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NUMERICAL-EXPERIMENTAL ANALYSIS OF THE POST-BUCKLING STATE OF A MULTI-SEGMENT AND MULTI-MEMBER THIN-WALLED STRUCTURE SUBJECTED TO TORSION

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A three-segment ten-member thin-shell structure with flat walls made of a material with an instantaneous characteristic approximated by means of an ideally elastic-plastic material model is considered. The structure material (polycarbonate) demonstrates the temporary double refraction effect in polarized light. The system is subject to twisting resulting in the state of local post-critical deformation of skin segments within the structure area. As a result of non-linear numerical analysis in the course of which conformance of equilibrium paths obtained numerically and by means of the experiment is assured, the stress field is determined taking into account the flexural and membrane state of the structure.

Key words: shell, nonlinear analysis, finite elements, constrained torsion, buckling

1. Introduction

Thin-shell load-bearing systems of aviation structures are characterised by admissibility of certain local loss of stability of skin elements in conditions of operational loads (Arborcz, 1985; Lynch, 2000). That results from the fact that the commonly adopted static model of a structure composed of a framing and a skin represents a semi-monocoque structure in which it is assumed that the function of the skin consists in transfer of shear interactions only. The framing, composed of transversally situated ribs (frames) characterised with large stiffness in their planes and members demonstrating large stiffness for normal forces and relatively small stiffness for bending, is a mechanism. Joined with the skin, the framing creates a structure able to transfer any loads resulting from any potentially arising admissible flight phases.

As for the transfer of loads by individual structure elements, the cases in which twisting is the dominant form of load generating the pure shear state in isolated skin elements between neighboring frames and members turn out to be the dimensioning ones for the skin elements. These elements, when subject to shearing, loose quickly their stability at relatively low critical compressive stress values. An important stage in the design work on an aircraft load-bearing structure with significant effect on relation between its mass, stiffness and load capacity limit, consists in choice of the number of frames and members resulting in certain level of internal load exerted on the skin elements in post-critical deformation conditions (Kopecki and Dębski, 2007).

This paper presents a concept concerning analysis of a multi-segment semimonocoque structure subjected to dominant twisting, resulting in a local post-critical deformation state in skin elements, based on an example of a 3-segment 10-member structure.

The considered structure is a simplified part of a torsion box applied in constructional solutions of wings, fuselages and tail planes of aircraft. The mentioned structures often contain various discontinuities, such as cut-outs, usually circular or rectangular with rounded corners, the presence of which is justified for exploatational reasons. They are, for example, inspection openings or cut-outs allowing quick replacements of equipment parts. In case of emergency events, there is a possibility of apperance of irregurar shapes holes, which can be a consequence of collisions with birds or any other obstacles. Then the best temporary method to minimize crash effets is a correction of shape of the damaged zone to the circular opening with possible small radius. In spite of reinforcements of edges, cut-outs, especially localised in the zones including longerons, significantly reduce stiffnes of the structure causing local stress redistributions, which can be a reason of fatique degradation. Aiming at the determination of the extremal values of the stress field, in order to determine fatique life, the considered structure was subject to nonlinear numerical analysis in the finite element approach verified by means of experimental work. The results of experimental research created the possibility to perform corrections of the numerical model in such a way that, at any stage of the structure deformation advancement, the conformance of equilibrium paths and deformation form was assured. The conformance of the results constituted a base for acknowledgement of the obtained structure stress patterns.

2. Subject and scope of research

The subject of research is a three-segment thin-shell structure with ten stringers, the general view of which is presented in Fig. 1. Joints between the structure elements were realised by means of densely distributed bolts (pitch $t = 15\,\mathrm{mm}$).

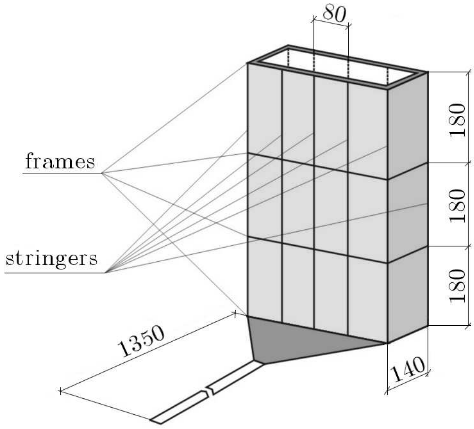

Fig. 1. Schematic view of the structure (dimensions in millimeters)

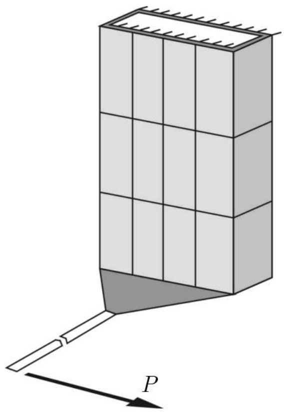

Fig. 2. Schematic view of the structure mounting and load application

The experimental research was carried out by mounting the structure on a specially designed test stand (Fig. 4) making it possible to introduce a load in the form of dominant twisting with negligible bending effect and transversal force. One of the structure boundary frames was fixed, while the other was connected by means of a stiff rib closing the cross-section, with a lever, by means of which the load was applied gravitationally. A schematic view of the mounting and the introduction of load to the structure is presented in Fig. 2.

The structure was made of polycarbonate a material with an instantaneous characteristic presented in Fig. 3.

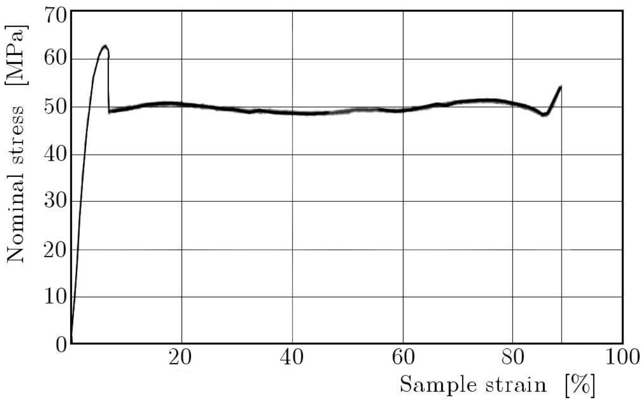

Fig. 3. Structure material (polycarbonate) tensile graph

Fig. 4. View of the test stand

The permanent deformations range, resulting from changes of the polymer molecules position and shape, corresponds in its nature to the plastic range of an elastic-plastic material. That enabled one to approximate the actual characteristic with an ideally elastic-plastic material model in the course of numerical analysis. Moreover, polycarbonate demonstrates a temporary double refraction effect. Observation of optical effects in circularly polarized light creates a possibility to obtain qualitative information about the existence and location of strong stress concentration zones (Kopecki, 1991; Laerman, 1982). In order to enable observation of the above-mentioned effects, the inner surface of skin elements were coated with a reflexive layer. The observations were carried out using the reflected light method.

In the course of experiment, the load was increased gradually, with very small increment values and the angle of torsion being measured accordingly. As a result, the dependence between the twisting moment and the structure total angle of torsion was obtained, i.e. the parameters determining representative equilibrium path of the system (Fig. 9).

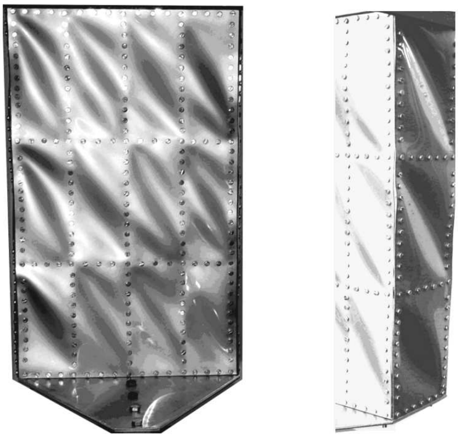

Fig. 5. Advanced phase of deformation of the structure

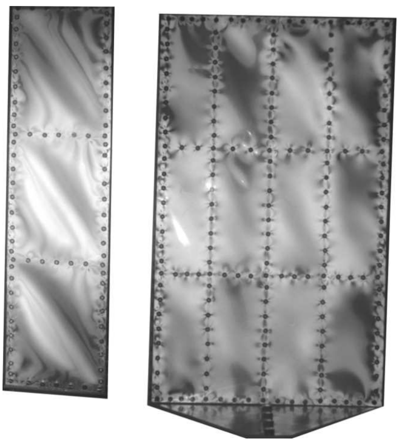

Fig. 6. Optical effect patterns

Already at relatively small values of the twisting moment, all skin segments reached the post-critical deformation state. After full release of the load, the structure returned to its initial form. Therefore, despite significant deformations in the advanced phase, a permanent set did not occur. In Fig. 5, an advanced phase of deformation of the structure is presented. Figure 6 shows corresponding patterns of optical effects.

3. Nonlinear numerical analysis

In nonlinear analysis of load-bearing structures, relations between a set of static parameters and the corresponding set of geometric parameters can be presented in the form of matrix equation

$$\boldsymbol{g} = \mathbf{K}^{-1}(\boldsymbol{g})\boldsymbol{f} \tag{3.1}$$

where $\boldsymbol{g}$ is a set of geometric parameters describing the system deformation state caused by the load, $\boldsymbol{f}$ is a set of static parameters, while $\mathbf{K}$ is the stiffness matrix depending on the set of geometric parameters determining the current deformation state and a nonlinear constitutive relation.

In view of the occurrence of permanent deformations observed in the course of experiments, the physical characteristic of the structure material determined through uniaxial tensile stress tests (Fig. 3) was approximated by an ideally elastic-plastic body model (Fig. 7).

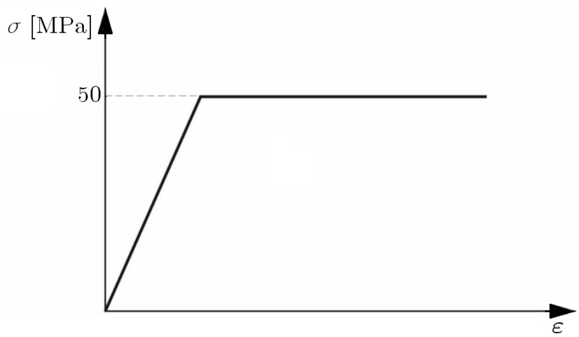

Fig. 7. Constitutive model of the material

In the constitutive equation, in the description related to the linear-elastic range

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{v}\boldsymbol{e} \tag{3.2}$$

an assumption about invariance of normal segment ($ve_z = 0$) is kept in force. Therefore, the plate stress state is represented by the vector $\boldsymbol{\sigma} = \{\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{zx}\}^\top$, while

$$\mathbf{D} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1-\nu}{2k} & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1-\nu}{2k} \end{bmatrix} \tag{3.3}$$

is the material constants matrix in which the effect of non-dilatational strain on the plate elastic energy was accounted for by introducing the correction coefficient $k = 1.2$

$$\boldsymbol{ve} = \begin{Bmatrix} ve_x \\ ve_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\Big[\Big(\dfrac{\partial u}{\partial x}\Big)^2 + \Big(\dfrac{\partial v}{\partial x}\Big)^2 + \Big(\dfrac{\partial w}{\partial x}\Big)^2\Big] \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\Big[\Big(\dfrac{\partial u}{\partial y}\Big)^2 + \Big(\dfrac{\partial v}{\partial y}\Big)^2 + \Big(\dfrac{\partial w}{\partial y}\Big)^2\Big] \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \Big(\dfrac{\partial u}{\partial x}\Big)\Big(\dfrac{\partial u}{\partial y}\Big) + \Big(\dfrac{\partial v}{\partial x}\Big)\Big(\dfrac{\partial v}{\partial y}\Big) + \Big(\dfrac{\partial w}{\partial x}\Big)\Big(\dfrac{\partial w}{\partial y}\Big) \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial y} + \Big(\dfrac{\partial u}{\partial y}\Big)\Big(\dfrac{\partial u}{\partial z}\Big) + \Big(\dfrac{\partial v}{\partial y}\Big)\Big(\dfrac{\partial v}{\partial z}\Big) + \Big(\dfrac{\partial w}{\partial y}\Big)\Big(\dfrac{\partial w}{\partial z}\Big) \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} + \Big(\dfrac{\partial u}{\partial x}\Big)\Big(\dfrac{\partial u}{\partial z}\Big) + \Big(\dfrac{\partial v}{\partial x}\Big)\Big(\dfrac{\partial v}{\partial z}\Big) + \Big(\dfrac{\partial w}{\partial x}\Big)\Big(\dfrac{\partial w}{\partial z}\Big) \end{Bmatrix} \tag{3.4}$$

is a vector containing deformation state components corresponding to the Green-Saint-Venant deformation tensor (Marcinowski, 1999); and u, v, w are displacement vector components in local system of coordinates x, y, z.

Numerical representations of the system nonlinear deformations are based on the assumption that at any solution stage with the corresponding load, the deformed system always retains a static equilibrium state. Thus, for a defined discrete system it is possible to formulate a system of equilibrium equations that, with respect to nonlinear structural analysis in its displacement-based representation, can be expressed in the form of matrix residual force equation

$$\boldsymbol{r}(\boldsymbol{u}, \boldsymbol{\Lambda}) = \mathbf{0} \tag{3.5}$$

where $\boldsymbol{u}$ is the state vector containing displacement components of the structure nodes corresponding to current geometrical configuration, $\boldsymbol{\Lambda}$ is a matrix composed of control parameters corresponding to the current load state, while

$\boldsymbol{r}$ is the residual vector containing uncompensated components of forces related to the current system deformation state (Felippa, 1976; Felippa *et al.*, 1994).

In numerical algorithms, the components of matrix $\boldsymbol{\Lambda}$ are expressed as functions of the parameter λ described as the state control parameter. It is a measure of the increase of load indirectly or directly related with the pseudo-time parameter t. Thus, the system of equilibrium equations (3.5) can also be presented in the form

$$\boldsymbol{r}(\boldsymbol{u}, \lambda) = \boldsymbol{0} \tag{3.6}$$

The above equation is known as the monoparametric residual force equation. Its solution includes a finite number of consecutive structure deformation states, where each state corresponds to a combination of varying control parameters related to the system load, expressed by a single state control parameter λ. Transition from the current state to the consecutive one, representing the increment step, is initialized by a change of the control parameter to which a new geometrical form is related determined by a new state vector.

Development of numerical methods reflected in contemporary algorithms implemented in professional commercial codes resulted in constitution of their two fundamental types. The first one includes purely incremental methods known also as prediction methods, while the other type encompasses correction methods, called also prediction-correction or increment-iteration methods. The first of the mentioned groups is characterised with limited, often unsatisfactory accuracy of obtained results. Moreover, they do not provide possibility to continue calculations after crossing critical points on the equilibrium path. Introduction of the iteration phase is therefore aimed at reduction of the solution error and possibility of determination of critical points. That enables analysis of a structure in advanced deformation states.

A feature common for both types of the method consists in the presence of the incremental phase. With respect to arbitrary increment, at the transition from n-th to $(n+1)$-th state, the undetermined quantities are

$$\varDelta \boldsymbol{u}_n = \boldsymbol{u}_{n+1} - \boldsymbol{u}_n \qquad\qquad \varDelta \lambda_n = \lambda_{n+1} - \lambda_n \tag{3.7}$$

In order to determine them, an additional increment control equation is formulated, known as the equation of constraints, expressed in the form of a condition

$$c(\varDelta \boldsymbol{u}_n, \varDelta \lambda_n) = 0 \tag{3.8}$$

The fundamental component of the increment phase consists in its prediction step, determining a point in the state hyperspace corresponding to the consecutive state configuration defined by determination of the increment $\Delta \boldsymbol{u}$ for the assumed $\Delta\lambda$, with equation (3.8) satisfied at the same time. The solution error at each increment step depends on the increment control equation and the adopted extrapolation formula. In each consecutive increment step, the value of total error may increase, resulting in the occurrence of the so-called drift error. Its minimization is ensured by the iteration phase.

The fundamental method used in the solution of structural mechanics nonlinear problems is the Newton-Raphson method with numerous program realisations and variations constituting a family of methods (Crisfied, 1997; Felippa, 1976; Felippa *et al.*, 1994; Kopecki and Dębski, 2007). The core idea of these methods consists in expansion of the residual forces equation, $\boldsymbol{r} = \boldsymbol{0}$, and the increment control equation, $c = 0$, into Fourier series.

Assuming that, as a result of the k-th correction iteration step, values $\boldsymbol{u}^k$ and λ^k are obtained, the equations will take the following forms

$$\begin{aligned} \boldsymbol{r}^{k+1} &= \boldsymbol{r}^k + \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{u}}\boldsymbol{d} + \frac{\partial \boldsymbol{r}}{\partial \lambda}\eta + \boldsymbol{H} = \boldsymbol{0} \\ c^{k+1} &= c^k + \frac{\partial c}{\partial \boldsymbol{u}}\boldsymbol{d} + \frac{\partial c}{\partial \lambda}\eta + H = 0 \end{aligned} \tag{3.9}$$

where

$$\boldsymbol{d} = \boldsymbol{u}^{k+1} - \boldsymbol{u}^k \qquad\qquad \eta = \lambda^{k+1} - \lambda^k \tag{3.10}$$

Terms H in both equations include the neglected residual values of higher orders. In the iteration process, consecutive values of $\boldsymbol{d}$ and η are determined with respect to which the solution convergence condition is checked at the assumed tolerance. The set obtained that way, constituting a solution to the nonlinear algebraic equations with respect to the unknown nodal displacements, creates a base for determination of the equilibrium path. The path, representing the relation between static parameters corresponding to the structure and geometrical parameters related to displacements of its individual points creates a hyper-surface in a multidimensional space with the number of dimensions corresponding to the number of degrees of freedom of the system taken into account. In practice, representative relations between the two parameters are usually developed.

The numerical analysis was performed by means of MSC MARC 7 software. In the modeling of the skin of the examined structure, one used a bilinear thin-shell element. This is a four-node thin element with global displacements and rotations as the degrees of freedom. The bilinear interpolation is used for the coordinates, displacements and rotations. The membrane strains were obtained from the displacement field and the curvatures from the rotation field. In the modeling of frames and loading system four-node, bilinear thick-shell elements were used. As in the case of thin-shell element, membrane strains were obtained from the displacement field and the curvatures from the rotation field. The transverse shear strains were calculated at the middle of the edges and interpolated to the integration points.

The stringers were represented by means of beam-type elements based on the Euler-Bernoulli method. All the above-listed elements had six degrees of freedom in the node. The total of 25300 nodes was obtained.

The nonlinear analysis was based on the Newton-Raphson prediction method and the Crisfield hyperspherical correction (Bathe, 1996; Doyle, 2001; Crisfied, 1997; Felipa, 1976; Felippa *et al.*, 1994; Rakowski and Kacprzyk, 1993). Reliability of the obtained results was assessed by comparing both equilibrium path shapes and deformation geometries. The two elements created a base for repeated corrections of the numerical model.

A series of tests was performed leading to a development of numerical models in which the nature of deformation fully qualitatively corresponded to deformations obtained in the course of experiment. For all model versions, likewise in the course of experimental research, a dependence was determined between the overall angle of torsion and torque, constituting representative equilibrium paths.

Figure 8 presents the geometrical model of the structure obtained by means of MSC PATRAN software and the finite element grid adopted for calculations with the MSC MARC program.

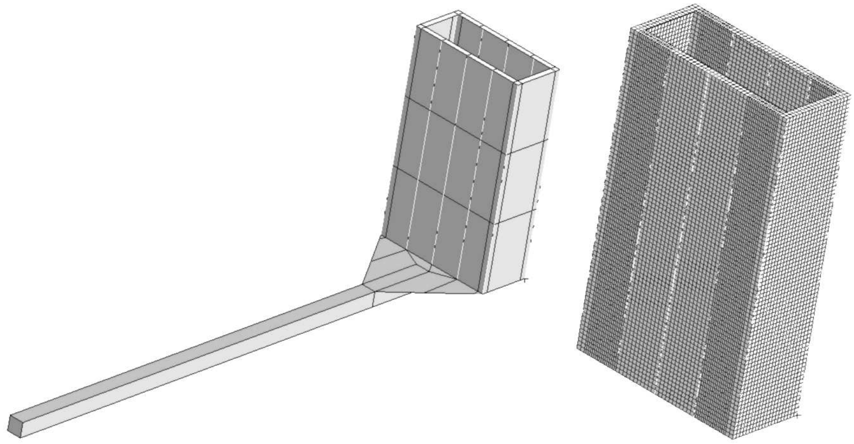

Fig. 8. Geometrical model (left) and the finite element grid (right)

The comparison of representative equilibrium paths is presented in Fig. 9.

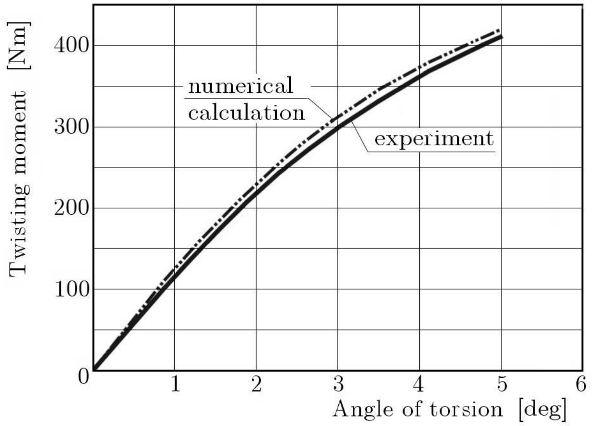

Fig. 9. Comparison of representative equilibrium paths

In Fig. 10, the reduced stress distribution according to H-M-H hypothesis is presented corresponding to the advanced deformation phase.

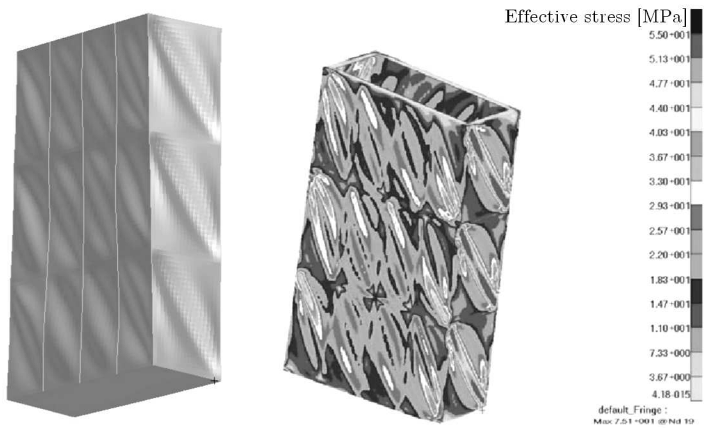

Fig. 10. Calculation results obtained with MSC MARC 7 software: the numerical model deformation picture and the effective stress distribution according to H-M-H hypothesis

The comparison of equilibrium paths and the deformation type enables one to acknowledge the results of nonlinear numerical analysis as satisfactory. The refinement of numerical models was possible in view of systematical comparison of results of calculations with those obtained in the experiment. Even minimal corrections to stiffness of the structure made significant variance in the equilibrium paths course, and even the lack of the solution convergence.

On the grounds of the solution uniqueness rule, providing that a specific deformation form can correspond only to one stress distribution, the effective stress distributions determined numerically can be acknowledged as corresponding to the actual ones. The qualitative form of verification of the calculation results consists in comparison of the obtained effective stress distribution patterns with pictures of optical effects observed in the course of experiments.

Nonlinear analysis was carried out with the use of identical iteration parameters.

Using the test stand described above, an experiment was carried out with the use of a model provided with circular opening. Figure 11 presents an advanced deformation state of the structure corresponding to twisting moment equal to 350 Nm.

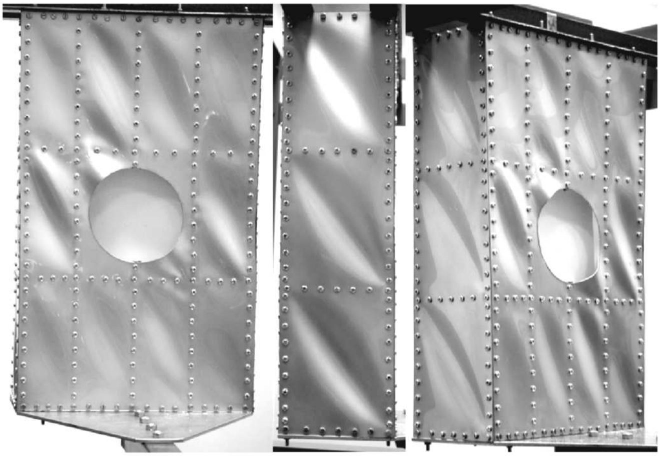

Fig. 11. Advanced phase of post-critical deformations in the structure with a cut-out – result of experiment

Figure 12 presents a comparison of pictures of the structure post-critical deformation obtained from the experiment on one hand and nonlinear numerical analysis on the other, while Fig. 13 shows a comparison of the corresponding representative equilibrium paths.

The presented results confirm full relevance of the deformation form and the representative equilibrium paths. The effective stress pattern presented in Fig. 14 can be considered as a reliable one, so it can constitute the base for fatique life calculations.

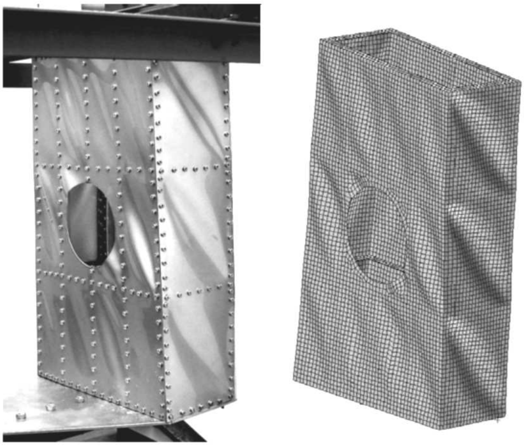

Fig. 12. Advanced phase of post-critical deformations in the structure with a circular cut-out: result of the experiment (left) and nonlinear numerical analysis

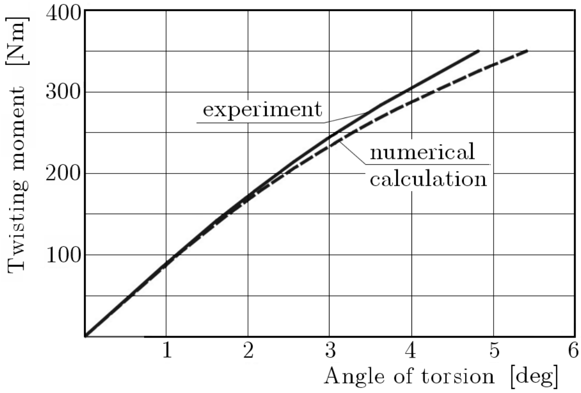

Fig. 13. Comparison of representative equilibrium paths – the model with circular opening

4. Conclusions

In the final conclusion, it is important to emhasise the difficultties in reproducing the stiffnes of the considered structure model, identified during the experiment. In fact, this problem comes down to the method of modeling the connection between the skin, longerons and frames. Making attempts to realise discrete connections, as have been practiced in the structure subjected to experimental tests, in the case of numerical modeling it led to strong local stress concentrations, disabling nonlinear numerical procedures. In that ca-

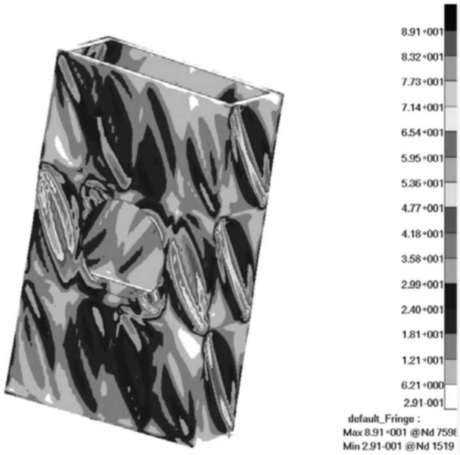

Fig. 14. Effective stress distribution according to H-M-H hypothesis for the model with circular opening

se, one decided to model the mentioned connections in a continuous form. It turned out to be an effective way to provide the conformity of deformation forms and equilibrium paths. In the succesfull realisation of such research, the problem with modeling of the object assigned to experimental tests seemed to be very significant. These problems do not confine to the compatibility of measurements of structure parts, but also to the maintaining of the technological limits during the assembly process. They significantly influence the state of initial deformations, as they led to incompatibility between the results of nonlinear numerical calculations and experimental tests.

It seems to be important that the presented methodology allows one to make modifications to the structure in the virtual environment. It enables refinement of the structure before very expensive and laborious prototype making.

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Numeryczno-eksperymentalna analiza stanu deformacji zakrytycznej wielosegmentowej, wielopodłużnicowej konstrukcji cienkościennej poddanej skręcaniu

Streszczenie

Rozważano trzysegmentową, dziesięciopodłużnicową strukturę cienkościenną o ściankach płaskich, wykonaną z materiału o charakterystyce natychmiastowej przybliżanej modelem materiału idealnie sprężysto-plastycznego. Materiał konstrukcji (poliwęglan) wykazuje efekt dwójłomności wymuszonej w świetle spolaryzowanym.

Konstrukcję poddawano skręcaniu, wskutek czego w obszarze struktury pojawiał się stan lokalnej deformacji zakrytycznej segmentów pokrycia. W wyniku nieliniowej analizy numerycznej, w trakcie której zachowywano zgodność ścieżek równowagi otrzymanych na drodze numerycznej oraz badań eksperymentalnych, wyznaczano pole naprężeń, uwzględniające stan giętny i błonowy ustroju.

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