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THE INFLUENCE OF LOADING PROGRAM ON THE COURSE OF FATIGUE DAMAGE CUMULATION

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In the paper, results of a comparative analysis of cyclic properties of specimens made of 30HGSA steel under constant-amplitude and programmed loading were presented. The analysis was carried out with the use of parameters of the characteristic hysteresis loop in function of the degree of fatigue damage. The analysis showed that courses of cyclic properties changes for chosen strain levels are very similar and do not depend on the loading program. It was shown that during damage cumulation it is possible to take into account the changes of cyclic properties.

Key words: fatigue life, low cycle fatigue, damage accumulation, programmed loading

1. Introduction

Fatigue life calculations of construction elements are inseparably connected with the problem of the fatigue damage cumulation. Since 1924 till the present day, there have appeared about 40 various fatigue damage cumulation hypotheses. A comparative analysis of the fatigue damage cumulation hypotheses was performed among others in the papers by Manson and Halford (1986), Fatemi and Yang (1998), Szala (1998). The oldest one is the Palmgren-Miner linear hypothesis (Palmgren, 1924; Miner, 1945). According to the Palmgren-Miner rule, fatigue damage cumulation can be performed with the use of various fatigue descriptions (stress, strain, energy). Analysis of particular fatigue descriptions allows one to conclude that the energy description of the fatigue is more complete than the stress or strain description. It takes into account the mutual interactions between stress and strain. For the energy description, numerous proposals of cumulation hypotheses, alternative to the linear hypothesis were formulated (Kujawski and Ellyin, 1984; Gołoś and Ellyin, 1988;

Leis, 1988; Duyi and Zhenlin, 2001; Mroziński and Topoliński, 1999). Despite good agreement of the experimental results with the fatigue life calculation ones obtained with the use of new hypotheses, they have not found widespread application in fatigue life analysis so far.

One of the reasons for lasting popularity of the linear hypothesis is its simplicity and still satisfactory agreement of the obtained calculation and test results, as far as the engineers', opinion is concerned. In the linear hypothesis, it is accepted that in the case of a constant-amplitude loading each loading cycle, independently of the phase of the fatigue process, contributes to damage in the same degree. For example, after carrying out n_i cycles of the constant-amplitude loading, the degree of fatigue damage can be determined from the relation

$$D_i = \frac{n_i}{N_i} \tag{1.1}$$

where n_i is the number of cycles of the constant-amplitude loading, N_i – number of cycles until failure at the given level of loading.

In the case of a multistep loading program, the failure will occur if the following condition is met

$$D_i = \lambda \sum_{i=1}^{k} \frac{n_i}{N_i} = 1 \tag{1.2}$$

where k denotes the number of steps in the program, λ – number of program iterations until failure.

The course of damage cumulation according to the linear hypothesis and found with the use of plastic strain energy $\varDelta W_{pl}$ dissipated in the material during one loading cycle as the criterion value, is shown in Fig. 1.

At A point, after carrying out n_{b1} cycles of the constant-amplitude loading with energy $\varDelta W_{pl3}$, the damage degree is

$$D_{i(s)} = \frac{n_{b1}}{N_3} \tag{1.3}$$

However, in the case of multistep loading, after carrying out $n_1 + n_2 + n_3$ cycles on the successive levels of the program, the degree of fatigue damage $D_{i(p)}$ at A point will be

$$D_{i(p)} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} \tag{1.4}$$

According to the linear hypothesis of damage cumulation, the damage degrees $D_{i(s)}$ and $D_{i(p)}$ at A point should be the same

$$D_{i(s)} = D_{i(p)} \tag{1.5}$$

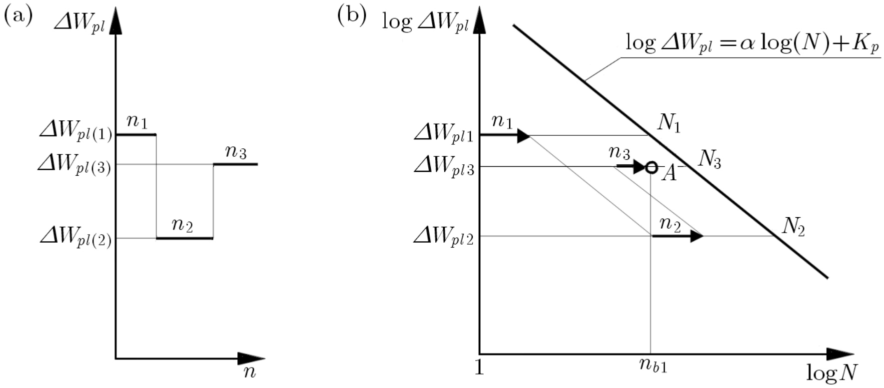


Fig. 1. Damage cumulation according to the Palmgren-Miner hypothesis in the energy approach: (a) loading program, (b) course of damage cumulation

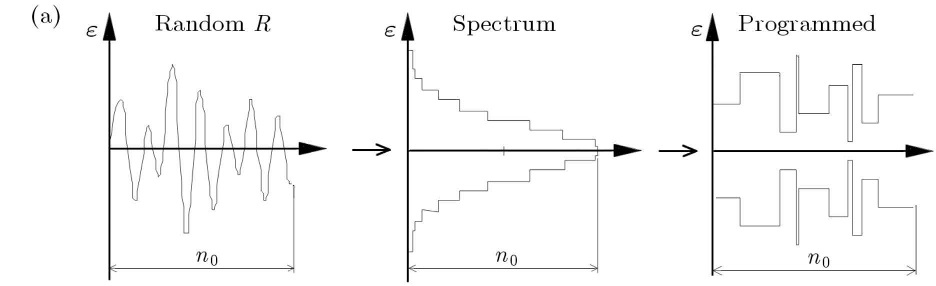
In the context of assumptions of the fatigue life calculation method, in which material data were determined in the low-cycle fatigue area, it should also mean the same criteria for ΔW_{pl} at A point after n_{b1} cycles of the constant-amplitude loading and after $n_1 + n_2 + n_3$ cycles of the programmed loading. Such a case was shown at the diagram in Fig. 1. In order to make it simpler, it was accepted that during the tests changes of the criterion value ΔW_{pl} at individual levels of the loading program are not observed. Such a case, however, takes place only when the energy ΔW_{pl} is the controlling value during the test ($\Delta W_{pl} = \text{const}$). Because of problems concerning the strength machines control, such tests are hardly ever performed, however. They were presented, for example, in the papers by Boroński and Mroziński (2007), Kasprzyczak and Macha (2006).

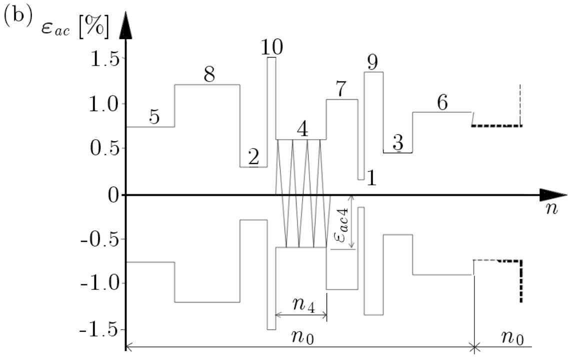
Most often, the energy ΔW_{pl} is the resultant value calculated after realisation of tests under controlled stress ($\sigma_a = \text{const}$) or strain ($\varepsilon_a = \text{const}$, $\varepsilon_{ap} = \text{const}$). As a result of changes of cyclic properties (weakening or hardening of the material) there occur changes of the criterion value ΔW_{pl} in function of the number of loading cycles at individual levels of the constant-amplitude loading (Koh, 2002; Li *et al.*, 1997; Mroziński, 2008). The changes of cyclic properties may be one of the reasons of the discrepancy between fatigue life calculations and experimental results if they are not considered in the calculations.

The basic aim of this paper is valuation of the possibility of taking into account changes of cyclic properties of the material during fatigue damage cumulation obtained with the use of the linear method. An additional aim is to determine the influence of the loading program on the course of damage cumulation.

2. Tests description

Low-cycle fatigue tests were carried out under constant-amplitude and programmed loadings. Constant-amplitude loadings were applied at five levels of total strain ($\varepsilon_{ac} = 0.5$, 0.6, 0.8, 1.0, 1.2%). The tests were performed under controlled strain ($\varepsilon_{ac} = \text{const}$) according to the guidelines defined in the standard (ASTM E606-92). Programmed loadings were set in form of repeated blocks with an irregular sequence of steps. The block of programmed loading was obtained on the base of schematization of the loading with a random sequence of cycles (Fig. 2a).





Level	ε_{ac} [%]	n_i	other
1	0.15	2	$n_0 = 100$, $\zeta = 0.56$
2	0.3	8	
3	0.45	9	
4	0.6	15	
5	0.75	14	
6	0.9	17	
7	1.05	9	
8	1.2	19	
9	1.35	5	
10	1.5	2	

Fig. 2. Loading programs: (a) methodology of program elaboration, (b) loading program parameters

Each cycle in the block of programmed loading (Fig. 2b) was an oscillatory cycle ($R = -1$). The loading program was described with the value of maximum total strain amplitude $\varepsilon_{ac\,max}$ and with the coefficient of spectrum density ζ. For the used loading program, the applied values of these parameters were: $\zeta = 0.56$ and $\varepsilon_{ac\,max} = 1.5\%$. The values of strain amplitudes ε_{ac} and numbers of cycles on each step of the program are presented in Fig. 2b. The specimens used in the fatigue tests were made of 30HGSA alloy steel according to the ASTM standard [1]. The strength parameters of 30HGSA steel are: $R_m = 1030\,\text{MPa}$, $E = 207000\,\text{MPa}$, $A_5 = 9.5\%$. A general view to the applied specimen is presented in Fig. 3

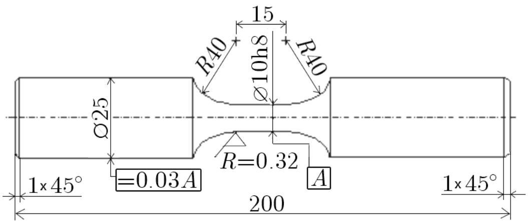


Fig. 3. The specimen used in the tests

The fatigue tests were performed with the use of the Instron 8501 strength machine. During the tests, a constant growth rate of relative strain for the measuring part equal to 1%/s was accepted. The controlling parameter both during the programmed and constant-amplitude loading was the total strain of the measuring part obtained with the use of an extensometer. During the tests under the constant-amplitude loading momentary values of the loading force and strain for chosen loading cycles were recorded. In the case of programmed loading, values of these parameters for the whole loading blocks (100 cycles) were recorded.

3. Test results

3.1. Constant-amplitude loading

Momentary values of the loading force and strain recorded during the tests at respective strain ε_{ac} levels were used for calculation of hysteresis loop parameters, i.e. stress amplitude σ_a, plastic strain amplitude ε_{ap} and plastic strain energy ΔW_{pl} described with the hysteresis loop area. In Fig. 4, examplary diagrams of changes of these parameters in function of the loading cycles number are shown.

Basing on the courses of parameters σ_a, ε_{ap} (Fig. 4a and 4b), it was found that the steel applied in the tests undergoes cyclic weakening. Confirmation of that fact is gradual (in function of the number of cycles) decreasing the stress amplitude σ_a and increasing the plastic strain amplitude with a constant level of the total strain amplitude ε_{ac}. The weakening refers to all strain levels.

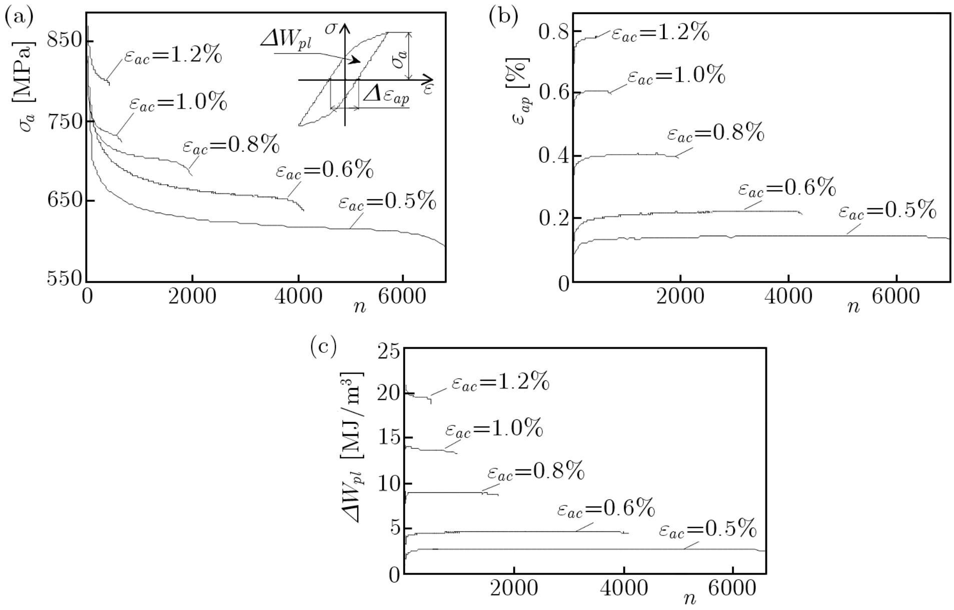


Fig. 4. Hysteresis loop parameters for the constant-amplitude loading: (a) σ_a, (b) ε_{ap}, (c) ΔW_{pl}

The result of mutual stress and strain interactions which occur in the energy description is that in the case of specimen made of 30HGSA steel this description not always reflects cyclic properties observed with the use of stress or strain description. Basing on the analysis of the courses of energy changes ΔW_{pl} (Fig. 4c), it can be found that cyclic properties of 30HGSA steel in the energy approach depend on the level of total strain. For the levels of $\varepsilon_{ac} = 0.5\%$, $\varepsilon_{ac} = 0.6\%$ and $\varepsilon_{ac} = 0.8\%$, the energy ΔW_{pl} slightly increases with the number of loading cycles, which indicates cyclic weakening of the material. For the remaining two strain levels, the ΔW_{pl} energy decreases, which indicates slight hardening of the tested steel.

Basing on the test results, a fatigue diagram in the bilogarithmic coordinates system: plastic strain energy ΔW_{pl} – number of cycles until failure N was made. The fatigue diagram was approximated with a straight line described with the equation

$$\log \Delta W_{pl} = \alpha \log(N) + K_p \tag{3.1}$$

The values of energy ΔW_{pl} obtained at five strain levels during the last realised cycle before fatigue failure were approximated with the diagram line.

3.2. Programmed loading

Similarly like during the constant-amplitude loading, for the recorded succeeding blocks of loading, values of the basic hysteresis loop parameters, i.e. stress amplitude σ_a, plastic strain amplitude ε_{ap} and plastic strain energy ΔW_{pl} were defined. In Fig. 5, examplary diagrams of stress σ_a changes in chosen blocks of the programmed loading realised in various periods of fatigue life are shown.

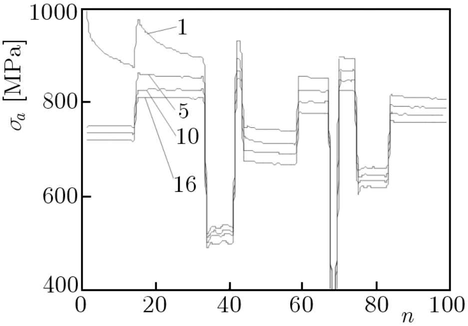


Fig. 5. Changes of stress σ_a in the block of loading program

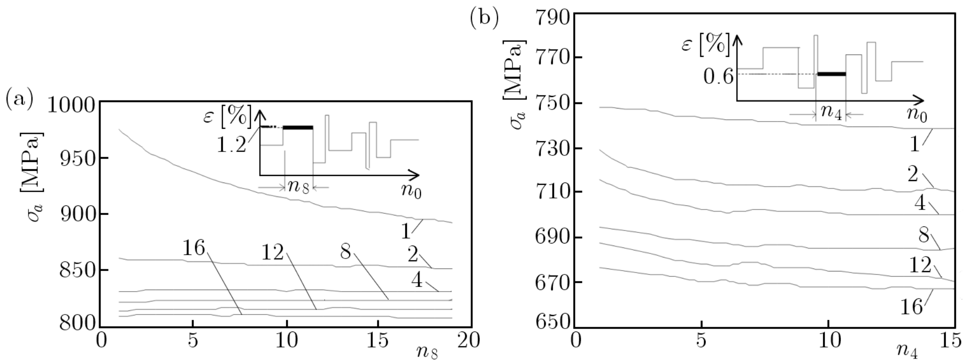


Fig. 6. Changes of stress σ_a during programmed loading: (a) at the step with amplitude $\varepsilon_{ac} = 0.6\%$, (b) at the step with amplitude $\varepsilon_{ac} = 1.2\%$

Basing on the analysis of the course of stress σ_a in the succeeding blocks of loading, it can be found that independently of the loading level, 30HGSA steel also undergoes cyclic weakening. This is proved by the decreasing stress σ_a on the same steps in succeeding block iterations of the loading program. In the paper, detailed analysis of the courses of σ_a, ε_{ap} and ΔW_{pl} changes at individual steps of the realised programs was carried out. Because of the limited volume of this paper, the presentation of the obtained results is limited only to changes of the stress amplitude σ_a and two steps of the program ($\varepsilon_{ac} = 0.6\%$ and 1.2% – Fig. 6).

Analysis of diagrams presented in Fig. 6 shows that changes of σ_a at individual steps depend in little degree on the loading program. Changes of the strain amplitude from lower to higher ones and vice versa lead to momentary weakening of the material at the succeeding step and the occurence of a new level of momentary stress stabilization σ_{as}. This stress is lower than stress stabilization obtained at the given step in the former block of loading program.

4. Analysis of test results

A comparative analysis of the course of changes of the basic hysteresis loop parameters under the constant-amplitude and programmed loading was carried out in function of the fatigue damage degree D_i. The analysis referred to these levels of strain ε_{ac} which were realised during the constant-amplitude and programmed tests ($\varepsilon_{ac} = 0.6\%$ and $\varepsilon_{ac} = 1.2\%$). Values of the obtained plastic strain energy $\Delta W_{pl(i)}$ in succeeding cycles of the constant-amplitude and programmed loading enabled one to define from the equation of fatigue diagram (6), the corresponding numbers of cycles until specimen failure N_i, and then to carry out damage cumulation according to relations (1.1) and (1.2) for succeeding cycles of the constant-amplitude and programmed loading. The presented procedure during damage cumulation is explained in Fig. 7. An

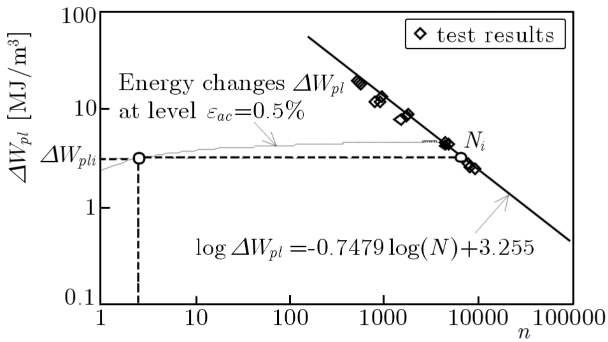


Fig. 7. Procedure during fatigue damage cumulation

exemplary course of energy changes at one strain level in function of the number of cycles and the fatigue diagram in the energy approach are presented there. The idea of the presented procedure is to connect the process of damage cumulation with the course of changes of cyclic properties.

The obtained calculation results are presented in form of diagrams of hysteresis loop parameters in function of the fatigue damage degree D_i. Examplary diagrams of changes of the loop parameters obtained at two strain levels ($\varepsilon_{ac} = 0.6\%$ and 1.2%) were shown in Fig. 8 and Fig. 9.

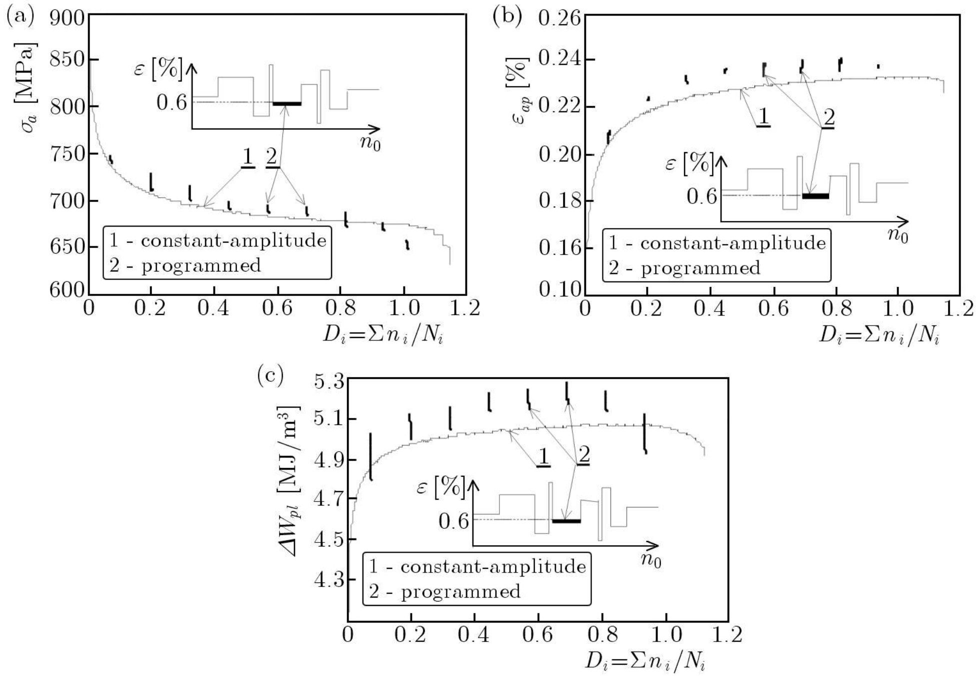


Fig. 8. Changes of σ_a (a) ε_{ap} (b) and $\varDelta W_{pl}$ (c) under the constant-amplitude and programmed loading at the strain level $\varepsilon_{ac} = 0.6\%$

Basing on the comparative analysis of diagrams presented in Figs. 8 and 9, one can observe the qualitative and quantitative similarity in the course of analysed parameters under the constant-amplitude and programmed loading. Independently of the kind of loading, momentary values of the hysteresis loop parameters (σ_a, ε_{ap}, $\varDelta W_{pl}$) for the same steps of fatigue damage D_i are comparable. The hysteresis loop parameters obtained in the terminal step cycles of the programmed loading with amplitudes $\varepsilon_{ac} = 0.6\%$ and $\varepsilon_{ac} = 1.2\%$ reach a level comparable to that observed under the constant-amplitude loading for the same steps of damage.

Moreover, it results from the diagrams that in spite of the disturbances in the stabilization process due to strain changes at successive steps, the steel

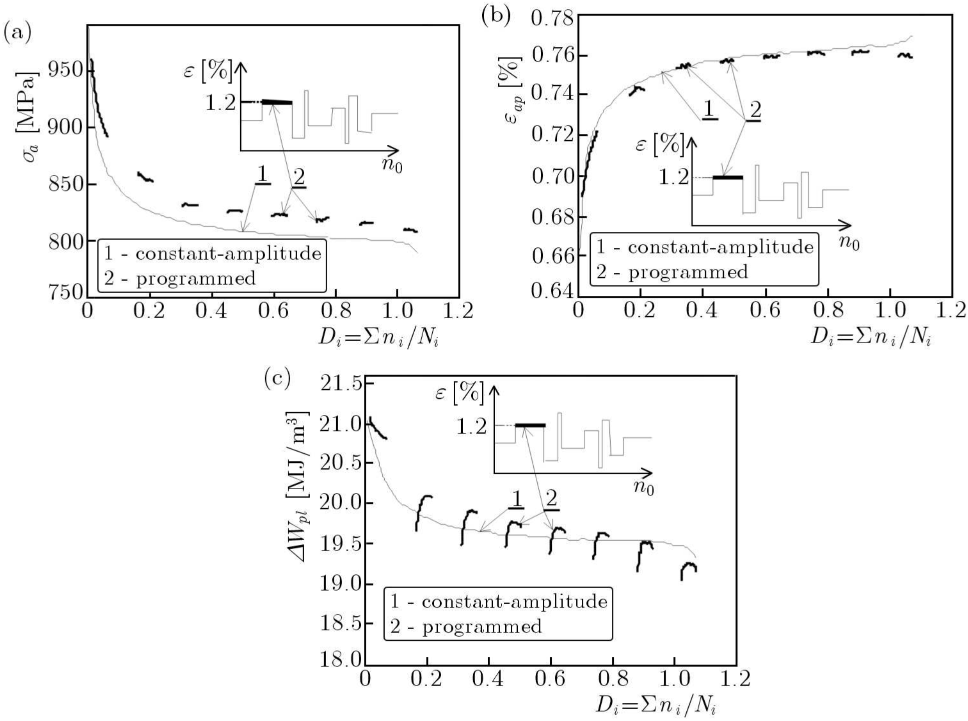


Fig. 9. Changes of σ_a (a) ε_{ap} (b) and ΔW_{pl} (c) under the constant-amplitude and programmed loading at the strain level $\varepsilon_{ac} = 1.2\%$

seems to remember the course of this process observed under the constant-amplitude loading. In the diagrams of changes of three parameters under the programmed loading, a trend of changes of cyclic properties is very clearly visible. It is similar to the course of changes of cyclic properties under the constant-amplitude loading. The above observation has capital practical consequences since it shows the possibility of predicting the course of changes of cyclic properties of a material under operating loading on the basis of known course under the constant-amplitude loading. The above mentioned conclusion confirms the test results presented by Mroziński (2008).

Basing on the determined diagrams, it can also be found that the degree of fatigue damage for the moment of failure in little degree depends on the loading program. The values of the total damage D_i close to one for both loading programs (constant-amplitude and programmed) are a proof of good agreement between the fatigue life found in calculations and experimentally.

5. Conclusions

The carried out analysis of the obtained experimental results allows one to formulate the following conclusions:

- Both under the programmed block loading and constant-amplitude loading of the specimens made of the alloy steel no period of cyclic properties stabilization was observed. Having considered this problem, some doubts concerning the results of fatigue life calculations based uppon the constant material data determined during the tests under constant-amplitude loading have arisen.
- The process of cyclic weakening which proceeds under a constant-amplitude and programmed loading of 30HGSA steel analysed with the use of such hysteresis loop parameters as σ_a, ε_{ap} and ΔW_{pl} showed qualitative similarity concerning the nature of changes of cyclic properties and quantitative similarity concerning momentary values of the mentioned parameters for the same degree of fatigue damage.
- Connection of the changes of cyclic properties with the process of damage cumulation may lead to improvement of the agreement between analytical and experimental results. Such an approach to the problem of fatigue life calculation can be of special importance in the case of fatigue life determination of construction elements made of materials characterised with the lack of stabilization period (aluminium and copper alloys). Realisation of such an approach in fatigue life calculations is possible if the courses of cyclic properties changes in function of the fatigue damage degree are known. The proposal of a method of determination of the course of cyclic properties changes under a constant-amplitude loading was presented by Mroziński (2008).

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Wpływ programu obciążenia na przebieg kumulacji uszkodzeń zmęczeniowych

Streszczenie

W pracy przedstawiono wyniki analizy porównawczej właściwości cyklicznych próbek ze stali 30 HGSA w warunkach obciążenia stałoamplitudowego i programowanego. Analizę prowadzono z wykorzystaniem charakterystycznych parametrów pętli histerezy w funkcji stopnia uszkodzenia zmęczeniowego. Przeprowadzona analiza wykazała, że przebiegi zmian właściwości cyklicznych na wybranych poziomach odkształcenia są bardzo podobne i nie zależą od programu obciążenia. W pracy wykazano, że podczas sumowania uszkodzeń istnieje możliwość uwzględniania zmian własności cyklicznych.

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