

## The recurrent algorithm for area discretization using the trapezoidal mesh method

## Rekurencyjny algorytm dyskretyzacji obszaru z wykorzystaniem siatki trapezowej

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**Key words:** electronic chart, area discretization, trapezoidal mesh, recurrent discretization algorithm, navigational decision support system

### Abstract

The problem considered in this article refers to such representation of data from an electronic navigational chart that fast and effective identification of areas meeting certain criteria, e.g. depths, will be possible. This is of importance in such processes as situation analysis and assessment or the determination of a safe trajectory of ship movement. The recurrent algorithm for area discretization herein presented makes use of the trapezoid mesh. Its properties and requirements, as well as the methodology of area discretization are presented. The results of an experiment based on that algorithm are described.

**Słowa kluczowe:** mapa elektroniczna, dyskretyzacja obszaru, siatka trapezowa, rekurencyjny algorytm dyskretyzujący, nawigacyjny system wspomagania decyzji

### Abstrakt

Rozpatrywany w artykule problem dotyczy reprezentacji danych z nawigacyjnej mapy elektronicznej umożliwiających szybkie i skuteczne identyfikowanie obszarów spełniających określone kryteria, np. głębokości. Ma to istotne znaczenie między innymi w procesach analizy i oceny sytuacji czy wyznaczania bezpiecznej trajektorii ruchu statku. W artykule zaprezentowano rekurencyjny algorytm dyskretyzacji obszaru z wykorzystaniem siatki trapezowej. Przedstawiono jego właściwości oraz wymagania. Zaprezentowano metodykę postępowania w procesie dyskretyzacji obszaru. W oparciu o opracowany algorytm przeprowadzono eksperyment i przedstawiono wyniki.

## Introduction

One of the basic tasks of navigation is safe conduct of a ship from a departing point to destination. This problem comes down to the determination of position and of the proper course to be followed by the vessel being steered. The determination of the appropriate course has to account for other vessels movement in the area (Collision Regulations), restrictions of the area itself (shape of lands, maximum depth, other obstruction), or the dynamics of own ship.

An increased intensity of vessel traffic and necessity to navigate at various restrictions (time of

day or night, hydro-meteorological conditions etc.) called for constructing other systems supporting actions of the navigator in the process of ship conduct. As a result, the quantity of information available on board ship has increased, which creates difficulties in its selection and processing.

For correct operation of navigational information systems and increasingly developed decision support systems, it is necessary to provide access to data describing a present situation of the ship. Apart from such devices and systems as ARPA, GPS and AIS, the electronic navigational chart (ENC) is a source of such data. However, the format of ENC data is not suitable. Therefore, the

information has to be represented in such a manner that its quick analysis and use in a decision system will be possible.

One solution is the application of a recurrent algorithm of area discretization, using trapezoidal meshes. It allows to transform data from an ENC of S57 standard into a form suitable for entering data and their use in a navigational decision support system.

### Assumptions for the area discretization algorithm

Basic requirements concerning the operation of the algorithm are imposed by modules working within a navigational decision support system [1, 2] and result directly from functions performed by the system [3, 4]. These requirements are mainly related to the assurance of proper speed of operation and such configuration of the algorithm and generated mesh that its modification will be possible depending on a change in moving objects positions and appearance of new objects (vessels). Also, the mesh configuration has to provide a sufficient amount of data for accurate calculations, analysis of a navigational situation and performance of own ship's manoeuvres.

In typical cases, there is no need to take land-covering mesh elements into account in detailed calculations. Such elements may be designated as prohibited and eliminated from further analysis, which allows to shorten computing time. Similarly, areas excluded from navigation, waters with insufficient depths, protruding wrecks etc. may also be regarded as prohibited areas.

The mesh configuration has to enable such representation of chart structure and charted objects that the relevant data will be presented with sufficient accuracy. In this connection, the algorithm has been developed, so that each mesh element includes a uniform area – with the same properties at each point (affiliation to a permitted / prohibited area etc.).

To obtain a possibly short operation time, the algorithm design should provide for a time-minimized computing of the mesh structure. Many operations are carried out using indexes for tables containing algorithm input data (parameters of nodes and vectors determining objects). In this way, the number of variables and necessary computations have been reduced. One necessary step is the determination of coordinates  $\varphi$  of some vertices of trapezoids making up the mesh.

In order to correctly identify charted areas, we have to ensure the right format of input data. It has been assumed that vectors determining area boundaries do not cross, and individual areas do not touch each other if they are of the same type (e.g. land). Should a situation like this occur, such areas are joined into one polygon at the preliminary stage of analysis, before the process of mesh formation.

### Transformation and selection of input data

The primary function of creating a representation of ENC data (points, triangles, rectangles, trapezoids and others) in the form of a mesh is their ordering, so that it will respond possibly fast to an enquiry concerning a position of a point or points on a plane and their position relative to other

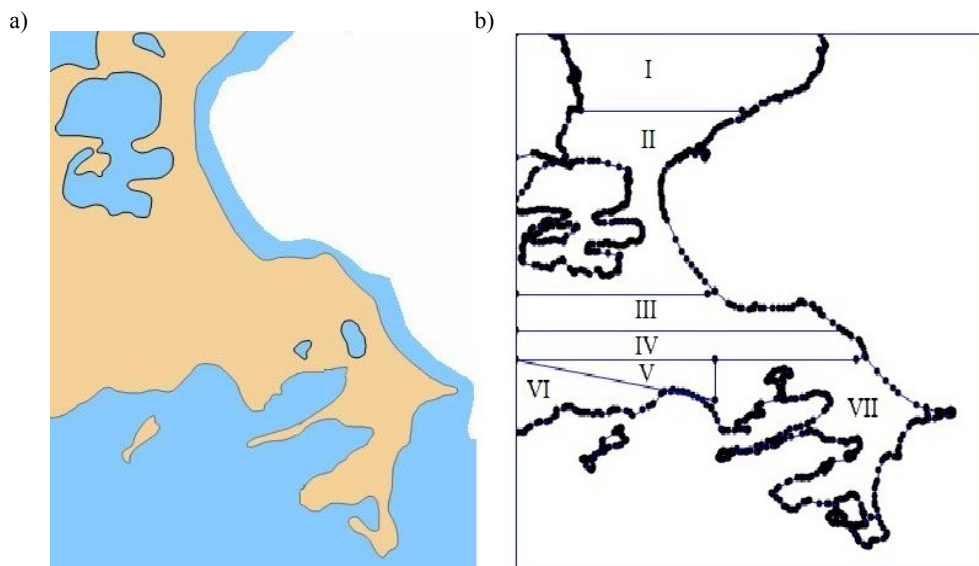


Fig. 1. An electronic chart and its representation – input data for an algorithm: a) chart, a land fragment, b) seven subareas representing the presented fragment

Rys. 1. Mapa elektroniczna oraz jej reprezentacja – dane wejściowe dla algorytmu: a) mapa, fragment lądu, b) siedem podobszarów, które tworzą prezentowany fragment

points and areas lying on the same plane. In reference to the mesh, this issue comes down to the determination of the number of mesh element in which the looked-for point is located. In the particular case, it will be an edge or angle (node) making up a boundary between mesh elements.

The proposed algorithm allows to create a discrete representation of the analyzed area in the form of a trapezoidal mesh. All mesh elements are trapezoidal in shape, or triangular in particular cases, where the length of one of the vertical walls is equal to zero (degenerated trapezoid).

Input data consist of an ordered set of points representing the endpoints of vectors marking individual objects found in the processed fragment of a plane. These points subsequently become nodes of a generated mesh.

Entered data are analyzed in order to determine the position of each object in relation to the others. It often turns out in the process of extracting data from an ENC that an area (mostly land), a whole from the viewpoint of the user, consists of a few parts adjacent to or overlapping each other. Such representation of data raises significant problems in mesh generation and may be a source of serious errors. Therefore, input data undergo a preliminary processing, in which adjacent or overlapping areas are identified. Then such areas are integrated into one area (Fig. 1).

Data from an ENC are divided into a number of groups, depending on the feature they refer to land, depth contours etc. The proper grouping is very important for the correctness of subsequent qualification of mesh elements as safe or dangerous (permitted / prohibited), and of the whole analysis of a navigational situation.

### Description of the algorithm

The mesh is generated on the basis of processed input data. The creation of the mesh begins by generation of a mesh for a selected rectangular area with one object located in it. Then further objects are added, and the previously generated mesh is locally modified to adjust to another object added. Mesh elements are created by projecting two vertical sections from each node, the endpoints of vectors defining object boundaries. This node for the first section is the lower end, for the second section is the upper end. The second end of each section is:

- the nearest point of crossing the boundary of any object on the chart, or
- boundary of the area covered by the mesh.

Each mesh eye is unequivocally defined by:

- indexes of two nodes from which sections are projected; these make up vertical sides of mesh eye;
- indexes of two lines belonging to an object (objects) on the chart, bounding mesh elements from above and below.

When a given element is located by the boundary of mesh-covered area, the limits of the area may be sides of mesh element.

Such method of mesh generation allows to eliminate a situation where in one mesh element one can find chart fragments of different properties, e.g. with various depths or fragments of an allowed area and a prohibited area.

The mesh modification process, where further objects are added, requires that at an initial stage it should be specified which of the existing mesh eyes must be modified. These elements are removed and replaced by new ones, incorporating an object being added. The algorithm contains 21 defined basic variants of mutual position of vectors determining a newly added object and currently modified / removed mesh elements. Depending on the qualification of a situation, a mesh element is divided and replaced by new ones.

As a result it can obtain:

- graphical representation of a mesh (Figs 2, 3a);
- ordered table including all mesh elements (trapezoids);
- neighbourhood matrix depicting possible transitions between successive trapezoids and its graph representation (Figs 3b, 4).

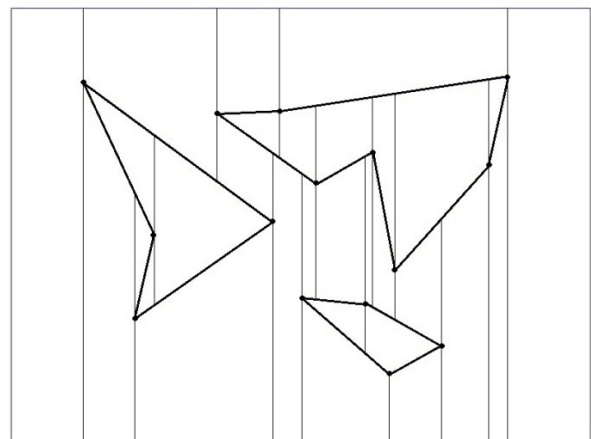


Fig. 2. A trapezoidal mesh  
Rys. 2. Siatka trapezowa

The table, including data on individual trapezoids, contains, inter alia:

- indexes of nodes from which sections, vertical trapezoid sides (coordinates  $\lambda$  of vertical sides), are projected;

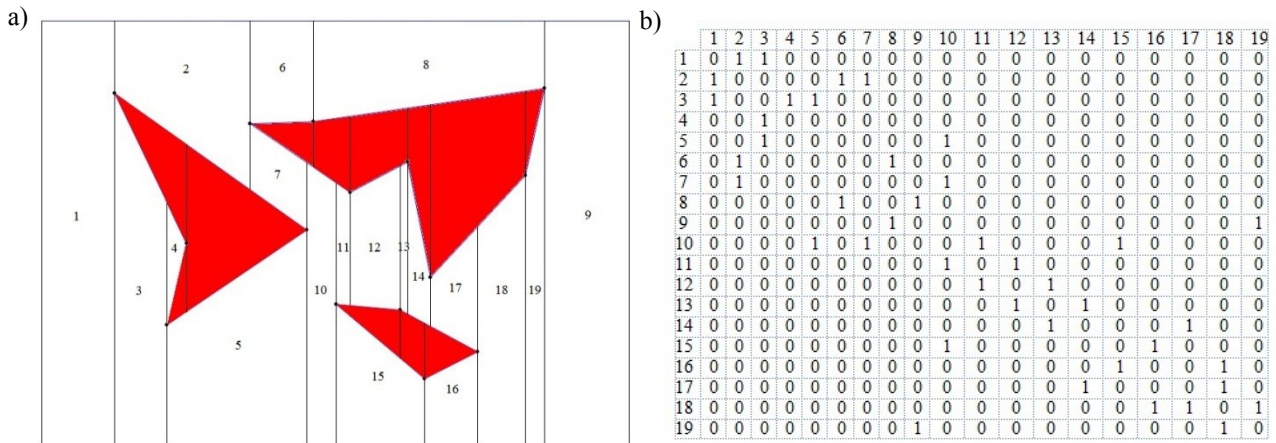


Fig. 3. a) Trapezoid mesh, prohibited areas are colored, b) neighbourhood matrix for the mesh in figure 3a  
 Rys. 3. a) siatka trapezowa, kolorem zaznaczone elementy zabronione, b) macierz sąsiedztwa dla siatki z rysunku 3a

- indexes of lines bounding a trapezoid from above and below;
- angle coordinates;
- indexes of neighbouring trapezoids (contacting a given trapezoid from left or right with its vertical side created by projecting sections from a mesh node);
- trapezoid class: prohibited / permitted.

Trapezoids are regarded as neighbouring only when they touch each other along a vertical side created by projecting sections from a mesh node. Each of the trapezoids may have a maximum of four neighbours, two on the left and two on the right-hand sides; these neighbours are referred to as [5]:

- top left;
- bottom left;
- top right;
- bottom right.

The determination of mutual position of mesh elements is one the requirements for a correct analysis of a navigational situation within the mesh-covered area. The neighbourhood matrix (Fig. 3b) makes up a basis for constructing a transition

graphused in algorithms of choice and route optimization.

### Determination of prohibited areas

The procedure of prohibited area determination is performed at the stage of mesh generation. It is not necessary to determine those areas at a later stage, unlike in other algorithms, where for instance, meshes are formed on the basis of quadratic tree [2, 6]. The identification of prohibited mesh elements is based on an analysis of lines (vectors) bounding a given element from above and below. The prerequisite for a given element to be initially preliminarily classed by the algorithm as prohibited (belonging to a prohibited area) is that the vectors bounding this element from above and below should belong to the same area and have opposite senses. Then trapezoids neighbouring to the left and right are checked, followed by checks of further neighbours until a successive trapezoid being checked happens not to have a neighbour or will be bounded from above, or below by vectors of another object. A trapezoid lies in a prohibited area only, if after searching its neighbours to the left and right

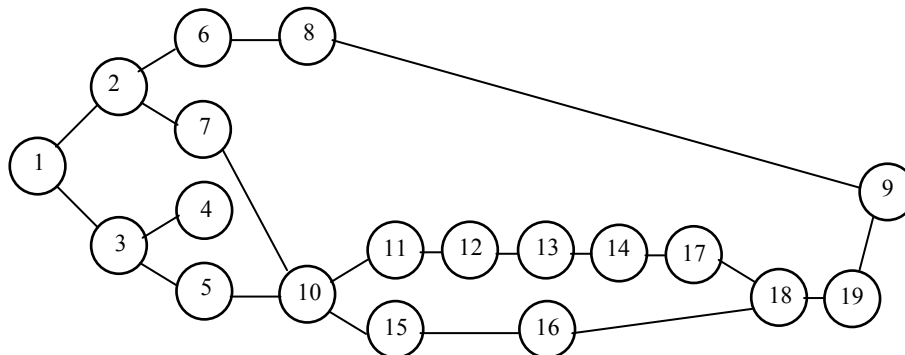


Fig. 4. Neighbourhood graph corresponding to the mesh and matrix from figure 3  
 Rys. 4. Graf sąsiedztwa odpowiadający siatce oraz macierzy sąsiedztwa z rysunku 3

those mesh elements are reached that do not have a neighbour: searching to the left – no neighbour on the left, and while searching to the right – there is no neighbour on the right (Fig. 3a).

To obtain information on the properties and position of any selected point  $p$  relative to other objects all that is needed is the identification of that mesh element that contains the coordinates of that point. Because all mesh elements are entered in a table ordered against coordinates  $\lambda$ , searching consists in identifying such elements whose coordinates  $\lambda$  of the left and right vertical edge are,

respectively, larger and smaller than the coordinate  $\lambda$  of point  $p$ . Then coordinates  $\varphi$  of point  $p$  and those of identified elements are checked in order to identify the element containing that point. Knowing the index of the looked for element, it can read out data referring to the area encompassed by that element.

**Execution**

A fragment of an electronic navigational chart of the southwest Baltic Sea was transformed by the above presented algorithm. The transformation

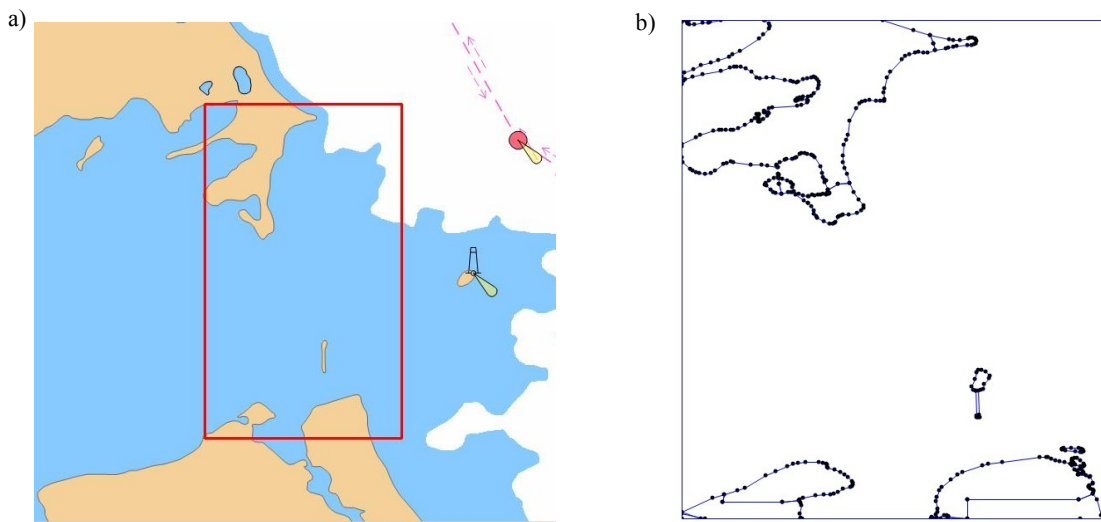


Fig. 5. a) Transformed part of an ENC, b) nodes and vectors determining objects – input data for the algorithm

Rys. 5. a) transformowany fragment mapy elektronicznej, b) węzły oraz wektory wyznaczające poszczególne obiekty – dane wejściowe dla algorytmu

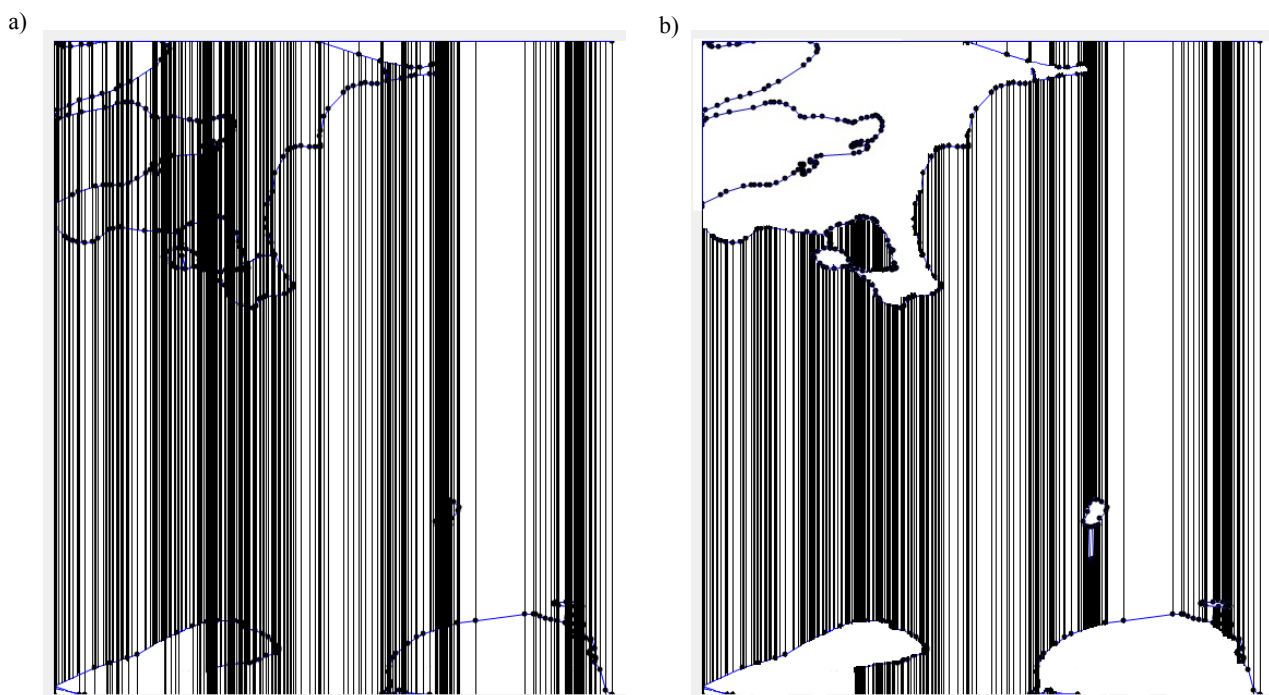


Fig. 6. Generated mesh: a) whole, b) removed elements in prohibited areas

Rys. 6. Wygenerowana siatka: a) całość, b) usunięte elementy znajdujące się w obszarach zabronionych

covered a rectangular area bounded by: 54°9'N – 54°21'N and 13°39'E – 13°51'E (54.15°N – 54.35°N and 13.65°E – 13.85°E) (Fig. 5). The area consisted of 11 objects with a total of 450 lines.

As a result, a mesh consisting of 1241 elements was obtained (Fig. 6). Once the prohibited areas were taken into account and the trapezoids included in those areas were eliminated, 768 elements remained. The total time needed for area analysis, mesh generation and area identification was 141 ms.

The obtained mesh is a basis for an analysis of a situation in a restricted area in a Navigational Decision Support System, and the neighbourhood matrix is used for constructing a transition graph, which in turn makes up a basis for route determination and optimization.

### Summary and conclusions

The presented method of the representation of data from an electronic navigational chart enables quick and effective identification of areas meeting certain criteria. The obtained research results confirm the effectiveness of the presented algorithm.

The application of the proposed method allows to carry out discretization in less than one second, which satisfies the relevant requirement of a navigational decision support system.

The basic advantage of this algorithm is short operation time and the construction of elements along to eliminate cases where fragments of two areas are located in one mesh element. Local modifications of mesh structure are also simple and need a very short time, depending on the present position of objects in motion. The basic disadvantage, on the other hand, is that the structure of generated mesh can hardly be changed. This results directly from the construction and principle of operation of the algorithm – mesh composition depends solely on the structure of an area being transformed.

Slight possibilities of altering the mesh structure may raise difficulties when probabilistic methods are used in vessel route optimization. A restricted possibility of decreasing the number of allowed elements by defining additional criteria may lead to excessively large number of nodes and mesh elements. Consequently, the maximum operation time of the optimization algorithm will be exceeded.

The analysis presented in this article makes up a basis for further research aimed at shortening the algorithm operating time, improvement of procedures for detecting prohibited areas, and development of rules for creating a transition graph based on algorithm-generated neighbourhood matrix. Besides, the analysis is a starting point for research on other methods of ENC data representation for use in a navigational decision support system.

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