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EFFECTS OF GEOMETRICAL AND DIMENSIONAL ERRORS ON KINEMATICS AND DYNAMICS OF TRACTA COUPLING

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The paper deals with the investigation of the influence of geometrical and dimensional errors on performance of the Tracta coupling. It is one of the most simple constant-velocity joints which has been used in front-wheel drive automobiles, all-wheel drive military vehicles, heavy vehicles and various other applications. The proposed study is based on building and simulating a parametric spatial multibody model of the full joint. Both kinematic and dynamic behaviour have been investigated. Five different linear and angular errors have been studied. The sensitivity of kinematics and dynamics irregularities to these errors has been computed and summarized in design charts. The results can be used in optimal tolerance allocation and accurate vehicle system simulation with the coupling joint sub-model.

Key words: Tracta joint, tolerance, mechanical error, multibody model

1. Introduction

Coupling joints are widely used in mechanical and industrial applications in order to transmit torque and motion between two misaligned shafts (Wagner, 1991; Schmelz *et al.*, 1991). Scientific literature, patents and commercial catalogues include a lot of different types of joints with different structure, performance and complexity. The most important features that a designer wants in coupling joints are two. The first is the exact kinematic coupling (i.e. homokinetic or constant velocity joint). This condition is fulfilled when the angular velocity of the input shaft is equal to that of the output one, whatever the angular misalignment is. The second important feature is the capability to transfer an adequate amount of torque between the two shafts without irregularities.

It means that the structure has to be designed to support loads, including transient phases and impulsive forces due to impacts. Among different joints, there is the Tracta coupling. Originally developed by the French inventor Jean Pierre Gregoire in 1924, this type of joint was the first constant velocity joint and enabled the development of the first front wheel drive cars in the 30s. Due to its robustness and simplicity, the Tracta joint gained widespread usage all over the world for civil and military vehicles. On the other hand, it presents some engineering problems. Due to the occurence of sliding contact between cooperating parts, its mechanical efficiency is lower than other coupling joints with rolling contacts. Moreover, it is affected by a considerable wear rate, and the presence of mechanical errors affects its performance in sensible way.

In Figures 1 and 2 a typical Tracta assembly is shown. The entire joint is composed of two shafts, two intermediate links and a housing system. At the ends of the shafts, there are two forks that are connected to intermediate links by means of revolute joints. The two intermediate links are connected together with a planar slot with an axis which is perpendicular to those of the revolute joints. This contact between planar surfaces ensures the transmission of torque. The housing system, connected with two revolute joints to the two shafts, ensures correct relative positioning of the forks and the sealing for the lubricant.

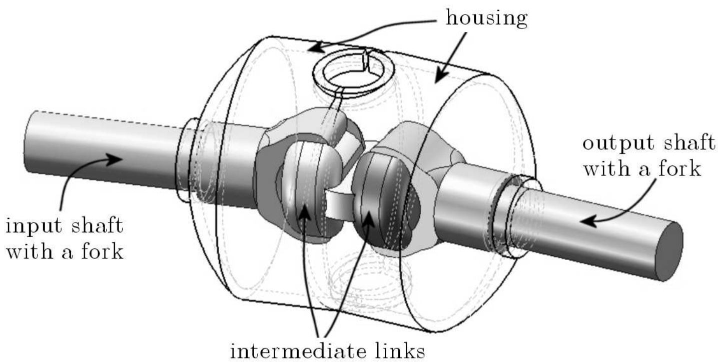

Fig. 1. Tracta joint assembly

Due to the presence of the housing system, the axis of the two shafts always intersect in the midpoint O (see Fig. 3). Moreover, the distances between the axes of the revolute joints and point O have to be the same. This accurate positioning ensures that the intersection point K between the axes of the revolute joints is always placed in the bisector plane between the two misaligned shafts. According to Myard's theorem (Myard, 1931; Pennestrí and Valentini, 2008), the fulfilment of this geometrical condition is sufficient to ensure an

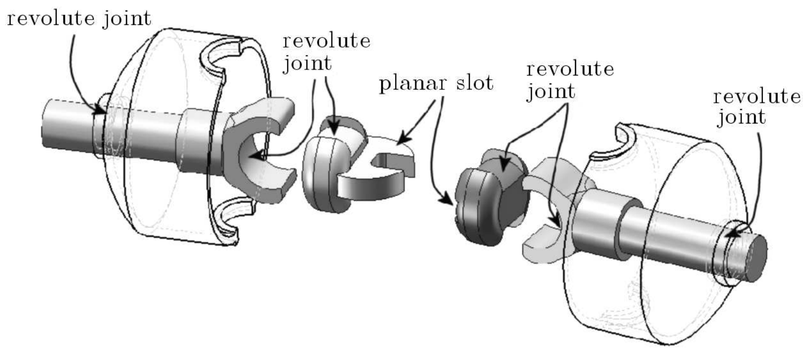

Fig. 2. Tracta joint exploded view

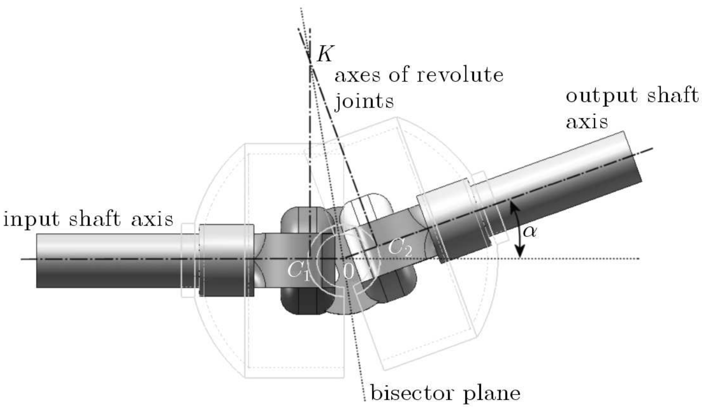

Fig. 3. Geometrical condition for a homokinetic Tracta joint

homokinetic joint, where the input and output rotations occur at the same speeds.

The scientific literature includes some investigations about features and kinematics of the Tracta joint. The structure was investigated by Freudenstein and Maki (1979), the general theory by Hunt (1979) and a deep kinematic investigation including transmission errors and internal displacement between links by Fisher (1994, 1999). Static forces in planar joints have been investigated by Fisher (2007).

With the increasing use of computer-aided simulation in almost all engineering fields, there is the need for building accurate and complex models to support the designer in optimization and verification of his choices. In many cases, the presence of coupling joints in a mechanical system is modelled with simple constraint equations, relating the kinematics between input and output shafts. For example, in the simulation of the front wheel system of a ground vehicle, a homokinetic joint is usually simulated by introducing a velocity constraint which imposes that rotations of the input and output shafts are the same. This approach is useful for building huge models, because it reduces the number of equations to be solved. On the other hand, it is not suitable for investigating local dynamic actions because computation of reaction forces is only approximated. Moreover, the real joints, due to inevitable assembling errors, may present irregularities in transmitting motion and variable load transfers which can be simulated only with accurate sub-modelling. The motivations of the presented study come from all these reasons.

The investigation on the effects of errors on both kinematics and dynamics of the joints is also useful to allocate the right manufacturing tolerances. In fact, the presence of mechanical errors may affect the performance of the mechanism, mining correct functioning, reducing the efficiency and reliability (Pezzuti *et al.*, 2006).

2. Dimensional and geometrical errors

The structural parts of the Tracta joint are manufactured with a common technological process. For this reason, the achieved precision in the coupling depends on geometrical and dimensional tolerances related to these operations.

The first step through the investigation of the influence of these errors is about the choice of the functional features of each part. The kinematic and dynamic behaviour of the joint is supposed to depend on these features. A dimensional or geometrical error which affects them, may cause an important modification of joint performance, mining correct functioning. According to the observation addressed in the introduction, one of the most relevant properties of the Tracta coupling is the homokineticity. Since this condition is obtained thanks to precise placement of the two shafts, all the features which constrain the relative position between them can be defined as functional. In particular, it is important to fulfil the condition $OC_1 = OC_2$ (see Fig. 3). This requirement depends on the dimensional error a which affects the position of the lateral surface of the bearing housing with respect to the centre of the joint (see Fig. 4).

Another important error is about the misalignment between the axis of shafts and the axis of its revolute joint at the fork. In the ideal condition, the two axes are perpendicular. Errors may affect this requirement producing an angle of misalignment (β, see Fig. 5).

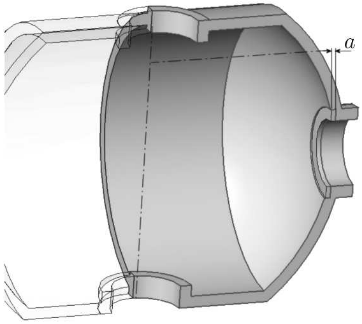

Fig. 4. Position error of the lateral surface of the bearing housing

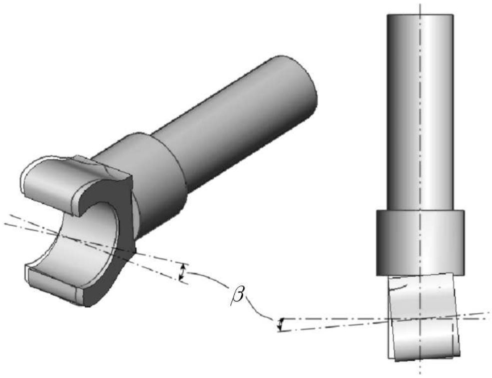

Fig. 5. Angular error of the fork axis

The geometrical features of the intermediate links are also very important for correct functioning of the joint. The ideal condition is when the axis of the revolute joint is perpendicular to the axis of the planar slot. This requirement can not be fully satisfied in the case of planarity or angular error on the surface of the slot. This kind of error is more complex to study, because it affects two degrees of freedom of the surface. They are two possible rotations about two independent axes which are perpendicular to that of the revolute joint (Figs. 6 and 7).

In more general cases, the two rotations of the surface occur combined, producing a lot of possible imprecise configurations (Fig. 8).

Table 1 summarizes five possible errors which have been included in the investigation.

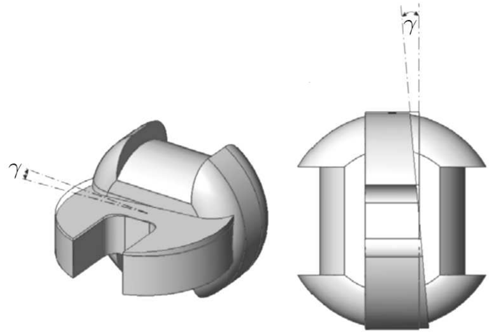

Fig. 6. First degree of freedom of the slot surface due to planarity or an angular error

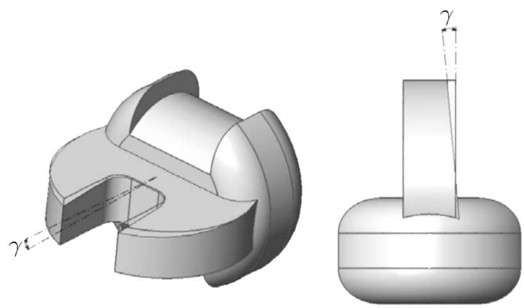

Fig. 7. Second degree of freedom of the slot surface due to planarity or an angular error

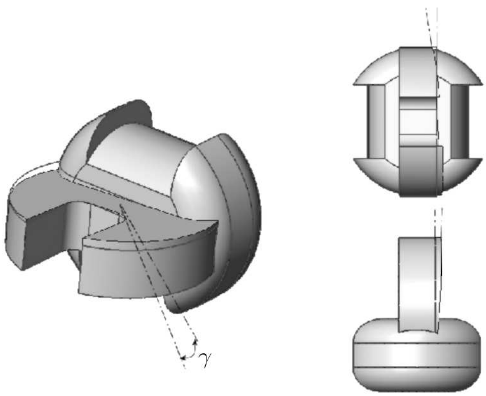

Fig. 8. General angular error combines the effects of the two degrees of freedom

Table 1. Investigated errors

Definition	Error	Type
E1	Misfit of the lateral surface of the bearing housing	linear − α
E2	Misalignment of the fork axis	angular − α
E3	Misalignment of the plane of the slot surface (rotation of the plane about the shaft axis)	angular − γ
E4	Misalignment of the plane of the slot surface (rotation of the plane about an axis which is perpendicular to that of the shaft)	angular − γ
E5	Combined error E3+E4 (Misalignment of the plane of the slot surface (rotation of the plane about an axis which is 45° with respect to that of the shaft)	angular − γ

3. Multibody model

In order to investigate both kinematic and dynamic performance of the Tracta coupling, a parametric multibody model has been built. The assembly, depicted in Fig. 2, is made of 5 rigid bodies: the housing cage (considered as the ground), two shafts and two intermediate links. Four revolute joints and a coincident planar joint have been included. They are described with 22 constraint equations (Haug, 1988). Friction and lubrication effects have been neglected. A torque of 1 Nm has been applied to the output shaft and a constant angular velocity of 360°/s has been imposed to the input one. The mass of each body is 1 kg. The analysis has been performed starting from a three dimensional CAD model. Thanks to solid modelling, it has been possible to simulate the presence of errors modifying the geometrical features defined by dimensional parameters, configuring several simulation scenarios. The results of simulations are presented in the next sections.

4. Results and discussion

The kinematic and dynamic performance of the Tracta coupling has been investigated considering two possible misalignments between the shafts: 10°

and 20°. Six error configurations have been taken into account: the ideal condition (without errors) and the five cases (E1,...,E5) shown in Table 1.

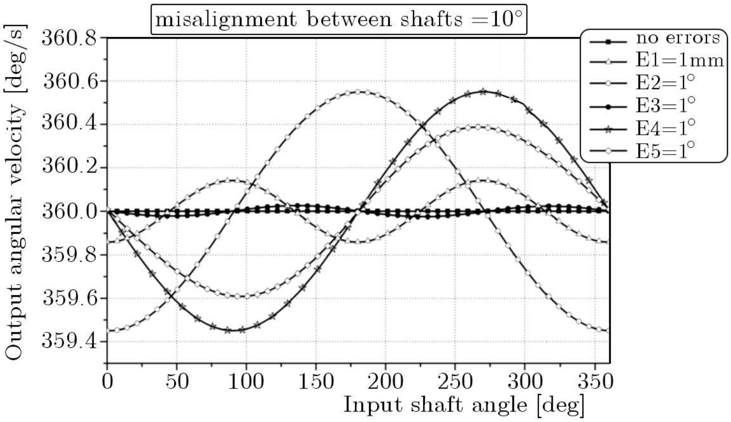

Fig. 9. Kinematic irregularities due to errors when the misalignment between the two shafts is 10°

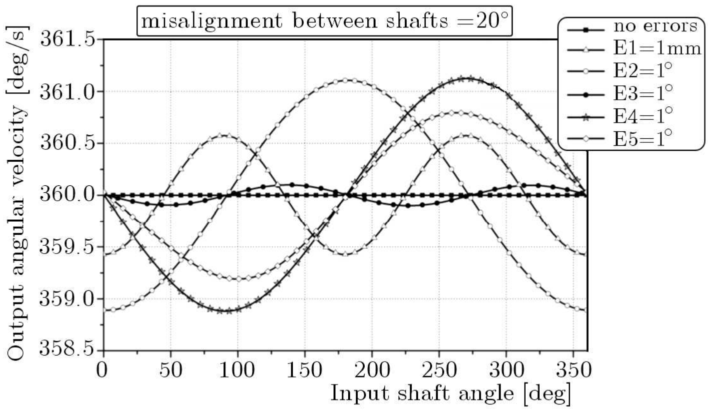

Fig. 10. Kinematic irregularities due to errors when the misalignment between the two shafts is 20°

Concerning the kinematics, Figures 9 and 10 show the computed angular velocity irregularities of the output shaft. In the ideal condition the joint is homokinetic and the output velocity is always equal to the input one. Due to the presence of errors several irregularities affect the joint. In some cases (presence of errors E2, E4) the irregularities occur with a frequency which is the same as that of rotation. With E1 and E3 errors, the irregularities occur with a frequency which is twice as much as the rotation. Error E5, that is a combination of E3 and E4, causes main irregularity with the same frequency of the shaft rotation and lower irregularity with a frequency that is twice of that of shaft rotation. With the same error amplitude (1°), E4 shows a lower irregularity magnitude. It means that the angular error on the slot plane causes irregularity whose amplitude and frequency depend on the direction of the misalignment. Its amplitude presents the maximum value when the error occurs with rotation of the plane about an axis which is perpendicular to that of the corresponding shaft. Considering medium manufacturing precision, according to ISO 2768, the angular errors affect the kinematics more than the linear ones.

When the shafts are more misaligned, the irregularities increase, keeping the same trend. For a precise comparison, a dimensionless kinematic irregularity factor (KIF) can be defined as

$$KIF\,[\%] = \frac{\omega_{out\,max} - \omega_{in}}{\omega_{in}} \cdot 100 \tag{4.1}$$

where $\omega_{out\,max}$ is the maximum amplitude of the irregularity on the output shaft angular velocity and ω_{in} is the input shaft angular velocity. Table 2 summarizes the KIFs for different errors and different misalignments between the shafts.

Table 2. KIFs due to errors

Misalignment between shafts	E1 = 1 mm	E2 = 1°	E3 = 1°	E4 = 1°	E5 = 1°
10°	0.04%	0.15%	0.007%	0.15%	0.11%
20°	0.16%	0.30%	0.027%	0.30%	0.21%

Concerning the dynamics, the presence of errors causes variation of the amplitude of reaction forces of joints. Figures 11 and 12 show plots of the magnitude of radial reaction forces of the revolute joint of the input shaft. It can be noted that even the ideal condition without errors presents a periodic trend of the force with a frequency which is twice the input shaft rotation. The presence of errors amplifies these irregularities, keeping the same frequency. Error E2 seems to produce higher dynamic irregularity, while the errors on planar slot have negligible effects on joint dynamics.

In order to compare their effects on the joint performance, a dimensionless dynamic irregularity factor (DIF) can be defined as

$$DIF\,[\%] = \frac{F_{error\,max} - F_{ideal\,max}}{F_{ideal\,max}} \cdot 100 \tag{4.2}$$

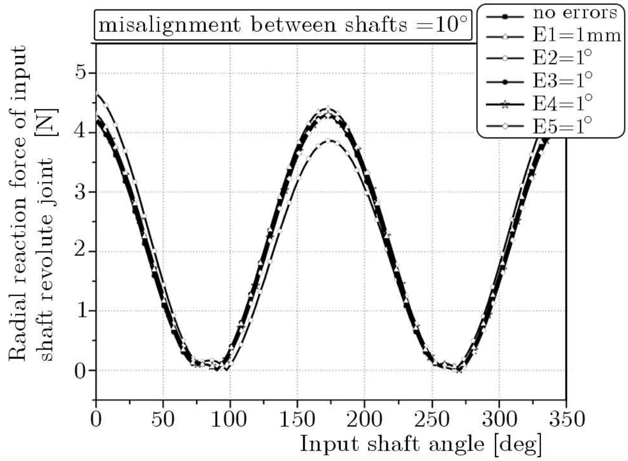

Fig. 11. Irregularities of the radial force of the input shaft revolute joint due to errors for the misalignment between shafts 10°

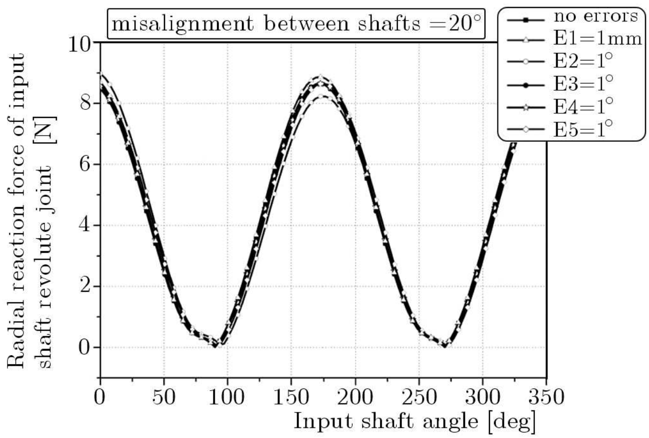

Fig. 12. Irregularities of the radial force of the input shaft revolute joint due to errors for the misalignment between shafts 20°

where $F_{error\,max}$ is the maximum amplitude of the reaction force computed in the presence of errors and the $F_{ideal\,max}$ is the maximum amplitude of the reaction force computed in the ideal model (without errors). The computed *DIF*s for the radial reaction force of the input shaft revolute joint are reported in Table 3.

Figures 13 and 14 show the axial thrust at the input shaft revolute joints. These axial reaction forces show similar behaviour with respect to the radial ones. Even the ideal condition without errors causes a periodic trend of

Table 3. *DIF*s of the radial reaction force of the input shaft revolute joint

Misalignment between shafts	E1 = 1 mm	E2 = 1°	E3 = 1°	E4 = 1°	E5 = 1°
10°	2.40%	11.54%	0.23%	0.09%	0.10%
20°	2.50%	6.67%	0.23%	0.10%	0.12%

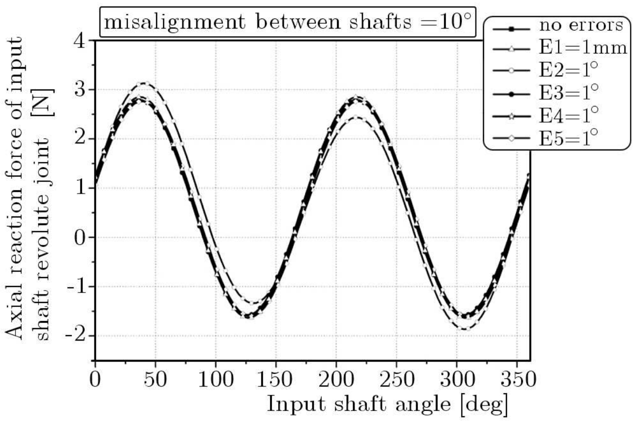

Fig. 13. Irregularities of the axial force of the input shaft revolute joint due to errors for the misalignment between shafts 10°

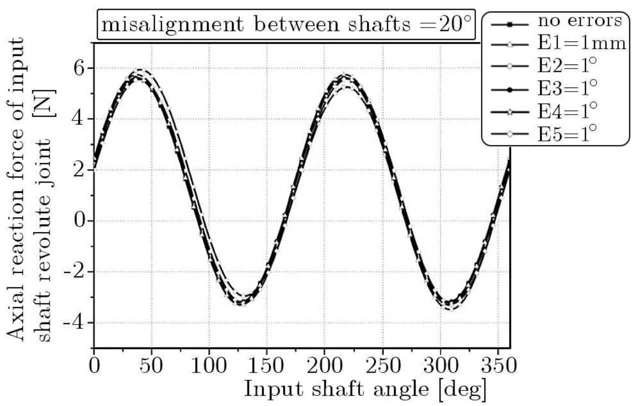

Fig. 14. Irregularities of the axial force of the input shaft revolute joint due to errors for the misalignment between shafts 20°

the axial force with a frequency which is twice the input shaft rotation. The presence of errors amplifies these irregularities, keeping the same frequency. E2 error seems to produce higher dynamic irregularity, while E3 and E4 show a negligible effect on joint dynamics, and their irregularities are not affected by the angular misalignment between shafts. The dynamic irregularities factors (*DIF*s) for the axial reaction forces are reported in Table 4.

Table 4. *DIF*s of the axial reaction force of the input shaft revolute joint

Misalignment between shafts	E1 = 1 mm	E2 = 1°	E3 = 1°	E4 = 1°	E5 = 1°
10°	2.40%	12.58%	0.57%	0.36%	0.72%
20°	2.50%	6.25%	0.56%	0.36%	0.71%

A similar trend concerning the dynamic irregularities can be observed also for the revolute joint of the output shaft (both radial and axial reaction forces).

5. Design charts for tolerance allocation

The results discussed in the previous section are about the presence of errors with a specific magnitude (1 millimetre for the linear dimension and 1 degree for the angular ones). In order to guide the designer to allocate the right tolerance, the influence of their amplitude has to be investigated too. The scientific literature includes works where linear sensitivity coefficients have been introduced (Valentini, in press). It means that the presence of an error influences the kinematic and dynamic performances in a linear way. It is a useful simplification when the closure loop equations are deduced algebraically. The modelling of the Tracta joint using multibody techniques allows one to study in depth the influence of errors and to investigate their nonlinear effects. For this purpose, the investigations discussed in the previous section have been extended to include variable errors (from 0.5 to 2 mm and from 0.5° to 3°) and different misalignments between shafts (from 5° to 20°). The results have been shown in the following Figures, collected as design charts. In order to compare them, the *KIF*s and the *DIF*s have been shown. As it can be noted from Figs. 15-20, the *KIF*s and *DIF*s shows a slightly non linear trend with respect to the increase of error amplitude. This nonlinearity is more evident with the increase of angular misalignment between shafts. The reaction forces of revolute joints of the input and output shafts are quite insensitive to the amplitude of E3, E4

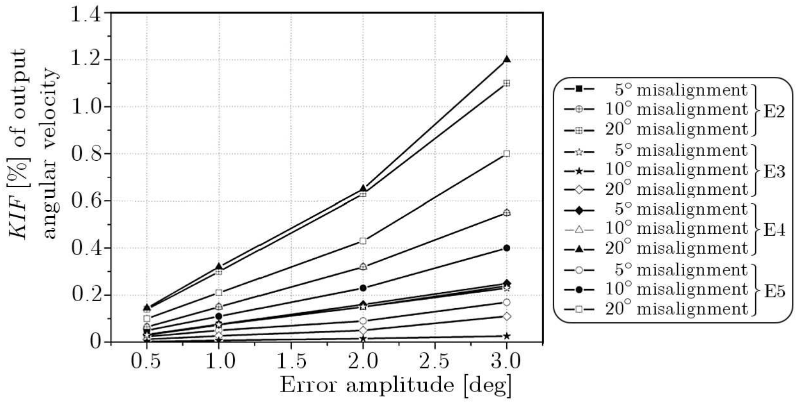

Fig. 15. Design chart of the angular errors *KIF*s of the output shaft angular velocity

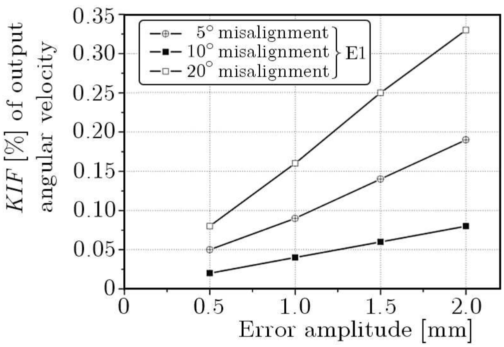

Fig. 16. Design chart of the linear error *KIF*s of output shaft angular velocity

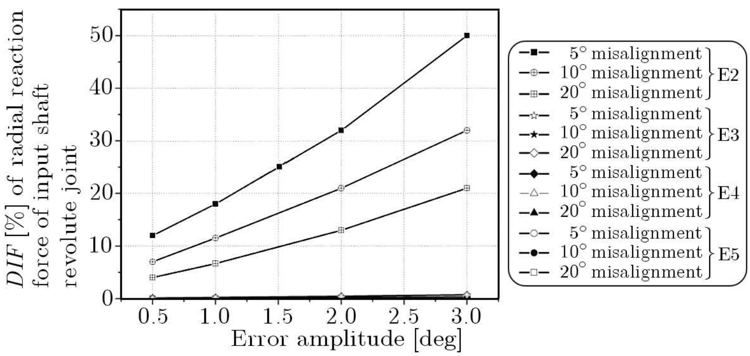

Fig. 17. Design chart of the angular errors *DIF*s of the radial reaction force of the input shaft revolute joint

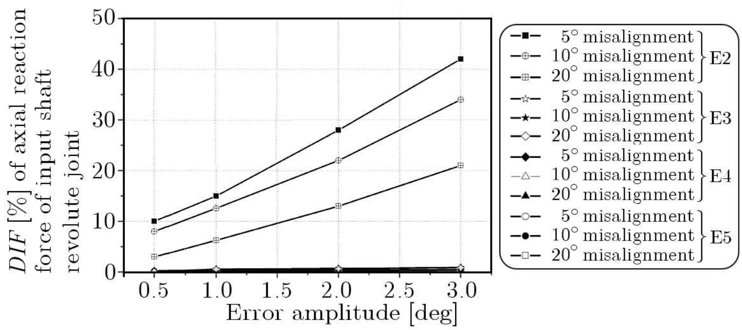

Fig. 18. Design chart of the angular errors *DIF*s on the axial reaction force of the input shaft revolute joint

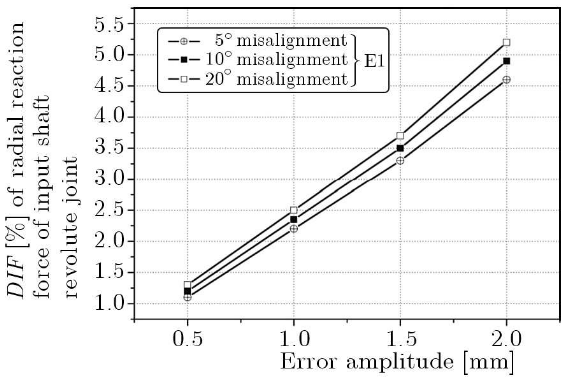

Fig. 19. Design chart of the linear errors *DIF*s on the radial reaction force of the input shaft revolute joint

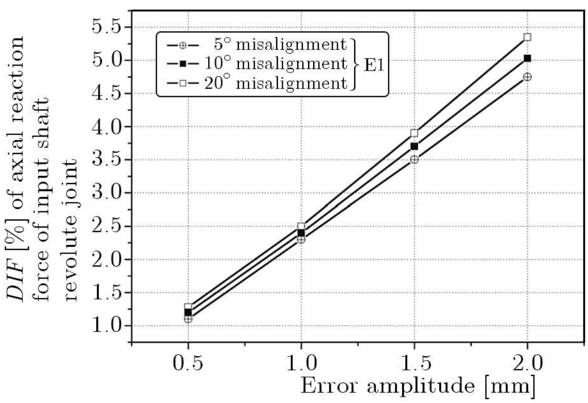

Fig. 20. Design chart of the linear errors *DIF*s of the axial reaction force of the input shaft revolute joint

and E5 errors and to the misalignment between shafts (Figs. 17 and 18). The presence of E2 error affects both kinematic and dynamics in a considerable way. Planar errors on the central slot affect more the kinematic performance than the dynamic one. The dynamic irregularities caused by the presence of linear error E1 are slightly influenced by the misalignment between shafts.

6. Conclusion

In this paper, an investigation about the influence of geometrical and dimensional errors on the kinematic and dynamic performance of the Tracta joint has been presented. Due to its plain architecture, the Tracta joint is one of the simplest homokinetic coupling. On the other hand, its usage is often limited by the influence of construction errors on its performances. In this paper a multibody virtual model has been proposed. The presence of one linear error and four different angular errors has been simulated. The influence of these errors has been computed and summarized in design charts. The results show that some errors cause kinematic irregularities with a frequency that is equal or twice of that of the input shaft rotation. This information is useful to address possible vibration phenomena. The presence of errors influences reaction forces, too, by amplifying their magnitude without modifying the frequency which is always twice that of the input shaft rotation. The correct assessment of internal loads is useful to optimize the structural dimensioning, preventing damage and fatigue phenomena. The discussed model and simulations can be useful not only for analysing the presence of possible errors, but also for the synthesis and allocation of adequate tolerance for every feature. In this case, the designer has to find the admissible error combination that allows the performance of the joint to be within an acceptable range. Given a threshold on the irregularities, he has to find the maximum value of possible errors, using the presented design charts.

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Wpływ błędów geometrycznych i wymiarowych na kinematykę i dynamikę przegubu równobieżnego typu Tracta

Streszczenie

W pracy przedstawiono badania wpływu błędów geometrycznych i wymiarowych na pracę przegubu typu Tracta. To jedno z najprostszych rozwiązań konstrukcyjnych przegubu równobieżnego, który szeroko stosowany jest w przednich osiach napędowych samochodów osobowych, pojazdach wojskowych z napędem na wszystkie osie, ciężkich pojazdach i wielu innych maszynach. Przedstawione badania oparto na symulacjach wielobryłowego i przestrzennego modelu przegubu. Przeanalizowano właściwości kinematyczne i dynamiczne modelu uwzględniającego pięć różnych błędów

geometrycznych odnoszących się do wymiarów liniowych i kątowych. Wrażliwość modelu na błędy opisano za pomocą charakterystyk kinematycznych i dynamicznych układu w funkcji wielkości tych odchyłek. Otrzymane wykresy mogą stanowić podstawę do optymalizacji obszaru tolerancji w projektowaniu przegubu Tracta oraz być pomocne przy właściwej symulacji zachowania się pojazdów wyposażonych w takie przeguby i opisanych modelem omówionym w pracy.

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