the transient flow the faster values of this function tend to zero (see Fig. 2).

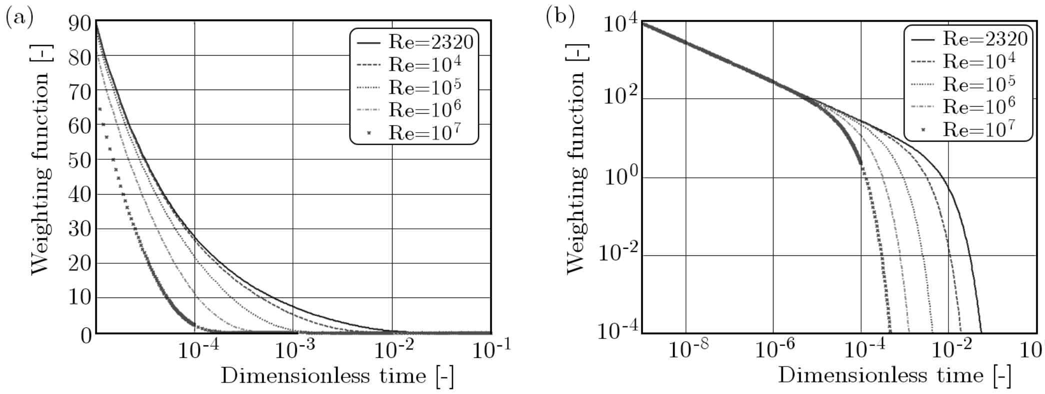

Fig. 2. Runs of the Vardy and Brown weighting function: (a) log-linear scale, (b) log-log scale

Figure 3 presents a schematic procedure for determination of the universal laminar-turbulent weighting function. In the case of laminar flow, the universal weighting function is characterized by a shape consistent with Zielke's weighting function (Zielke, 1968). As soon as there is a turbulent flow, the coefficients representing the effective function must be appropriately scaled. This process is modeled with the use of the procedure presented by Vitkovsky *et al.* (2004).

Assuming that the shape of the universal weighting function in the case of turbulent flow should be convergent to the original model of Vardy and Brown (2003), the process of finding the current coefficients should be carried out according to the scheme shown in Fig. 3 (an alternative procedure is presented in Appendix – there the function for the case of turbulent flow are convergent to the original model of Zarzycki (2000)).

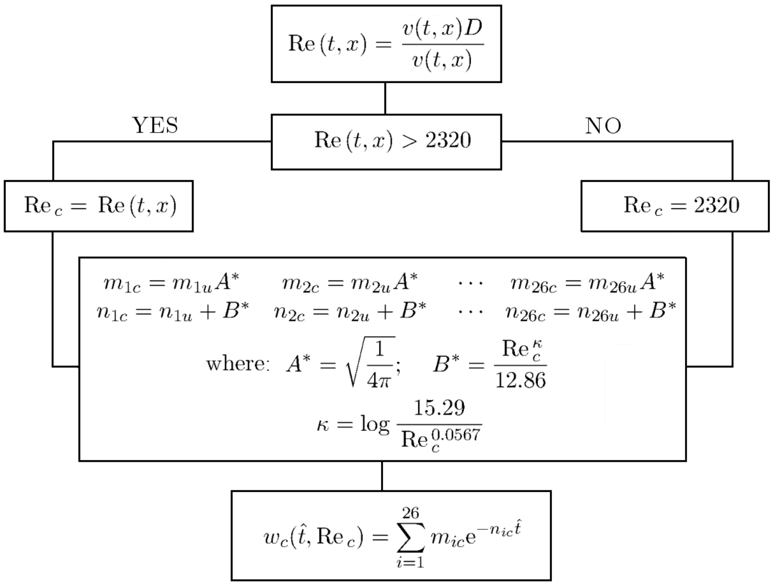

Fig. 3. Schematic determination of the current form of the universal weighting function w_c in the numerical process. It retains the shape consistent with the classical weighting function according to Vardy and Brown (2003)

Universal coefficients, needed for determination of the current form of the universal weighting function are calculated (for $\mathrm{Re}_{cr} = 2320$) before the start of the simulation using following formulas

$$
\begin{array}{llll}
m_{1u} = \dfrac{m_1}{A^*} & m_{2u} = \dfrac{m_2}{A^*} & \cdots & m_{26u} = \dfrac{m_{26}}{A^*} \\
n_{1u} = n_1 - B^* & n_{2u} = n_2 - B^* & \cdots & n_{26u} = n_{26} - B^*
\end{array}
\tag{3.2}
$$

where

$$
A^* = \sqrt{\frac{1}{4\pi}} \qquad B^* = \frac{\mathrm{Re}^{\kappa}}{12.86} = \frac{2320^{\kappa}}{12.86} \qquad \kappa = \log\frac{15.29}{\mathrm{Re}^{0.0567}} = \log\frac{15.29}{2320^{0.0567}}
$$

and m_i, n_i are the coefficients of the laminar effective weighting function presented in our previous paper (Urbanowicz and Zarzycki, 2012).

Degrees of match of the universal weighting function with the classical counterparts are shown by the graphs presented in Fig. 4.

Qualitative analysis of the new universal function matching shows that it is difficult to note any significant deviations, for Reynolds numbers $\mathrm{Re} \geqslant 10^4$ (Figs. 4b-4e), from its classical counterpart. The quantitative analysis for the dimensionless time range $10^{-9} \leqslant \hat{t} \leqslant 10^{-3}$, and Reynolds numbers $2320 \leqslant \mathrm{Re} \leqslant 10^7$ shows that the absolute percentage error does not exceed 11% (for the case presented in Appendix, the percentage absolute error in the same ranges did not exceed 14%).

The relative percentage error, as shown in Fig. 4f, is large but only for values of dimensionless time $\hat{t} > 10^{-3}$. The values of weighting function for the dimensionless time $\hat{t} > 10^{-3}$ are very small – as shown in Fig. 5. This means that if the dimensionless time step is significantly small during numerical calculations, these errors will not affect significantly the result of simulation of the wall shear stress.

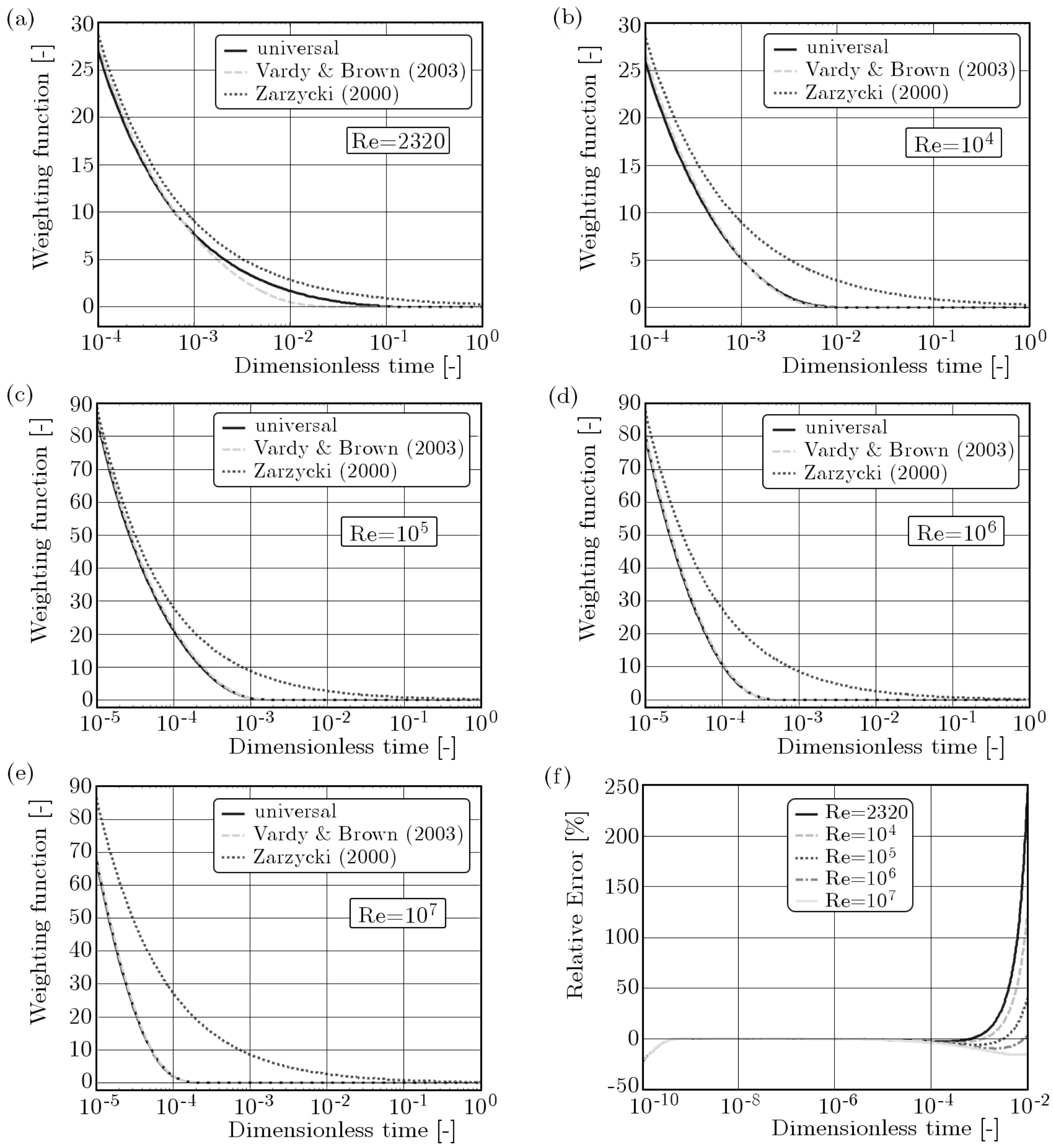

Fig. 4. Comparison of the universal weighting function (maintaining the shape consistent with the model of Vardy and Brown) with the classical models

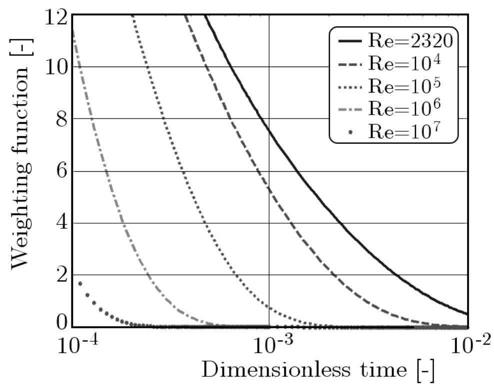

Fig. 5. The values of turbulent weighting function for the dimensionless time 10^{-3}

4. Numerical example

In order to compare the accuracy of unsteady (with the use of a universal weighting function) and quasi-steady models of friction in relation to experimental data, simulations of a simple waterhammer case (tank – long liquid line and cut-off valve) were conducted.

The computed results (using transient cavitating pipe flow models – CSM (Wylie and Streeter, 1993) and BCM (Shu, 2003)) were compared with the experimental data reported by Bergant and Simpson (1996, 1999).

Bergant and Simpson conducted an experiment on a test rig installed at the University of Adelaide (Fig. 6). The rig comprises of copper pipe with internal diameter $D = 0.0221$ m and length $L = 37.2$ m connecting two pressurized tanks. The liquid used in the experiment was water having kinematic viscosity $\nu = 1 \cdot 10^{-6}$ m^2/s. The measured sound speed was $c = 1319$ m/s and the initial flow velocity $v_0 = 1.40$ m/s (Re = 30940) and $v_0 = 1.50$ m/s (Re = 33150).

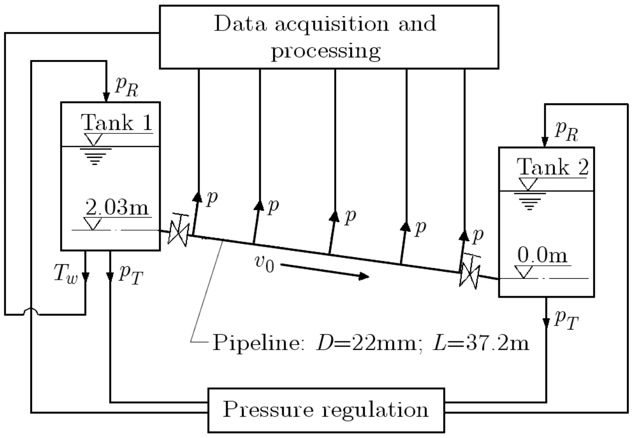

Fig. 6. Test rig

The downstream valve was rapidly closed in the pipe line during flow. Pressure fluctuation was measured at the endpoint of the line (near the valve). From the above parameters, it followed that it was a case of turbulent flow. The results of simulations compared to experimental data are shown in Figs. 7 and 8.

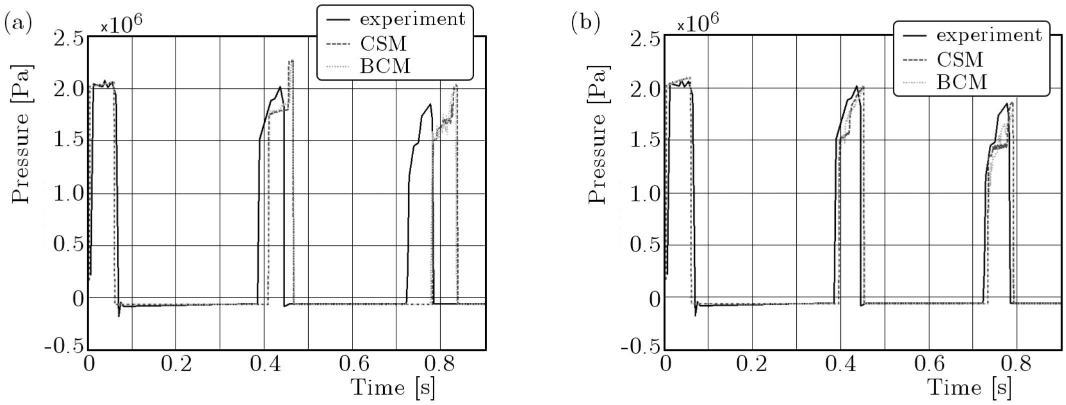

Fig. 7. Pressure variations at the valve: $v_0 = 1.4$ m/s (Re = 30940), $p_t = 2.158 \cdot 10^5$ Pa (a) quasi-steady friction model, (b) unsteady friction model

From the above graphs showing the runs of pressure changes (at the valve), it is clear that the use of unsteady hydraulic resistance in simulation brings the results much closer to the experimental observations. In the case of cavitating flow, the model of friction used affects

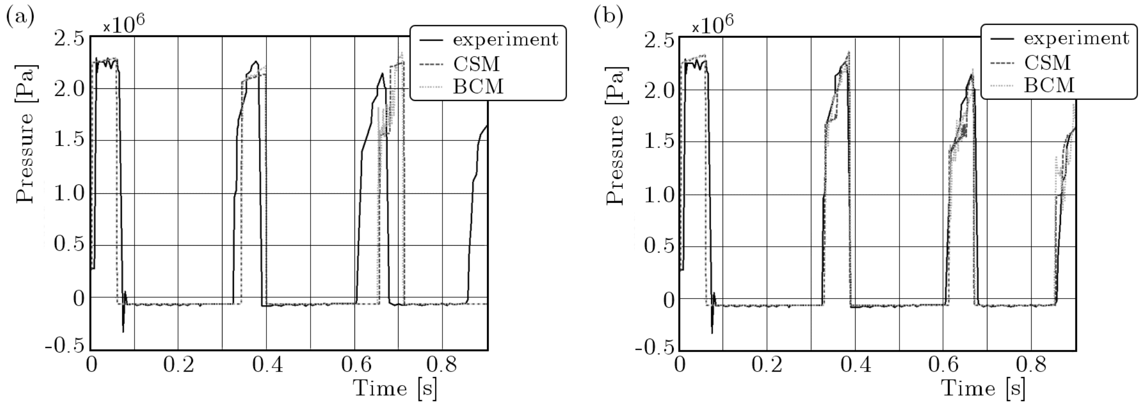

Fig. 8. Pressure variations at the valve: $v_0 = 1.5\,\mathrm{m/s}$ (Re = 33150), $p_t = 3.139 \cdot 10^5\,\mathrm{Pa}$ (a) quasi-steady friction model, (b) unsteady friction model

not only the maximum pressure values for consecutive amplitudes of pressure, but also the duration of the cavitation. Therefore, the role of the friction model is essential for the modeling of transients in the pressure lines with cavitation.

4.1. Quantitative analysis

A quantitative analysis of transient flow, especially valuation of maximum pressures values and times of its occurrences, is very important from the point of view of automatic control engineering.

In many available articles concerning transient flow with cavitation, there are no shown mathematical quantitative methods useful for comparison of simulated runs in relation to the experimental ones. All authors concentrated only on qualitative estimation.

The quantitative analysis in this research work is estimated by two parameters (p_p and t_p) characterizing the degree of matching with the experimental course.

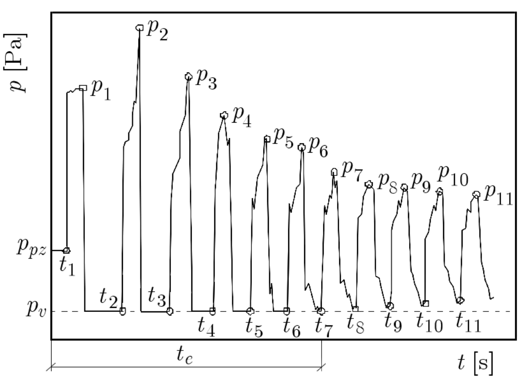

Fig. 9. An example of pressure course during transient pipe flow with cavitation

Parameter p_p is determined on the basis of maximum pressures (Fig. 9 – p_1 to p_n) of successive amplitudes of the analyzed course in the following way

$$p_p = \sum_{i=1}^{n} \frac{|p_{pi}|}{n} \tag{4.1}$$

where

$$p_{pi} = \frac{p_{is} - p_{ie}}{p_{ie}} \cdot 100\% \tag{4.2}$$

and p_{ie} is the value of maximum pressure on the analyzed i-th amplitude, based on the analysis of experimental results, p_{is} – maximum pressure on the analyzed i-th amplitude, based on the simulated results.

Similar analysis can be carried out for the time of occurrence of the pressure amplitudes (their maxima) (Fig. 9 – t_1 to t_n)

$$t_p = \sum_{i=1}^{n} \frac{|t_{pi}|}{n} \tag{4.3}$$

where

$$t_{pi} = \frac{t_{is} - t_{ie}}{t_{ie}} \cdot 100\% \tag{4.4}$$

and t_{ie} is the time of occurrence of the i-th amplitude of pressure, based on the analysis of experimental results, t_{is} – time of occurrence of the i-th amplitude of pressure, based on the analysis of simulated results.

When the values of p_p and t_p get smaller, then the discrepancy between the simulated and experimental results is smaller.

The results of quantitative analyses of the pressure runs, presented in Section 4, are shown in Tables 1 and 2.

Table 1. Quantitative analysis – time-dependent hydraulic resistance

Parameter	CSM model	BCM model
p_p [%] (Run I – Re = 30940)	0.7	1.2
p_p [%] (Run II – Re = 33150)	2.3	3.1
t_p [%] (Run I – Re = 30940)	2.7	2.7
t_p [%] (Run II – Re = 33150)	2.8	2.8

Table 2. Quantitative analysis – quasi-steady hydraulic resistance

Parameter	CSM model	BCM model
p_p [%] (Run I – Re = 30940)	7.5	8.0
p_p [%] (Run II – Re = 33150)	3.7	3.8
t_p [%] (Run I – Re = 30940)	7.0	6.4
t_p [%] (Run II – Re = 33150)	8.5	8.0

The quantitative analysis shows that the results of simulations with usage of the CSM model (with taking into account also time-dependent hydraulic losses) have the best aggrement in simulating maximum pressures. The presented analysis confirms also the conclusion from qualitative analysis (Figs. 7 and 8) that the results originated from models in which the time-dependent hydraulic resistance were included match definitely better with the experiment than models using of quasi-steady ones (see percentage results in Tables 1 and 2).

5. Summary

The presented method enables the determination of variable hydraulic resistance for laminar and turbulent flow. A new function includes a range of laminar and turbulent flow. The coefficients of the new function are updated every step of the calculation along with the change of Re.

Using the proposed approach eliminates numerical problems associated with the transition from laminar flow into turbulent one and vice versa. In view of the fact that in literature there are two weighting functions for the turbulent flow, two approaches are presented at the time of occurrence of turbulent flow.

From the numerical examples showing the runs of pressure changes (at the valve), it is clear that the use of time-dependent hydraulic resistance in simulation brings the results much closer to the experimental observations. Important is also the fact that in the case of cavitating flow, the used model of friction affects not only the maximum pressure values for consecutive amplitudes of pressure but also the duration of cavitation. Therefore, the role of the friction model is essential for modeling transients in the pressure lines with cavitation.

A. Appendix

Assuming that the shape of the universal weighting function remained convergent to the model by Zarzycki in the case of turbulent flow, the process of searching for current coefficients of the function can be done with the use of the scheme shown in Fig. 10. Universal coefficients, needed further for determination of the current form of the universal weighting function, are computed before starting the simulation process from the following relations

$$\begin{array}{llll} m_{1u} = Dm_1 & m_{2u} = Dm_2 & \cdots & m_{26u} = Dm_{26} \\ n_{1u} = n_1 & n_{2u} = n_2 & \cdots & n_{26u} = n_{26} \end{array} \tag{A.1}$$

where

$$D = \frac{\mathrm{Re}^{0.005535}}{0.299635} = \frac{2320^{0.005535}}{0.299635}$$

and m_i, n_i are the coefficients of the laminar effective weighting function presented in our previous paper Urbanowicz and Zarzycki (2012).

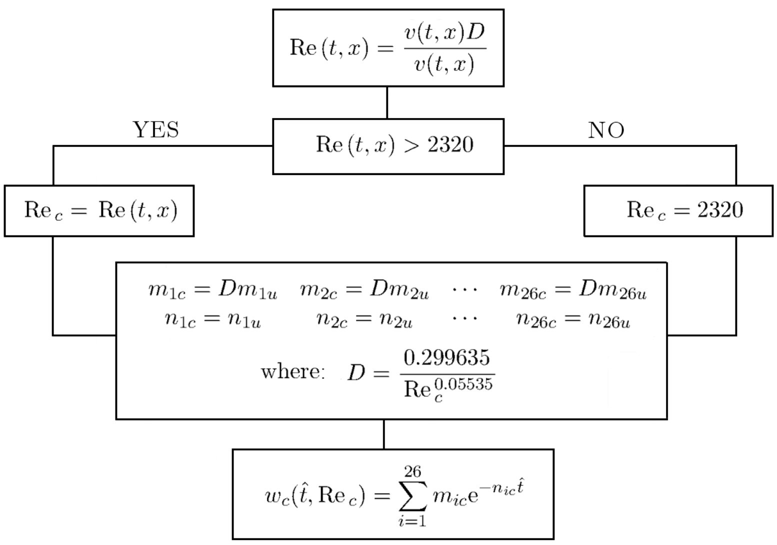

Fig. 10. Schematic determination of the current form of the universal weighting function w_c in the numerical process. It retains the shape consistent with the classical weighting function according to Zarzycki for turbulent flow (Zarzycki, 2000)

Degrees of match of the universal weighting function with the classical counterparts one shown by the graphs presented in Fig. 11.

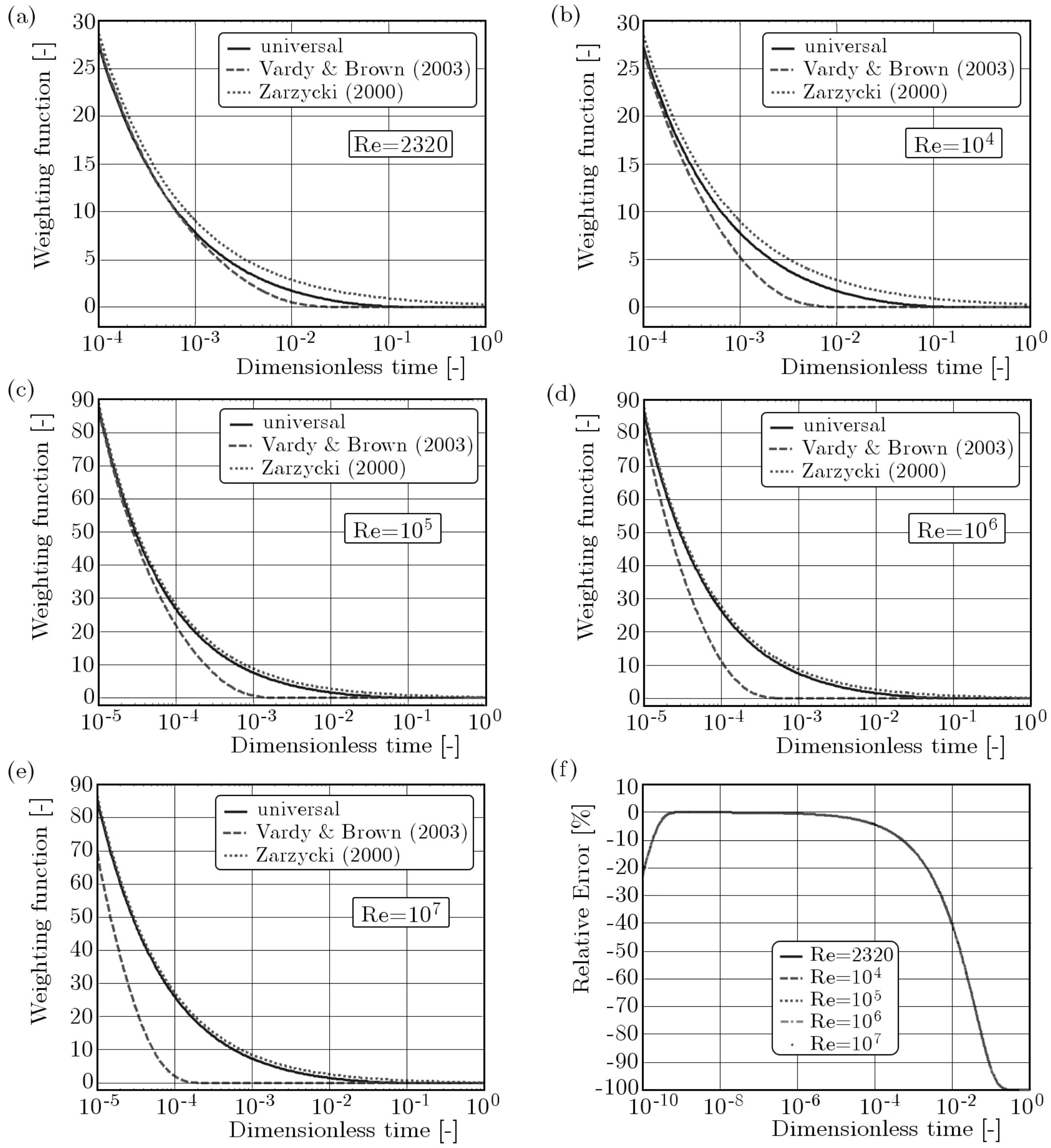

Fig. 11. Comparison of the universal weighting function (maintaining the shape consistent with the model of Zarzycki) with the classical models

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Uniwersalna funkcja wagi w modelowaniu niestacjonarnych przepływów z kawitacją występujących w przewodach ciśnieniowych

Streszczenie

Praca dotyczy modelowania zmiennych w czasie oporów podczas nieustalonego przepływu cieczy w przewodach ciśnieniowych. Naprężenie styczne na ściance przewodu przedstawione zostało w klasyczny sposób (wg. Zielke) jako całka splotowa z przyśpieszenia cieczy i pewnej funkcji wagowej.

Przedstawiono procedury wyznaczania dwóch uniwersalnych funkcji wagowych laminarno-turbulentnych (wg. Zielke i Vardy-Browna oraz wg. Zielke i Zarzyckiego). W oparciu o te funkcje przedstawiono metodę modelowania (symulacji) przebiegów przejściowych w przewodach ciśnieniowych w szerokim zakresie liczb Reynoldsa. Eliminuje on problemy natury numerycznej związane z przechodzeniem min. z przepływu laminarnego w turbulentny i odwrotnie. Podano przykład zastosowania opracowanej metody do symulacji uderzenia hydraulicznego. Wyniki obliczeń porównano z wynikami eksperymentalnymi wg. Berganta i Simpsona, otrzymując dobrą zgodność.

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