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# NONLINEAR VIBRATION ANALYSIS OF A FIXED-FIXED BEAM UNDER OSCILLATING AXIAL LOAD AND VIBRATING MAGNETIC FIELD

Tai-Ping Chang

*National Kaohsiung First University of Science and Technology, Department of Construction Engineering, Kaohsiung, Taiwan, ROC; e-mail: tpchang@ccms.nkfust.edu.tw*

The nonlinear vibration behavior of a fixed-fixed beam under oscillating axial load and vibrating magnetic field is investigated in the present study. The transverse magnetic force, transverse magnetic couple, axial force, uniform translation spring force, transverse surface force and the damper are considered in the system. Hamilton's principle is adopted to derive the equation of motion of the beam system under certain hypotheses, and then Galerkin's method is utilized to obtain the solution of the system. It can be concluded from the present study that under stable situations, the more the transverse magnetic field increases, the more the displacement and natural frequency of the beam system decrease.

*Key words:* nonlinear vibration, magnetic force, axial force, Hamilton's principle, Galerkin's method

## 1. Introduction

The development of electrical machinery, communications equipment and computers, which involve magnetic phenomena, play an important role in everyday life. In this study, the interactions among transverse magnetic fields, compressive axial forces, springs forces, and external transverse forces of a beam are investigated; the basic relation of magnetism adopted in this paper is given in Reitz *et al.* (1992). Shin *et al.* (1998) studied transient vibrations of a simply supported beam with axial loads and transverse magnetic fields. Moon and Pao (1969) presented vibration and parametric instability of a cantilever beam-plate in a transverse magnetic field, and provided theoretical and experimental results. Ambartsumian (1982) wrote one of the reviews on magneto-elasticity.

The buckling of a thin plate simply supported around its edges was investigated by Moon (1970), and magneto-elastic buckling of beams and plates was investigated by Van de Ven (1978), Wallerstein and Peach (1972). Moon and Pao (1968) studied the buckling of a cantilever beam-plate in a transverse magnetic field. A mathematical model was proposed with distributed magnetic forces and torque in their study. The critical value of buckling field of the elliptical plate was determined by Dalrymple *et al.* (1974). Dalrymple *et al.* (1977) also extended the Moon-Pao theoretical treatment to investigate the edge effect. Miya *et al.* (1978) applied experimental and finite element methods to study the magneto-elastic buckling of a cantilevered beam-plate. In the Moon-Pao theoretical treatment, the magnetic torque without axial load was considered. Kojima and Nagana (1985) investigated the parameter nonlinear forced vibrations of a beam with a tip mass subjected to alternating electromagnetic forces acting on the tip mass. Yang *et al.* (1999) adopted the energy method to investigate the effects of a magnetic field on vibration and instability of ferromagnetic thin plates. Zhou *et al.* (2000) investigated the buckling and post-buckling behavior of a soft ferromagnetic beam-plate with unmovable simple supports under an applied magnetic field. Chen and Yeh (2001) studied the parametric instability of a beam under an electromagnetic excitation experimentally and analytically. In addition, Zhou *et al.* (2003) performed the buckling and post-buckling analysis for magneto-elastic-plastic ferromagnetic beam-plates with geometrically nonlinear deformation. Zheng and Zeng (2004) investigated the influence of permanent magnets on vibration characteristics of a partially covered sandwich cantilever beam. Wu (2005) studied the dynamic instability and transient vibrations of a pinned beam with transverse magnetic fields and thermal loads. Pratiher *et al.* (2007) investigated the parametric instability of a cantilever beam with a magnetic field and a periodic axial load. More recently, the dynamic instability of a pinned beam subjected to an alternating magnetic field and a thermal load with the nonlinear strain, and made of physically nonlinear thermoelastic material was studied by Wu (2007). Wei *et al.* (2007) estimated the vibration frequency of a ferromagnetic beam subjected to an inclined magnetic field by employing formulations of the magnetic force. Yoo *et al.* (2009) proposed a dynamic modeling method to analyze an axially oscillating beam undergoing a periodic impulsive force.

A beam system involving compressive axial load, transverse magnetic field, transverse force, springs force and damping effect is considered in this paper. The equation of motion is derived by Hamilton's principle. The axial force and the transverse magnetic force are assumed to be periodic functions of time, meanwhile two frequencies associated with the axial force and oscillation transverse magnetic field are applied to the system. The relationship of the displacement versus time with the effect of magnetic force and axial force are determined from the modal equation by using the Runge-Kutta method.

## 2. Equation of motion

### 2.1. Statement of problem

In this paper, the beam system as shown in Fig. 1 is investigated. The beam of a linearly elastic material with width $d$, thickness $h$ and length $L$ is subjected to an oscillating compressive axial force $\boldsymbol{P} = [P_0 + P_1 \cos(\Omega_2 t)]\boldsymbol{i}$ in the $x$-direction and a transverse alternating magnetic field $\boldsymbol{B}_0 = [B_m \cos(\Omega_1 t)]\boldsymbol{j}$, transverse force, damper in the $y$-direction, and attached by the linear springs with the spring constant $k$.

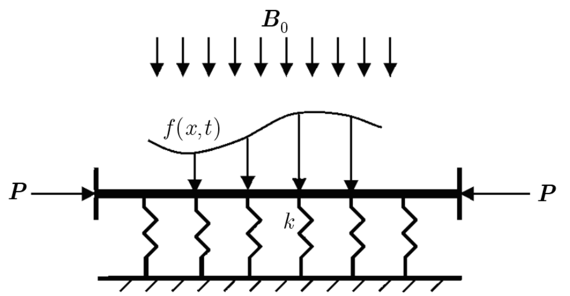


Fig. 1. The model of a beam with an axial load, spring constant and magnetic force

The following physical assumptions are made to simplify the analysis of the beam mentioned above:

(1) The length is much larger than the thickness $(L \geqslant h)$

(2) The cross section of the beam is symmetric and the thickness is much larger than the width $(h \geqslant d)$

(3) Cross sections of the beam remain to be plane and normal to longitudinal lines within the beam

(4) The material obeys Hooke's law

(5) The deflection of the beam is small

(6) The effects of shear and inertia are neglected.

## 2.2. Mathematical modeling

Hamilton's principle (Langhaar, 1962) is adopted to derive the equation of motion (Shin *et al.*, 1998) of the beam as follows

$$m\frac{\partial^2 y}{\partial t^2}+Cd\frac{\partial y}{\partial t}+EI\frac{\partial^4 y}{\partial x^4}+ky+P\frac{\partial^2 y}{\partial x^2} = f(x,t)+\frac{\partial c}{\partial x}+\frac{\partial}{\partial x}\Big[\Big(\int\limits_0^x p\,d\xi\Big)\frac{\partial y}{\partial x}\Big] \tag{2.1}$$

where $y(x,t)$ is the displacement function, $m$ is the mass per unit length, $C_d$ is the damping coefficient, $E$ is Young's modulus, $I$ is the moment of inertia of the cross section, $p$ is the body force of the beam per unit length in the $x$-direction, $k$ is the constant of spring and $f(x,t)$ is the distributed force in the $y$-direction. As shown in Fig. 2, $\boldsymbol{c}$ is the induced couple per unit length due to the magnetic field, and can be calculated as follows

$$\boldsymbol{c} = \int(\boldsymbol{M}\times\boldsymbol{B}_0)\,dV \tag{2.2}$$

where $\boldsymbol{M} = (\chi_m/\mu_0)|\boldsymbol{B}_0|\boldsymbol{n}$, and $\boldsymbol{M}$ is called the magnetic dipole moment per unit volume, or simply the magnetization, $\boldsymbol{B}_0$ is called the induced magnetic field, $\mu_0$ is the permeability of the vacuum, $\mu_0 = 1.2566\cdot 10^{-6}\,\mathrm{Ns^2/c^2} =$ const and $\chi_m$ is the magnetic susceptibility.

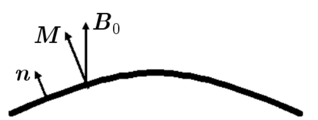


Fig. 2. The relationship between $\boldsymbol{M}$ and $\boldsymbol{B}_0$ in magnetic field

## 3. Analytical procedure

The boundary conditions of the beam are considered to be fixed-fixed, then the displacement function can be written as

$$y(x,t) = \sum_{n=1}^{\infty} q_n(t)[1-\cos(\lambda_n x)] \qquad 0 \leqslant x \leqslant L \tag{3.1}$$

where $\lambda_n = 2n\pi/L$.

Since the middle plane of the beam remains unstrained during bending, the length of the beam is supposed to be constant $\ell$, therefore

$$\int_0^x \sqrt{1+y'^2(\xi,t)}\,d\xi = \ell \tag{3.2}$$

Differentiating Eq. (3.2) with respect to $t$ yields

$$\int_0^x \frac{y'\dot{y}'}{\sqrt{1+y'^2}}\,d\xi + \sqrt{1+y'}\,\dot{x} = 0 \tag{3.3}$$

According to Eq. (3.3) for the small deflection theory, the velocity in the $x$-direction can be simplified to

$$\dot{x} = \frac{dx}{dt} \approx -\int_0^x y'\dot{y}'\,d\xi = \sum_{n=1}^{\infty} -\frac{1}{2}q_n\dot{q}_n\lambda_n^2\Big(x - \frac{1}{2\lambda_n}\sin(2\lambda_n x)\Big) \tag{3.4}$$

Also, the component of body force $\boldsymbol{p}$ contributed by the magnetic force $\boldsymbol{B}_0$ is shown as follows

$$\begin{aligned} p &= \Big(\int \sigma(\dot{\boldsymbol{r}}\times\boldsymbol{B}_0)\times\boldsymbol{B}_0\,dV\Big)_x \\ &= \sum_{n=1}^{\infty}\frac{\sigma}{2}\lambda_n^2 hdB_m^2\Big(x-\frac{1}{2\lambda_n}\sin(2\lambda_n x)\Big)\Big(\frac{1+\cos(2\Omega_2)}{2}t\Big)q_n\dot{q}_n \end{aligned} \tag{3.5}$$

Meanwhile, the component of couple $\boldsymbol{c}$ per unit length induced by the magnetic force can be written as follows

$$c = \Big(\int \boldsymbol{M}\times\boldsymbol{B}_0\,dV\Big)_x = \sum_{n=1}^{\infty}\lambda_n bhd\sin(\lambda_n x)\frac{1+\cos(2\Omega_1 t)}{2}q_n \tag{3.6}$$

where $b = (\chi_m/\mu_0)B_m^2$

## 4. Galerkin's method

### 4.1. Free vibration

Substituting Eqs. (3.4)-(3.6) into Eq. (2.1) leads to a nonlinear operator residual $R(q)$, and the residual function can be written as follows

$$
\begin{aligned}
R(q,t) &= \sum_{n=1}^{\infty}\Big\{m\ddot{q}_n[1-\cos(\lambda_n x)] + C_d\dot{q}_n[1-\cos(\lambda_n x)] - EI\lambda_n^4 q_n\cos(\lambda_n x)\\
&\quad + kq_n[1-\cos(\lambda_n x)] + [p_0 + p_1\cos(\Omega_2 t)]\lambda_n^2 q_n\cos(\lambda_n x)\\
&\quad - \frac{1}{2}\lambda_n^2 bhd\cos(\lambda_n x)[1+\cos(2\Omega_1 t)]q_n\\
&\quad - \frac{1}{2}\sigma\lambda_n^3 hdB_m^2[1+\cos(2\Omega_1 t)]q_n^2\dot{q}_n\Big(x - \frac{1}{2\lambda_n}\sin(2\lambda_n x)\Big)\sin(\lambda_n x)\\
&\quad + \Big(\frac{1}{2}x^2 + \frac{1}{4\lambda_n^2}\cos(2\lambda_n x) - \frac{1}{4\lambda_n^2}\Big)\lambda_n\cos(\lambda_n x)\Big]\Big\}
\end{aligned}
\tag{4.1}
$$

Assuming an approximate function $\phi_n(x) = 1-\cos(\lambda_n x)$ as the weighting function in the Galerkin Method (Goudjo and Maugin, 1985), it requires

$$
\int_0^L R(q,t)\phi_n(x)\,dx = 0 \tag{4.2}
$$

A time-dependent differential equation can be derived based on Eq. (4.2), the equation for the first mode $(n=1)$ is solved and presented in Eq. (4.3)

$$
\int_0^L R(q_1,\dot{q}_1,\ddot{q}_1,t)\phi_1(x)\,dx = 0 \tag{4.3}
$$

By carrying out the required mathematical operations in Eq. (4.3), we obtain a second-order nonlinear differential equation that can be presented as follows

$$
\frac{3}{2}m\ddot{q}_1 + \frac{3}{2}C_d\dot{q}_1 + \frac{1}{2}EI\lambda_1^4 q_1 + \frac{3}{2}kq_1 - \frac{1}{2}P\lambda_1^2 q_1 + Cq_1^2\dot{q}_1 + Dq_1 = 0 \tag{4.4}
$$

where

$$
\begin{aligned}
P &= p_0 + p_1\cos(\Omega_2 t)\\
C &= -\frac{1}{2}\sigma\lambda_1^3 hdB_m^2[1+\cos(2\Omega_1 t)]\frac{-1}{48}\frac{-15+4L^2\lambda_1^2}{\lambda_1}\\
D &= \frac{1}{4}\lambda_1^2 bhd[1+\cos(2\Omega_1 t)]
\end{aligned}
$$

### 4.2. Forced vibration

Assuming $f(x,t) = \sum_{n=1}^{\infty}\sin t[1-\cos(\lambda_n x)]$ in Eq. (2.1) is an external force which acts on the beam system when $t < 0.1\,\mathrm{s}$, and once again substituting Eqs. (3.4)-(3.6) into Eq. (2.1) leads to a nonlinear operator residual $R(q,t)$.

Eventually, the following nonlinear differential equation can be obtained as follows

$$\frac{3}{2}m\ddot{q}_1+\frac{3}{2}C_d\dot{q}_1+\frac{1}{2}EI\lambda_1^4q_1+\frac{3}{2}kq_1-\frac{1}{2}P\lambda_1^2q_1+Cq_1^2\dot{q}_1+Dq_1=\frac{3}{2}\sin t \qquad (4.5)$$

For the transient vibration, the displacement at the transient time can be determined; also it is necessary to assume a set of initial conditions for this model while the transverse force is set to be zero. The fourth order Runge-Kutta method is applied to solve Eq. (4.4) and Eq. (4.5), and the time step size here is chosen as 0.001 s.

## 5. Numerical results and discussions

A low-carbon steel is considered in this study, and the material parameters adopted here are as follows

$$E=194\,\mathrm{GPa} \qquad m=0.03965\,\frac{\mathrm{kg}}{\mathrm{m}} \qquad L=2\,\mathrm{m}$$

$$h=0.005\,\mathrm{m} \qquad d=0.001\,\mathrm{m} \qquad \mu_r=1.0001$$

$$\mu_0=1.26\cdot 10^{-6}\,\frac{\mathrm{H}}{\mathrm{m}} \qquad \sigma=10^7\,\frac{\mathrm{V}}{\mathrm{m}} \qquad \varOmega_1=\varOmega_2=15\,\frac{\mathrm{rad}}{\mathrm{s}}$$

$$k=1.0\,\frac{\mathrm{N}}{\mathrm{m}}$$

### 5.1. Free vibration analysis

It should be noted that only the first mode is considered in the numerical computations for simplicity. Initial conditions $q_1(0)=0.001\,\mathrm{m}$ and $\dot{q}_1(0)=0$ are chosen for all free vibration discussions in this study, and the displacement at the middle point of the beam is presented in Fig. 3. In order to investigate the effect of magnetic field, we choose $B_m$ value that is much greater than the practical value. With the magnetic field only, the relationships between the displacement and time are shown in Figs. 3a,b. Under a stable situation, the more the magnetic field increases, the more the displacement decays, and this phenomenon can be explained in Eq. (4.4) involving an expression $(Cq_1^2\dot{q}_1)$ that is similar to the damping effect. The result with the axial force and damping coefficient only is shown in Fig. 3c, it is well known that the natural frequency without an axial force system is higher than that with it. Under a stable situation, the more the axial force increases, the more the natural frequency of vibration decreases. With the magnetic field only, that varies from zero to ten T (Tesla), the result is shown in Fig. 3d at time $t = 1\,$s. As it can be seen from Fig. 3d, the displacement at the middle point of the beam decreases as the intensity of the magnetic field increases.

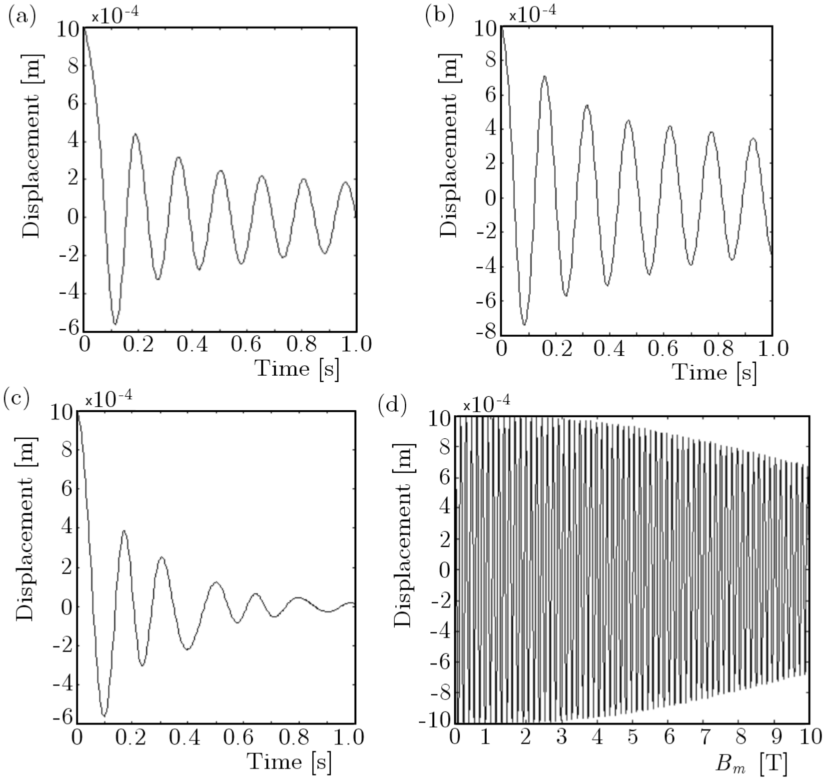


Fig. 3. Displacement at the middle point of the beam for the free vibration system; (a) $B_m = 100\,$T, $P_0 = P_1 = C_d = 0$, $k = 1\,$N/m, (b) $B_m = 50\,$T, $P_0 = P_1 = C_d = 0$, $k = 1\,$N/m, (c) $B_m = 0$, $P_0 = 1\,$N, $P_1 = 10\,$N, $C_d = 0.3$, $k = 1\,$N/m, (d) $P_0 = 1\,$N, $P_1 = 10\,$N, $C_d = 0.3$, $k = 1\,$N/m

### 5.2. Forced vibration analysis

Assume $f(x,t) = \sum_{n=1}^{\infty} \sin t[1 - \cos(\lambda_n x)]$ in Eq. (2.1) is an external force which acts on the beam system, where $t < 0.1\,$s, as mentioned before. The relationships between the displacement and time with the magnetic field only are shown in Fig. 4. Under a stable situation, the more the magnetic field increases, the more the displacement decays, and this phenomenon can be explained in Eq. (4.5) involving the expression $(Cq_1^2\dot{q}_1)$ that is similar to the damping effect. The result with the axial force and damping coefficient only is shown in Fig. 5a. The compressive axial force decreases the stiffness of the beam system, so that under a stable situation, the more the compressive axial force increases, the more the natural frequency of vibration decreases, which is quite reasonable. With the presence of the magnetic field, the axial force and damping coefficient, displacement at the middle point of the beam is presented in Fig. 5b. As it can be found from Fig. 5b, the displacement of the beam is smaller than that of the system without the magnetic field as depicted in Fig. 5a. Obviously, the presence of magnetic field increases the damping effect, which is very similar to the phenomena detected in the free vibration analysis.

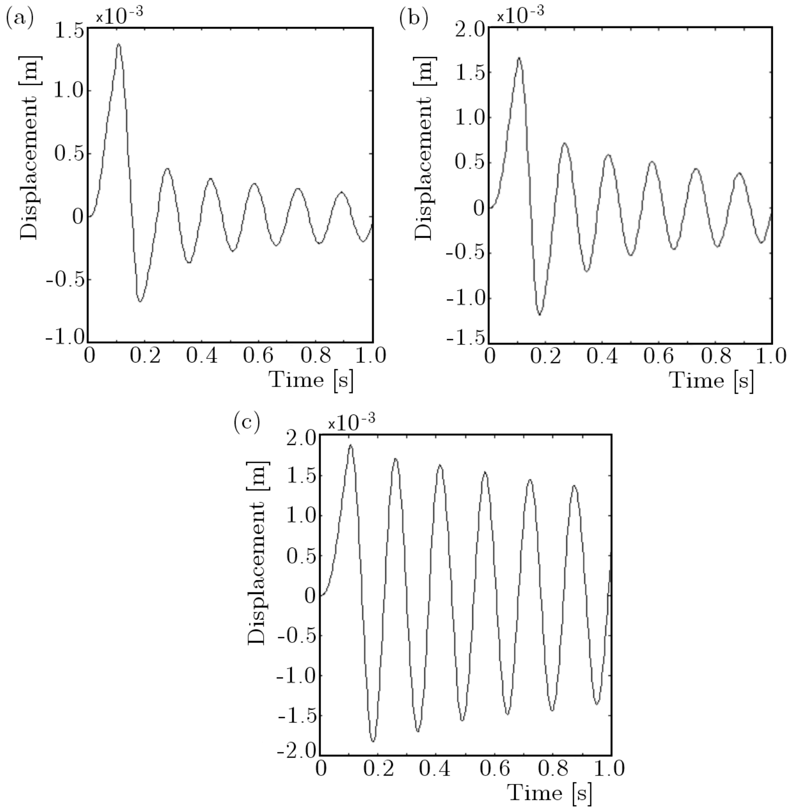


Fig. 4. Displacement at the middle point of the beam for the forced vibration system; (a) $B_m = 100\,\mathrm{T}$, $P_0 = P_1 = C_d = 0$, $k = 1\,\mathrm{N/m}$, (b) $B_m = 50\,\mathrm{T}$, $P_0 = P_1 = C_d = 0$, $k = 1\,\mathrm{N/m}$, (c) $B_m = 10\,\mathrm{T}$, $P_0 = P_1 = C_d = 0$, $k = 1\,\mathrm{N/m}$

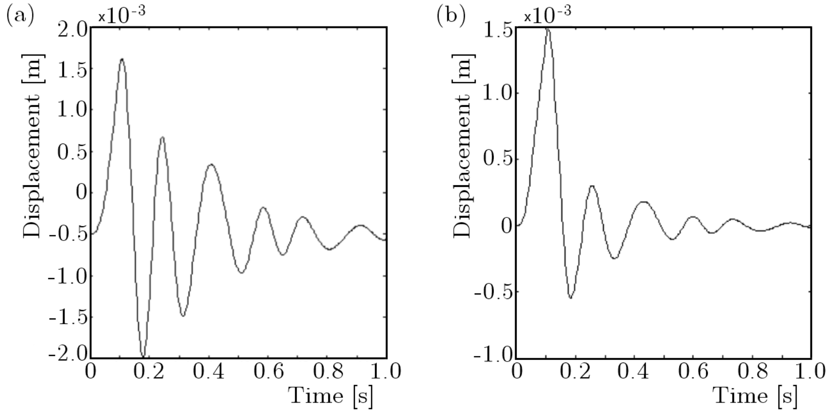


Fig. 5. Displacement at the middle point of the beam for the forced vibration system; (a) $B_m = 0$, $P_0 = 1\,\mathrm{N}$, $P_1 = 10\,\mathrm{N}$, $C_d = 0.3$, $k = 1\,\mathrm{N/m}$, (b) $B_m = 100\,\mathrm{T}$, $P_0 = 1\,\mathrm{N}$, $P_1 = 10\,\mathrm{N}$, $C_d = 0.3$, $k = 1\,\mathrm{N/m}$

Finally, it can be seen from the figures that the displacement of the beam decays while damping is considered, which is reasonable, the compressive axial force decreases the stiffness of the beam, also decreases the natural frequency of the vibration of the beam. It should be concluded that the effect of the magnetic field is similar to the damping effect.

## 6. Conclusions

The nonlinear vibration behavior of a fixed-fixed beam is investigated in the present study. The transverse magnetic force, transverse magnetic couple, axial force, uniform translation spring force, transverse surface force and the damper are considered in the system. Hamilton's principle is adopted to derive the equation of motion of the beam system under certain hypotheses. Based on the assumption of an inextensible beam, motion of the beam in the transverse magnetic field leads to a nonlinear damping effect that is proportional to the square of the amplitude, so the effect of the magnetic field is similar to the damping effect. It can be concluded from the present study that under stable situations, the more the transverse magnetic field increases, the more the displacement and the natural frequency of the beam system decrease. As long as the parameters of the beam system are practical and reasonable, we can also investigate the behavior of the micro beam system in a magnetic field by the proposed approach.

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## Analiza drgań nieliniowych obustronnie zamurowanej belki poddanej wymuszeniu siłą harmoniczną w obecności zmiennego pola magnetycznego

Streszczenie

Praca przedstawia analizę nieliniowej dynamiki belki obustronnie zamurowanej i poddanej zewnętrznemu obciążeniu siłą harmoniczną w obecności oscylacyjnie zmiennego pola magnetycznego. W badanym układzie uwzględniono poprzeczną

siłę magnetyczną, poprzeczny moment magnetyczny, siłę mechaniczną w kierunku osiowym, jednorodnie rozłożoną poprzeczną siłę sprężystości, poprzeczne obciążenie powierzchniowe oraz tłumienie. Do wyznaczenia równań ruchu zastosowano zasadę Hamiltona przy założeniu pewnych hipotez, a następnie użyto metody Galerkina w celu rozwiązania tych równań. W wyniku przeprowadzonej analizy zaobserwowano, że w stabilnych warunkach wzrost indukcji poprzecznego pola magnetycznego powoduje ograniczenie drgań belki oraz spadek częstości własnych układu.

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