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ANALYTICAL APPROACH TO ESTIMATE AMPLITUDE OF STICK-SLIP OSCILLATIONS

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The objective of the present work is to evaluate alternative approximate techniques to determine the amplitudes of the limit cycles that evolve from stick-slip vibrations based on a mass-on-moving-belt model. The control of self-excited systems is a very interesting problem because of friction-induced self sustained oscillations which result in a very robust limit cycle that characterizes stick-slip motion. This motion should be avoided because it creates unwanted noise, diminishes accuracy and increases wear. The stick-slip motion produced by a mass-spring-damper on a moving belt is analyzed using the Liapunov second method, which is based on constructing a positive definite function and checking the condition for which its time derivative is negative semi-definite. From this condition, an estimate of the amplitude of the velocity of the limit cycle of stick-slip motion is obtained. This estimate is found to be the zero of a certain function derived from the Coulomb friction model. An estimate of the amplitude of the displacement is also found. It is shown that the simulation results of the amplitude and the estimated amplitude are indistinguishable.

Key words: stick-slip oscillations, limit cycles, Liapunov second method

1. Introduction

This work presents a technique that will allow useful estimation of the amplitude of displacement and velocity of the limit cycle of stick-slip motion produced by a mass-spring-damper on a moving belt and analyzed by using the Liapunov second method. Often in engineering practice, there is a need for minimization or maximization of the friction force like in rolling and breaking processes. The problem of control of self-excited systems is very difficult because of friction-induced vibrations. Self-excited vibrations can be frequently noticed in everyday situations, not only in the engineering practice. Noise and wear appear to be their undesirable and avoided results. Modern systems require a very high operating precision as necessary for working, namely, the proper operation of various types of manipulators in modern automatic control system requires a very high operating precision. Hence, there is a strapping need to reduce the amplitude of such vibrations. This problem can be solved by additionally influenced external harmonic excitation for the amplitude vibration minimization. In some cases, it is impossible to use absorbers and the emerging stick-slip vibration is unavoidable. It is characterized by a displacement-time evolution which clearly defines stick and slip phases in which the two surfaces in contact respectively slip over each other. The motion is governed by a static friction force in the stick phase and a velocity dependent kinetic friction force in the slip phase. The present work outlines a study of the estimate of the amplitude of velocity of the limit cycle of stick-slip motion, and also simulation results of the amplitude and the estimated amplitude are compared.

2. Brief overview of the state of the art

In oscillatory motion both phenomena, i.e. stick and slip, take place successively, resulting in a stick-slip mode. Since friction characteristics consist of two quantitatively different parts with non-smooth transition, the resulting motion also shows non-smooth behavior. Thus, stick-slip systems belong to the class of non-smooth systems, where discontinuities occur on a surface in the state space (Utkin, 1978). Numerous mechanical interfaces are characterized by some form of dry friction where the force-velocity curve has a negative slope at low velocities (Popp and Rudolph, 2004). The stick-slip vibrations are well known in many kinds of engineering systems and everyday life, e.g. like sounds form when a violin is played, squeaking chalks and shoes, creaking doors, squealing tramways, chattering machine tools, drillstrings, car steering systems, grating brakes and various other systems. For ease of the setup and interpretation, an idealized physical system consisting of a mass sliding on a moving belt has been considered very often. For self-excited friction induced oscillations, essentially four different instability mechanisms have been described in literature. First, the friction coefficient decreasing with relative sliding velocity may lead to negative damping and consequently to oscillatory instability of the steady sliding state. Second, mode-coupling (sometimes also referred to as binary flutter or displacement dependent friction force instability) may destabilize the steady sliding state also for constant friction coefficients. Third, sprag slip, and fourth the follower force nature of the friction force have been identified as fundamental mechanisms for friction self-excited vibrations. All of these mechanisms are amply described in literature (Spurr, 1961; Popp and Stelter, 1990; Ibrahim, 1994; Wallaschek *et al.*, 1999; Gaul and Nitsche, 2001; Gasparetto, 2001; Hoffmann *et al.*, 2002; Hoffmann and Gaul, 2003; Abdo and Al-Yahmadi, 2009; Abdo *et al.*, 2010), a further discussion is therefore not given here. Also when it comes to the system nonlinearities, a lot of work has already been conducted.

Mainly, there are four different models of stick-slip friction: Rough surfaces or surface topology model (McClelland, 1989; Meyer *et al.*, 1998), distance-dependent or creep model (Sampson *et al.*, 1943), velocity dependent friction model (Carlson and Langer, 1989; Nasuno *et al.*, 1997), and phase transition model (Thompson and Robbins, 1990; Robbins and Thompson, 1991). An approximate analysis of the stick-slip vibration amplitude is conducted by Thomsen and Fidlin (2003). The analysis is based on dividing the motion into two phases. The stick phase in which the velocity of motion is constant and the slip phase in which motion is approximated as circular (pure sinusoidal) motion with a constant amplitude. The solutions of the two phases are batched together. The exact solution for this model is given in Wensrich (2006) for a simple Coulomb friction model consisting of a sign function and a linear term in the relative velocity with no cubic term. One sided solution (i.e. when the relative velocity is negative) is only considered. This is also done by Thomsen and Fidlin (2003).

Same model with friction is considered by McMilan (1997). The model treats velocity and acceleartion in a discontiuous manner. The experimentally observed phenomenon is explained and aproximated by hystersis. The model is numerically integrated and the results are compared with the experimental ones to validate the friction model. Leine *et al.* (1998) proposed a numerical method known as the shooting method for calculation of the period of vibration of a one degree of freedom mechanical model. Two degrees of freedom for stick-slip motion is studied in Awrejcewicz and Olejnik (2007) in which the mass-spring- damper on a belt is used with allowable vertical motion.

In this paper, stick-slip vibration is analyzed for one degree of freedom depending on the model of Thomsen and Fidlin (2003). However, a novel approach for estimating the amplitude of vibration is introduced. A prediction of oscillations is done using the Liapunov second method (Khalil, 1992), which is usually used to check the stability of equlibrium points. The Liapunov direct method, also known as the Liapunov second method, represents an approach to the problem of stability of dynamical systems not requiring the solution to the differential equations of motion. The condition for the derivative of a negative definite or semi-definite Liapunov function is examined from which the amplitude of the velocity of the limit cycle is estimated. The friction model of Thomsen and Fidlin (2003) is used here. It is shown that the amplitude of the velocity is a zero function obatined from the friction model. The estimated amplitude is thus related to the model parameters allowing for controlling the amplitude of vibration by adjusting these control parameters. Simulation results show matching between the predicted amplitude and the simulation results.

3. Analysis using the Liapunov direct method

The equation of motion of a mass-spring-damper on a moving belt, shown in Fig. 1, is given by

$$M\ddot{X} + C\dot{X} + KX = F_s \tag{3.1}$$

where X is the displacement, M is the mass, C is the viscous damping, K is the spring constant, and F_s is the friction force, where the dot indicates derivation w.r.t. t. Figure 2 shows a typical friction force as given in Thomsen and Fidlin (2003), Ibrahim (1992), Navaro-Lopezz *et al.*, 2004.

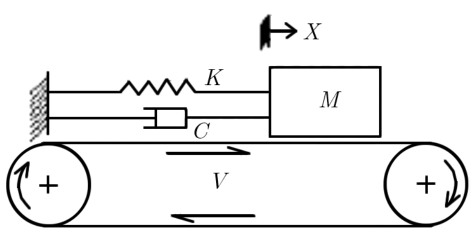

Fig. 1. Mass-spring-damper on a moving belt

Let $V_r = \dot{X} - V$ be the relative velocity of the mass, for $|C\dot{X} + KX| \leqslant \leqslant \mu_s Mg$ (g is the gravity vector) and $V_r = 0$, the pulling force is smaller than the friction force and there is no motion (stick phase), hence

$$M\ddot{X} = 0 \qquad \text{for} \qquad |C\dot{X} + KX| \leqslant \mu_s Mg \quad \text{and} \quad V_r = 0 \tag{3.2}$$

otherwise

$$F_s = -\mu_s Mg \operatorname{sgn}(\dot{X} - V) + K_1(\dot{X} - V) - K_3(\dot{X} - V)^3 \tag{3.3}$$

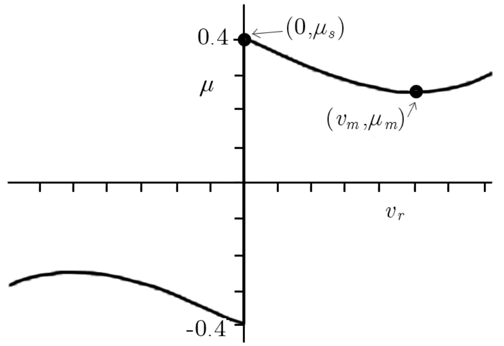

Fig. 2. Friction function as given by (3.2)

where

$$\operatorname{sgn}(s) = \begin{cases} 1 & \text{for} \quad s > 0 \\ 0 & \text{for} \quad s = 0 \\ -1 & \text{for} \quad s < 0 \end{cases}$$

and the equation of motion is

$$M\ddot{X} + C\dot{X} + KX = -\mu_s Mg \operatorname{sgn}(\dot{X} - V) + K_1(\dot{X} - V) - K_3(\dot{X} - V)^3 \tag{3.4}$$

with

$$\mu_s = 0.4 \qquad \mu_m = 0.25 \qquad v_m = 0.5$$
$$K_1 = \frac{3(\mu_s - \mu_m)}{2v_m} \qquad K_3 = \frac{\mu_s - \mu_m}{2v_m^3}$$

Transformation of the equation of motion into a dimensionless form is done in Thomsen and Fidlin (2003).

Let

$$L = \frac{Mg}{K} \qquad \omega_0 = \sqrt{\frac{K}{M}} \qquad t_0 = \frac{1}{\omega_0} \qquad k_1 = \frac{K_1}{M\omega_0}$$
$$k_3 = \frac{K_3\omega_0 L^2}{M} \qquad 2\beta = \frac{C}{\sqrt{KM}} \qquad v = \frac{Vt_0}{L} \qquad \tau = \frac{t}{t_0}$$
$$x = \frac{X}{L} \qquad v_r = \frac{V_r L}{t_0}$$

equations of motion (3.2), (3.4) read

$$\begin{aligned} &\ddot{x} = 0 && \text{for} \quad \dot{x} = v \quad \text{and} \quad -\mu_s \leqslant 2\beta v + x \leqslant \mu_s \\ &\ddot{x} + 2\beta\dot{x} + x = -\mu_s \operatorname{sgn}(\dot{x} - v) + k_1(\dot{x} - v) - k_3(\dot{x} - v)^3 && \text{for} \quad \dot{x} \neq v \end{aligned} \tag{3.5}$$

where the dot indicates derivation w.r.t. τ.

Analysis of motion is carried out using the Liapunov second method (Banakov and Gubanov, 1965; Leine *et al.*, 1998), by splitting it into two phases, the stick and slip phase. Partition the state space R^2 into three regions

$$\begin{aligned} \Omega &= [-\mu_s - 2\beta v, \mu_s - 2\beta v] \times \{v\} \\ \Omega_- &= R \times \{\dot{x} < v\} \cup [-\mu_s - 2\beta v, -\infty] \times \{v\} \\ \Omega_+ &= R \times \{\dot{x} > v\} \cup [\infty, \mu_s - 2\beta v] \times \{v\} \end{aligned}$$

In Ω, motion is linear

$$x(\tau) = x(0) + v\tau \tag{3.6}$$

In Ω_-, motion is described by

$$\ddot{x} + 2\beta\dot{x} + x = \mu_s + k_1(\dot{x} - v) - k_3(\dot{x} - v)^3 \tag{3.7}$$

and in Ω_+, motion is described by

$$\ddot{x} + 2\beta\dot{x} + x = -\mu_s + k_1(\dot{x} - v) - k_3(\dot{x} - v)^3 \tag{3.8}$$

We consider two cases:

Case 1: The motion starts in Ω

Case 2: The motion starts in Ω_- or Ω_+.

Case 1: Assume that the motion starts in Ω, we show that the trajectory described by (3.6) and (3.7) returns to Ω. Thus the solution maps into itself. Since the exit point from Ω is unique $(x = x_0, \dot{x} = v)$, $(x\omega_0 = \mu_s - 2\beta v)$ this is a unique trajectory, call it the stick-slip limit cycle. However to be a true limit cycle it should have the property that any trajectory starting in its neighborhood must converge to (or diverge from) it Leine *et al.* (1998). This stick-slip limit cycle is the union of two motions; one given in (3.6) and the other being the solution to (3.7). To analyze the solution to Eq. (3.7) starting from (x_0, v), using the Lyapunov method, a change of variables is conducted to move the equilibrium point to the origin. The equilibrium displacement $\overline{x} = \mu_s - k_1 v + k_3 v^3$. Let $y = x - \overline{x}$, hence $\dot{y} = \dot{x}$, and

$$\ddot{y} + (2\beta - k_1 + 3k_3 v^2)\dot{y} - 3k_3 v\dot{y}^2 + k_3\dot{y}^3 + y = 0 \tag{3.9}$$

Let

$$h(\dot{y}) = (2\beta - k_1 + 3k_3 v^2)\dot{y} - 3k_3 v\dot{y}^2 + k_3\dot{y}^3 \tag{3.10}$$

then

$$\ddot{y} + h(\dot{y}) + y = 0 \tag{3.11}$$

Make the following change of variables: $\dot{y} = z$, hence

$$\dot{z} = \ddot{y} = -y - h(z) \qquad \ddot{z} + h'(z)\dot{z} + z = 0 \tag{3.12}$$

where $h'(z)$ is the derivative of h w.r.t z. Let $x_1 = z$, $x_2 = \dot{z} + h(z)$, then

$$\dot{x}_1 = x_2 - h(x_1) \qquad \dot{x}_2 = -x_1 \tag{3.13}$$

Note that $x_1 = \dot{y}$, $x_2 = -y$. Consider the Liapunov function

$$W = \frac{1}{2}(x_1^2 + x_2^2) \tag{3.14}$$

Time differentiating W along (3.13), we obtain

$$\dot{W} = -x_1 h(x_1) \tag{3.15}$$

Figure 3 shows the function $h(x_1)$. It has three roots $r_2 = \frac{3}{2}v + \sqrt{D} > 0$, $r_1 = \frac{3}{2}v - \sqrt{D} < 0$ and $r_0 = 0$, if the belt velocity v is less than

$$v_{max} = \sqrt{\frac{k_1 - 2\beta}{3k_3}} \tag{3.16}$$

where

$$D = \Big(\frac{3v}{2}\Big)^2 - \frac{2\beta - k_1 + 3k_3v^3}{k_3} \tag{3.17}$$

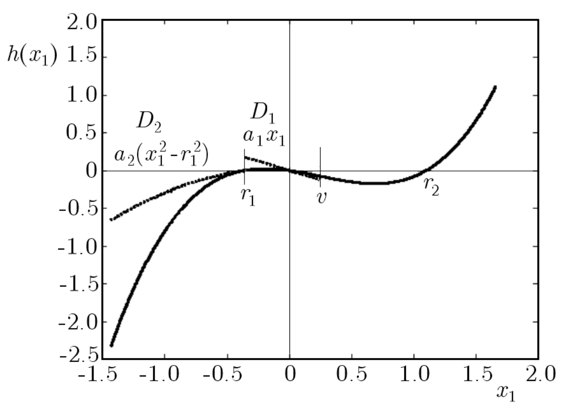

Fig. 3. The polynomial $h(x_1)$ and its approximations in D_1 and D_2

The range of $h(x_1)$ is divided into three intervals $x_1 > r_2$, $r_2 \geqslant x_1 \geqslant r_1$ and $x_1 < r_1$. Since the trajectory starts with $x_1 = v < r_2$, we analyse motion in the intervals $D_1 : \{v \geqslant x_1 \geqslant r_1\}$ and $D_2 : \{x_1 < r_1\}$. In D_1, $\dot{W} \geqslant 0$ and the trajectory spirals away from the origin which is an unstable focus point and in D_2, $\dot{W} < 0$ hence the trajectory converges to the origin. Also shown in Fig. 3 the two functions $\widehat{h}_1(x_1)$ in $, D_1$ and $\widehat{h}_2(x_1)$ in D_2 where $\widehat{h}_1(x_1)$ is chosen linear and satisfies:

$$0 \geqslant x_1 h(x_1) \geqslant x_1 \widehat{h}_1(x_1) \tag{3.18}$$

hence

$$\dot{W} \leqslant -x_1 \widehat{h}_1(x_1) \tag{3.19}$$

in D_1 and $\widehat{h}_2(x_1)$ is chosen parabolic such that $x_1 h(x_1) > x_1 \widehat{h}_2(x_1)$ in D_2 and

$$\dot{W} \leqslant -x_1 \widehat{h}_2(x_1) \leqslant 0 \tag{3.20}$$

Since $0.5r^2 = x_1^2 + x_2^2 = \dot{y}^2 + y^2$ it follows that in D_2: $dr^2/dt \leqslant -x_1 \widehat{h}_1(x_1)$ or $r^2 \leqslant -\int x_1 \widehat{h}_1(x_1)\, dt$ and the trajectory of motion is contained inside the trajectory described by DE: $\ddot{y} + \widehat{h}_1(\dot{y}) + y = 0$ starting at (y_0, v), $(y_0 = x_0 - \overline{x}) \in D_1$.

In D_2, $dr^2/dt \leqslant -x_1 \widehat{h}_2(x_1)$ or $r^2 \leqslant -\int x_1 \widehat{h}_2(x_1)\, dt$ and the trajectory of motion is contained inside the trajectory described by DE: $\ddot{y} + \widehat{h}_2(\dot{y}) + y = 0 \in D_2$.

We choose $\widehat{h}_1(x_1) = -a_1 x_1$, $a_1 > 0$ in D_1 and $\widehat{h}_2(x_1) = -a_2(x_1^2 - r_1^2)$, $a_2 > 0$ in D_2.

We analyze the trajectory of motion in the two regions D_1, D_2.

In D_1

$$\ddot{y} - 2a_1 \dot{y} + y = 0 \tag{3.21}$$

which is a linear DE and has two complex poles in the right half plane at $a_1 \pm j$ for $a_1 \ll 1$.

The solution to which is

$$y(t) = \mathrm{e}^{a_1 t}(y_0 \cos t + (v - a_1 y_0) \sin t) \tag{3.22}$$

If $\dot{y} = r_1$, $y = y_1 \approx \sqrt{y_0^2 + (v - a_1 y_0)^2 - r_1^2}$ approximately for $a_1 \ll 1$.

In D_2

$$\ddot{y} - a_2(\dot{y}^2 - r_1^2) + y = 0 \tag{3.23}$$

or

$$\frac{d\dot{y}^2)}{dy} - 2a_2(\dot{y}^2 - r_1^2) + 2y = 0 \tag{3.24}$$

This is a first order linear differential equation $\dot{y}^2$ which has the solution

$$\dot{y}^2 = \mathrm{e}^{2a_2(y-y_1)}\Big(\dot{y}_1^2 - \frac{2a_2y_1 + 2a_2^2r_1^2 + 1}{2a_2^2}\Big) + \frac{2a_2y + 2a_2^2r_1^2 + 1}{2a_2^2} \tag{3.25}$$

starting from $(y_1, \dot{y}_1)$. If $\dot{y}_1 = r_1$ then the trajectory will move clockwise until $\dot{y} = r_1$ again and y reaches the value y_2 which satisfies the equation

$$(1 + 2a_2y_2)\mathrm{e}^{-2a_2y_2} = (1 + 2a_2y_1)\mathrm{e}^{-2a_2y_1} = c \tag{3.26}$$

For a solution to exist $c < 1$. Solving for y_2 and y_1, we get using the Padé approximation of $\mathrm{e}^{-x} = (1 + x + x^2/2)^{-1}$, $a_2y_2 = (1 - c - \sqrt{1-c^2})/c < 0$ and $a_2y_1 = (1 - c + \sqrt{1-c^2})/c > 0$.

Eliminating c, we get

$$a_2y_2 + a_2y_1 = 2\frac{1-c}{c} \qquad a_2^2y_2y_1 = -2\frac{1-c}{c} = -a_2y_1 - a_2y_2$$

Thus

$$y_2 = \frac{-y_1}{1 + a_2y_1} \tag{3.27}$$

Hence, the absolute value of y_2 is smaller than y_1. If $y = 0$, $\dot{y} = \dot{y}_m$

$$\dot{y}_m^2 = \mathrm{e}^{-2a_2y_1}\Big(\dot{y}_1^2 - \frac{2a_2y_1 + 2a_2^2r_1^2 + 1}{2a_2^2}\Big) + \frac{2a_2^2r_1^2 + 1}{2a_2^2} \tag{3.28}$$

When reaching (y_2, r_1), the trajectory enters D_1 again and has an equation

$$y(t) = \mathrm{e}^{a_1t}(y_2\cos t + (r_1 - a_1y_2)\sin t) \tag{3.29}$$

which hits the line $y_2 = v$ at approximately $y = y_3$

$$y_3 \approx -\sqrt{y_2^2 + (r_1 - a_1y_2)^2 - v^2} \tag{3.30}$$

at which the motion enters the stick phase again since $y_3 \geqslant -y_0$. The smallest slip velocity is greater than $r_1 + \dot{y}_m$ and the amplitude of the displeacement is smaller than $\sqrt{y_0^2 + (v - a_1y_0)^2}$. This completes the analysis of Case1.

Case 2: If the trajectory of motion starts in Ω_-, then it is described by Eq. (3.7). Shifting the equilibrium again, we obtain Eq. (3.9) which possesses a stable limit cycle in the whole plane (Leine *et al.*, 1998) (provided that there is no other type of motion). Any trajectory starting in the plane would converge to this limit cycle, call it the right limit cycle.

Similarly, if the motion starts in Ω_+ the equation of motion is (3.8) with the equilibrium point $\overline{\overline{x}} = -\mu_s - k_1 v + k_3 v^3$. Shifting this equilibrium point to the origin, we obatin the same Eq. (3.9) which also possesses a stable limit cycle in the whole plane (provided that there is no other type of motion). Any trajectory starting in the plane would converge to this limit cycle, call it the left limit cycle. The distance between the two origins is $2\mu_s$, which equals the length of Ω. If the right and left limit cycles intersect the set Ω, then a trajectory starting at any point in the plane must enter the set Ω in trying to reach one of the limit cycles. Figure 4 shows these limit cycles and their intersections with the set Ω. Thus, eventually every trajectory will enter the set Ω, and hence reach the slip-stick limit cycle. This completes the analysis of Case 2.

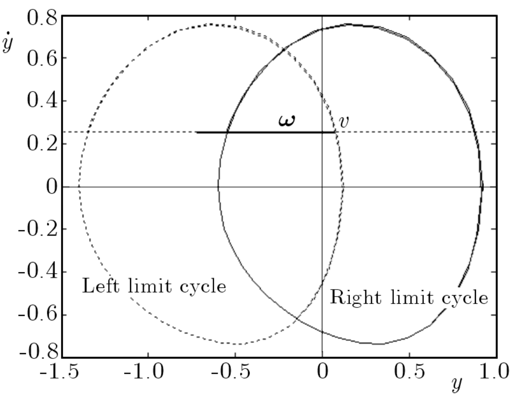

Fig. 4. The left and right limit cycles and there intersection with Ω

4. Simulation results

The dynamic model in Eq. (3.7) is simulated using the parameters used in Thomsen and Fidlin (2003), where $\beta = 0.05$, $v = 0.25$, $\mu_s = 0.4$, $\mu_m = 0.25$, $v_m = 0.5$, $k_1 = 0.45$, $k_3 = 0.6$. For the selected parameters, the zeros of $h(\dot{y})$ are 0, 1.1074, -0.3574, the negative zero is an estimate of the amplitude of velocity. Figure 5 shows the stick-slip limit cycle starting from $(0, 0.25)$. Figure 6 shows both the displacement and velocity, where the stick phase is obvious. Figures 7 and 8 show the the trajectories with stating points $(0, 0.125)$ and $(0, 0.5)$, respectively. Both converge to the slip-stick limit cycle. Note that the lower slip velocity predicted in Thomsen and Fidlin (2003) is equal to $A = 0.7265$, which is about two times the absolute value of our

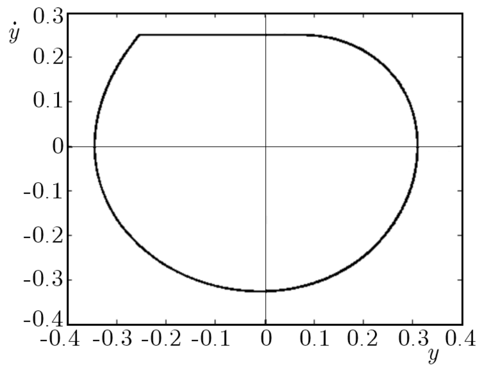

Fig. 5. Stick-slip limit cycle with the initial point $(0, 0.25)$

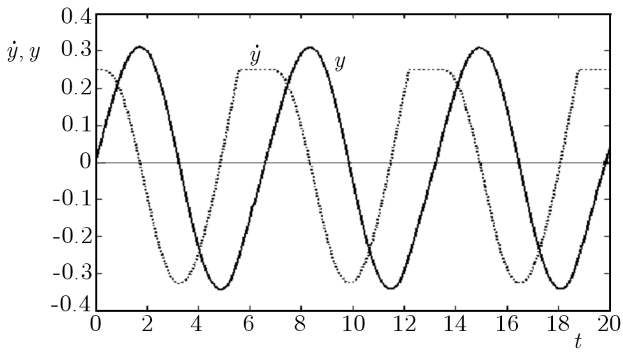

Fig. 6. Displacement and velocity with the initial values $(0, 0.25)$

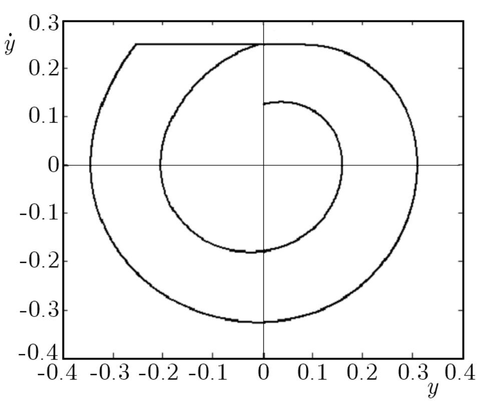

Fig. 7. Trajectory with the initial point $(0, 0.125)$ converges to the slip-stick limit cycle

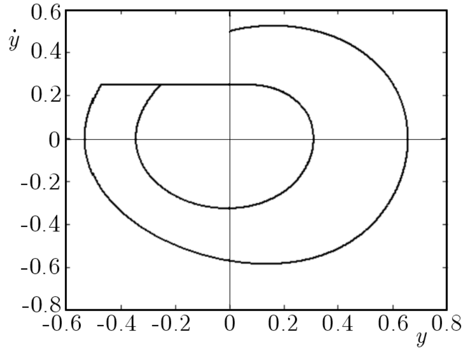

Fig. 8. Trajectory with the initial point $(0, 0.5)$ converges to the slip-stick limit cycle

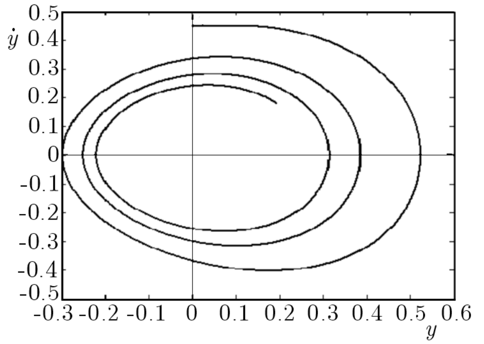

Fig. 9. Trajectory with the initial point $(0, 0.45)$ converges to the slip-stick limit cycle with belt velocity 0.45

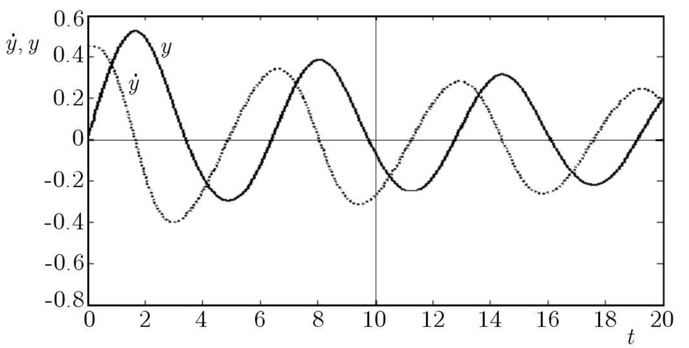

Fig. 10. Displacement and velocity with belt velocity 0.45 (initial point $(0, 0.45)$)

estimated value. This shows that our estimated amplitude is more accurate than the one presented by Thomsen and Fidlin (2003). The displacement range in the existed work is $-0.45 < x < 035$. Note that the maximum belt velocity is 0.44. Figure 9 shows the trajectory of motion with belt velovcity 0.45, which is greater than the maximum belt velocity. Figure 10 shows the displacement and velocity of this case.The obtained trajectory converges to the origin and there is no sick-slip limit cycle.

5. Conclusion

This work analyzes stick-slip motion using the Liapunov second method, which is based on constructing the positive definite energy like a function and testing the condition for which its time derivative is negative semi-definite or negative definite. From this condition, an estimate of the amplitude of velocity of the limit cycle of stick-slip motion is obtained. The estimate of the amplitude of the displacement is also found. It is shown that the simulation results match the estimated amplitude. Due to their vast realm in abundant engineering systems and severe impacts on their performance, the oscillations resulting due to stick-slip always confront smooth operation, and hence it is vital to have knowledge about their behavior and effects. The benefit of this estimate is that it presents a new model for examination of control parameters which affect the vibration amplitude and thus facilitate its control.

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Analityczna metoda określania amplitudy drgań ciernych (typu stick-slip)

Streszczenie

Praca przedstawia ocenę przydatności przybliżonych metod wyznaczania amplitudy cykli granicznych drgań ciernych (*stick-slip*) na podstawie modelu skupionej masy poruszającej się na ruchomym pasie. Sterowanie drganiami samowzbudnymi stanowi interesujące zagadnienie z racji ochrony układów mechanicznych przed znacznymi i silnie stabilnymi cyklami granicznymi drgań indukowanych tarciem. Ruch typu *stick-slip* powinien być eliminowany w maszynach i urządzeniach, gdyż wywołuje hałas, zmniejsza precyzję działania, zwiększa zużycie części. Zjawisko to opisane prostym modelem masy skupionej zamocowanej elementami sprężysto-tłumiącymi i usytuowanej na ruchomym szorstkim pasie przeanalizowano za pomocą drugiej metody Lapunowa opartej na konstrukcji dodatnio określonej funkcji i sprawdzeniu warunku, dla którego jej pochodna względem czasu jest pół-określona ujemnie. Na tej podstawie dało się oszacować amplitudę prędkości cyklu granicznego obserwowanych drgań ciernych. Oszacowanie to znaleziono dla zerowania pewnej funkcji zbudowanej dla modelu tarcia Coulomba. Wyznaczono także amplitudę przemieszczeń układu. Pokazano, że różnice pomiędzy otrzymanymi wartościami szacowanymi amplitud i wynikami symulacji numerycznych są pomijalne.

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