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APPLICATION OF ETR FOR DIAGNOSIS OF DAMAGE IN STEEL-CONCRETE COMPOSITE BEAMS

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The paper presents how ETR (energy transfer ratio) changes for a steel-concrete composite beam depending on the degree of damage. A numerical model of composite beam was constructed on the basis of research conducted on a real beam. The study presents how the frequency of natural vibrations, the damping ratio and ETR alter due to changes in the structure of the beam. The performed studies show that ETR is the most sensitive parameter to damage taking place in the beam. Consequently, this parameter can be used to diagnose damage in this kind of beams.

Key words: steel-concrete composite beams, nonproportional damping, energy transfer ratio (ETR)

1. Introduction

Modal analysis is ever more often used to detect damage in engineering structures. An early detection and ability to locate damage is particularly important in bridge structures. Steel-concrete composite beams are often used as main carrying girders in bridge constructions. This kind of beams is composed of a steel girder and a reinforced concrete slab placed onto it. This connection is made by welding steel elements to the top flange of a girder. These elements will be later embedded in concrete thus creating a fixed connection between the reinforced concrete slab and steel girder.

The observed increase of road transport intensity makes it necessary to monitor the technical condition of the existing bridge constructions. Particular attention should be paid to both static and dynamic properties of a structure. The applied modal analysis can be used to detect damage. An observed change of some selected modal parameters can be the evidence of damage which is developing in the structure. Early detection of defects allows one to undertake preventive steps, thus to avoiding risk of having to close off the construction in question. Traditional modal parameters, such as the frequency and modes of natural vibrations and their respective damping ratios, are not sensitive enough to some kinds of damage.

The aim of the present study was to apply the Energy Transfer Ratio – ETR to diagnostics of defects in steel-concrete connections of composite beams. ETR is a modal parameter defined by Liang and Lee (1991). According to the authors, it is much more sensitive to appearing damage than the traditionally defined modal parameters. The results of experiments conducted on a real composite beam have been used in this study to analyse and validate the effectiveness of ETR.

2. The state-of-the-art analysis

Commonly applied modal analysis is a technique of testing dynamic properties of a structure. The conducted analysis yields a modal model comprising a set of frequencies and modes of natural vibrations as well as damping ratios. Modal analysis is ever more often used to diagnose the technical condition of structures. This approach can be seen in the studies into steel-concrete composite beams carried out by the team of scientists from the University of Udine, Italy. A paper published in 2 parts by Dilena and Morassi (2003) and by Morassi and Rochetto (2003) presents the results of investigations conducted on composite beams with damage in the steel-concrete interface. The analysed beam had no supports and it was hung on four flexible ropes. The investigated damage was introduced at one of the free ends of the beams. The authors focused on the analysis of changes taking place in the frequency vibrations. Consequently, they obtained results, according to which longitudinal/axial vibrations of the beam were not very sensitive to damage in the connections. In contrast, in the case of flexural vibrations, the differences amounted to 38%.

Liang and Lee (1991) defined a new modal parameter – Energy Transfer Ratio – which describes the amount of energy transferred between various modes of vibrations. Their definition of ETR says that it is the ratio of modal energy transferred during a cycle to the total energy stored in the structure prior to the cycle. Energy transferred between various modes of vibrations exists for non-proportionally damped systems, which are majority in the civil engineering, and for these kinds of systems the ETR is possible to be determined. Details concerning this coefficient will be presented later in this paper.

Theoretical analyses usually look into two kinds of damping: proportional and non-proportional. Systems with proportional damping are extremely rare in the real world (it is an almost hypothetic case). However, scholars very often assume damping to be proportional. This is due to the fact that for proportionally damped systems and for those without damping vibration modes are identical and frequencies of natural vibrations are very similar. An assumption about proportional damping makes it possible to determine modal properties of a structure by analysing a system without damping, which is quite a simplification.

A system is proportionally damped if the following relation is met:
– according to Caughey and O'Kelley (1965)

$$\mathbf{C}\mathbf{M}^{-1}\mathbf{K} = \mathbf{K}\mathbf{M}^{-1}\mathbf{C} \tag{2.1}$$

– according to Ewins (1995)

$$\mathbf{C} = \beta\mathbf{K} \tag{2.2}$$

– according to Ewins (1995), Uhl (1997), Lee and Liang (1998)

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{2.3}$$

where: $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ – damping matrix, $\mathbf{K}$ – stiffness matrix. α and β are proportionality coefficients. Details concerning the issue of proportional damping can be found in the above mentioned studies.

In 1999 Liang and Lee were conducting research on a composite bridge model on a scale of 1:6, which consisted of a concrete slab resting on three single-span steel girders. They limited their study to two kinds of damage: 1) removal of the support at one end of the central girder, 2) crack of a stretched part of the girder in the middle span. On the basis of purely theoretical investigations and concrete assumptions, the authors determined a relation according to which ETR was 1000 times more sensitive to damage than frequency of natural vibrations. In order to validate their assumptions, they conducted experiments whose results are presented in Table 1.

The results presented there clearly show that ETR is a good identifier of changes taking place in beams both for the first and second kind of damage. ETR is much more sensitive to changes in a structure than any other parameter. Unfortunately, the experiment conducted by Liang and Lee did not confirm their assumption that ETR is 1000 times more sensitive to damage than frequency of natural vibrations.

Table 1. Changes of modal parameters [%]

Mode number	Damage 1			Damage 2		
	Frequency	Damping ratio	ETR	Frequency	Damping ratio	ETR
1	4.1	89.5	309	2.7	7.3	117
2	3.8	1.3	30	1.0	15.2	243
3	0.1	7.4	1892	7.8	129.0	236

Wang and Zong in their papers published in 2002 and 2003 analysed ETR as well. The model made on a scale of 1:6 consisted of a concrete slab resting on four steel girders. Two kinds of artificial damage were introduced into the model: 1) the first was a simulated lack of the support; 2) the second was buckling and cracking of the steel girder. The authors, similarly to Liang and Lee, introduced a relation according to which ETR is 1000 times more sensitive to damage than frequency of natural vibrations. However, also this time they failed to confirm their theoretical assumptions. The sensitivity analysis for the investigated modal parameters is presented in Table 2.

Table 2. Sensitivity analysis for damages

Mode number	Damage 1			Damage 2		
	Frequency	Damping ratio	ETR	Frequency	Damping ratio	ETR
1	5.7	38.4	592.5	7.5	1.3	1614.7
2	8.4	74.0	81.7	2.8	67.3	820.2
3	6.3	123.4	4524.0	3.5	18.9	68.2

Having analysed Table 2, it can be concluded that:

- ETR is much more sensitive to introduced damage than either frequency of natural vibrations or corresponding damping ratios.
- The form of vibrations for which ETR showed the biggest changes is not the form for which the frequency vibrations and/or damping ratio reached their biggest differences both prior to and after the introduced damage, e.g. for the first kind of damage the biggest change in the frequency of natural vibrations occurred for the 2nd form, while for ETR the biggest change took place for the 3rd form.
- ETR shows much bigger changes for damage resulting from the missing support than for damage such as cracking of the girder.

As shown above, procedures aiming at diagnosing damage in steel-concrete composite beams have been applied for several years now. Scientists have been trying to determine how the analysed parameters change given defects recorded in their structure. The studies conducted so far seem to point out that ETR is the parameter which is the most sensitive to introduced damage. There are, however, some discrepancies between relations that were introduced in a purely theoretical way and empirical tests. All these facts seem to suggest the need of conducting ETR analysis of steel-concrete composite beams.

3. Theory of complex damping

In structural dynamics, the equilibrium of a vibrating n-DOF system can be given by a set of differential equations

$$\mathsf{M}\boldsymbol{X}'' + \mathsf{C}\boldsymbol{X}' + \mathsf{K}\boldsymbol{X} = \boldsymbol{F} \tag{3.1}$$

where M, C, K are the mass, damping and stiffness matrices $(n \times n)$, respectively, $\boldsymbol{X}''$, $\boldsymbol{X}'$, $\boldsymbol{X}$ – acceleration, velocity and displacement vectors $(n \times 1)$, $\boldsymbol{F}$ – vectot of the external forcing function.

According to Lee and Liang (1998), the corresponding homogenous equation of the system can be written as

$$\boldsymbol{Y}'' + \mathsf{D}\boldsymbol{Y}' + \boldsymbol{\Omega}^2\boldsymbol{Y} = \boldsymbol{0} \tag{3.2}$$

where

$$\begin{aligned}
\mathsf{D} &= \boldsymbol{Q}^\top \mathsf{M}^{-\frac{1}{2}} \mathsf{C} \mathsf{M}^{-\frac{1}{2}} \boldsymbol{Q} \\
\boldsymbol{Y} &= \boldsymbol{Q}^\top \mathsf{M}^{\frac{1}{2}} \boldsymbol{X} \\
\boldsymbol{\Omega}^2 &= \boldsymbol{Q}^\top \mathsf{M}^{-\frac{1}{2}} \mathsf{K} \mathsf{M}^{-\frac{1}{2}} \boldsymbol{Q} = \operatorname{diag}(\omega_{ni}^2)
\end{aligned}$$

In the above equation, $\boldsymbol{Q}$ stands for the eigenvector of the generalized stiffness matrix $\overline{\mathsf{K}} = \mathsf{M}^{-\frac{1}{2}}\mathsf{K}\mathsf{M}^{-\frac{1}{2}}$ and ω_{ni} is the natural frequency of the system without damping. The above system may have n modes if the D matrix is diagonal, which means that system is proportionally damped. If the matrix D cannot be diagonalized together with the matrix $\boldsymbol{\Omega}^2$, the system cannot be decoupled into n isolated modes. There is some energy transfer between those 'modes'. In such a case, the system is non-proportionally damped. This type of damping exists in most real structures. If a system is non-proportionally damped, the Caughey criterion described in equation (2.1) will not be satisfied, and also the matrix D will not be diagonal.

We can assume that the n-DOF system described by equation (3.2) has a mode shape matrix $\mathbf{P}$, which is complex because of the non-proportional damping. It has to be noticed that the mode shape matrix $\mathbf{P}$ of the system is no equal to the eigenvector $\boldsymbol{Q}$ of the generalized stiffness matrix. The i-th eigenvalue of the system λ_i is also called the complex frequency

$$\lambda_i = -\xi_i\omega_i \pm j\sqrt{1-\xi_i^2}\omega_i \tag{3.3}$$

where ξ_i is damping ratio and ω_i is natural frequency of the non-proportionally damped system. It should be noted that $\omega_i \neq \omega_{ni}$.

Now, we can rewrite equation (3.2) as follows

$$\boldsymbol{Q}^\top\mathbf{P}\boldsymbol{\Lambda}^2 + \boldsymbol{Q}^\top\overline{\mathbf{C}}\mathbf{P}\boldsymbol{\Lambda} + \boldsymbol{\Omega}^2\boldsymbol{Q}^\top\mathbf{P} = \mathbf{0} \tag{3.4}$$

where

$$\overline{\mathbf{C}} = \mathbf{M}^{-\frac{1}{2}}\mathbf{C}\mathbf{M}^{-\frac{1}{2}} \qquad\qquad \boldsymbol{\Lambda} = \operatorname{diag}(\lambda_i)$$

The ii-th entry of each matrix of equation (3.4) becomes

$$\lambda_i^2\boldsymbol{Q}_i^\top\mathbf{P}_i + \lambda_i\boldsymbol{Q}_i^\top\overline{\mathbf{C}}\mathbf{P}_i + \omega_{ni}^2\boldsymbol{Q}_i^\top\mathbf{P}_i = \mathbf{0} \tag{3.5}$$

The above equation can be rewritten to the form

$$\lambda_i^2 + \lambda_i\Big[\frac{\boldsymbol{Q}_i^\top\overline{\mathbf{C}}\mathbf{P}_i}{\boldsymbol{Q}_i^\top\mathbf{P}_i}\Big] + \omega_{ni}^2 = \mathbf{0} \tag{3.6}$$

The term in the square brackets is called the generalized Rayleigh quotient. According to Lee and Liang (1998), we can notice that

$$\frac{1}{2\omega_i}\Big[\frac{\boldsymbol{Q}_i^\top\overline{\mathbf{C}}\mathbf{P}_i}{\boldsymbol{Q}_i^\top\mathbf{P}_i}\Big] = \xi_i + j\zeta_i \tag{3.7}$$

The term ζ_i is called to be the i-th energy transfer ratio – ETR. It can be also proven that

$$\omega_i = \omega_{ni}\exp(\zeta_i) \tag{3.8}$$

Detailed information and derivations of the above equations are presented elsewhere, see Liang and Lee (1991), Lee and Liang (1998).

4. Composite beam

The beam cross-section is presented in Fig. 1. The beam measured 3200 mm in overall length (only the I-bar was 20 mm longer at both ends). The beam consisted of a rolled steel I-bar IPE 160 made of S235JRG2 steel and a reinforced concrete slab 60×600 mm in section size made of C25/30 concrete. The reinforcement rods were placed in two layers in the way shown in Fig. 1.

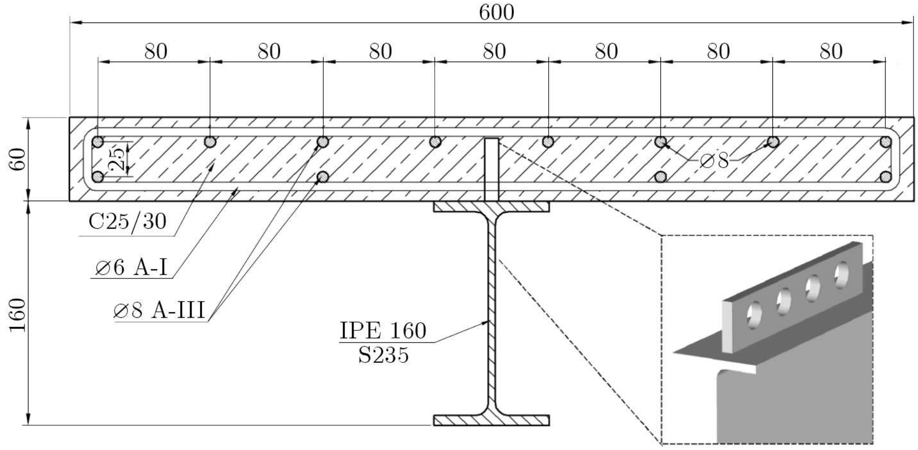

Fig. 1. Cross-section of the composite beam

Transverse reinforcement was made of rods, 6 mm in diameter, which were placed every 150 mm. The beam had a stiff connection consisting of perforated steel slats. The perforated slats are a new generation of connecting elements used in the bridge engineering. The slats were made from a flat bar, 10 mm thick, made from S235JRG2 steel. The distribution of connecting elements is shown in Fig. 2.

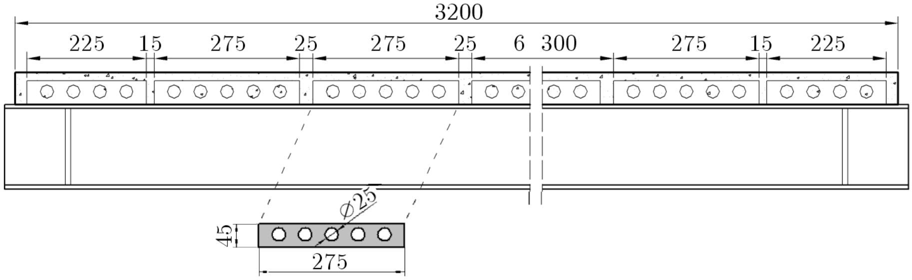

Fig. 2. Distribution of connecting elements

The program of experimental tests included realisation of the following operations: (i) initial static load tests; (ii) tests to determine basic dynamic characteristics; (iii) other additional tests (material tests).

During the initial static load tests, a simply supported beam was assumed. Some static load was applied in order to test the correctness of workmanship of the beams and to prepare them for dynamic tests. The obtained results showed no anomalies.

The tests whose aim was to determine basic dynamic characteristics, were conducted for a free beam. This kind of a beam was achieved by suspending the tested beam on two steel frames by means of four steel cables, 3 mm in diameter. A grid of measuring points and a system of coordinates used in the studies are presented in Fig. 3. The acceleration of vibrations was measured as the system response. Each beam was tested three times. In every test, the beam vibrations were excited in different points. An impulse excitation was used in the tests. The excitation points are marked in Fig. 3 as A, B and C. At the points A and B, the excitation was applied in the y-direction, whereas at the point C, in the x-direction.

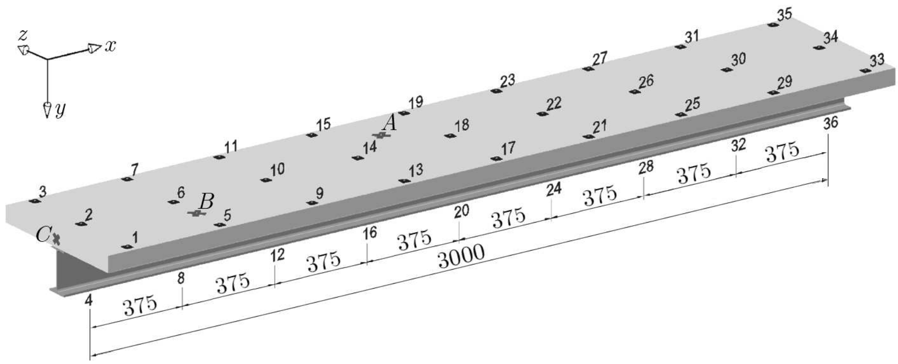

Fig. 3. A grid of measuring points

To process the results obtained in the dynamic tests, Time Domain MDOF ASM module implemented in CADA-X software package was used – LMS International (2000). This module uses a universal method of estimating parameters of a modal model LSCE/LSFD (Least Square Complex Exponential/Least Square Frequency Domain). The estimation of the model parameters in this method is global for all transfer functions. In order to determine frequencies of natural vibrations on the basis of the obtained transfer functions, SUM index was used. This index gives a normalized sum of amplitudes of the measured transfer functions at the selected measuring points and directions.

Table 3 contains vibration frequencies and values of their corresponding modal damping ratios for the first five flexural vibrations modes of the beam.

Table 3. Experimental natural vibration frequencies and their corresponding values of modal damping ratio

Frequency	1_{flex}	2_{flex}	3_{flex}	4_{flex}	5_{flex}
f_i [Hz]	77.75	186.65	304.72	417.57	531.13
ξ_i [%]	0.11	0.27	0.33	0.42	0.42

5. Rigid finite element model of the beam

The numerical model of the beam has been created in the convention of rigid finite element method – RFE model. The central idea of the method is the division of a real system into rigid bodies which are called rigid finite elements (RFE), which are then in turn connected by means of spring-damping elements (SDE).

For continuous parts of a structure it is customary to start creating a model with segmentation of a beam into equal or nearly equal segments. This segmentation is also called the primary division. A spring-damping element (SDE) is placed in the centre of gravity of every segment. This SDE is supposed to concentrate all spring and damping properties of a given segment. The next step is to connect SDEs created in the primary division by means of RFE. This is the so-called secondary division.

While modelling a composite beam, its steel and concrete parts should be treated separately. Figure 4a shows a composite beam with length L and divided into n segments of equal length ΔL (primary division). As it can be seen, two SDEs were placed in every segment of the beam resulting from the primary division: one concentrating the properties of the steel I-bar and another one concentrating the properties of the segment of a reinforced concrete slab. The RFEs modelling the beam (secondary division, Fig. 4b) were placed between SDEs. The initial and final RFEs are half the size of indirect RFEs of length ΔL. The SDEs, modelling the connection are the last elements of the model. These SDEs connect RFEs modelling steel and concrete parts of the composite beam.

Every RFE of a given number i has its own independent coordinate system $\widehat{X}_1^{(i)}, \widehat{X}_2^{(i)}, \widehat{X}_3^{(i)}$. The system is chosen so that it would overlap with the principal central axes of inertia of the given RFE. Having applied this assumption,

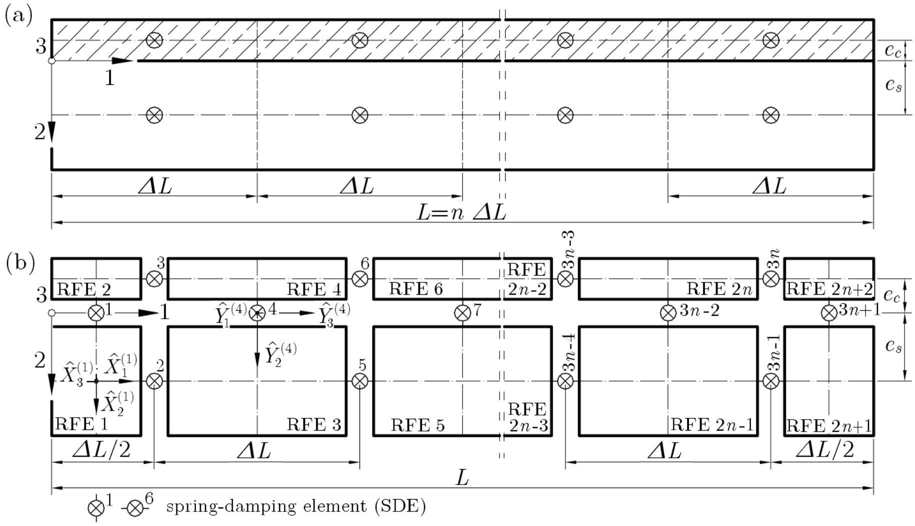

Fig. 4. RFE model: (a) segmentation of a beam into sections (primary division); (b) RFE system (secondary division)

the mass and moments of inertia are the only parameters necessary to describe any RFE. These quantities can be given in a form of a diagonal mass matrix

$$\mathbf{M}^{(i)} = \operatorname{diag}[m^{(i)}, m^{(i)}, m^{(i)}, J_1^{(i)}, J_2^{(i)}, J_3^{(i)}] \tag{5.1}$$

The first three terms of the matrix are equal to the mass of an RFE, while the other three are the mass moments of inertia of RFE relative to the axes $\widehat{X}_1^{(i)}, \widehat{X}_2^{(i)}, \widehat{X}_3^{(i)}$.

Every SDE of the k number has its own independent co-ordinate system of the main axes $\widehat{Y}_1^{(i)}, \widehat{Y}_2^{(i)}, \widehat{Y}_3^{(i)}$. The main axes of an SDE have a property that forces acting on the SDE in the direction compatible with these axes result in its translational deformations which take place only in the direction where these forces are applied.

The main parameters that describe an SDE of the number k are coefficients defining its spring and damping properties. The spring properties are described by means of two matrices: the matrix of translational stiffness coefficients $\mathbf{K}_Y^{(k)}$ and the matrix of rotational stiffness coefficients $\mathbf{K}_\varphi^{(k)}$. Both matrices are diagonal and they are 3×3 in dimension

$$\mathbf{K}_Y^{(k)} = \operatorname{diag}[k_{Y,1}^{(k)}, k_{Y,2}^{(k)}, k_{Y,3}^{(k)}] \qquad \mathbf{K}_\varphi^{(k)} = \operatorname{diag}[k_{\varphi,1}^{(k)}, k_{\varphi,2}^{(k)}, k_{\varphi,3}^{(k)}] \tag{5.2}$$

While investigating the question of motion of flat systems, the above discussed matrices reduce their dimensions respectively to: $\mathbf{M}^{(i)}$ (3×3), $\mathbf{K}_Y^{(k)}$ (2×2),

$\mathbf{K}_{\varphi}^{(k)}$ (1×1). Practical methods of creating the stiffness matrix $\mathbf{K}$ on the basis of translational $\mathbf{K}_{Y}^{(k)}$ and rotational $\mathbf{K}_{\varphi}^{(k)}$ stiffness coefficients as well as methods of creating the inertia matrix $\mathbf{M}$ on the basis of individual mass matrices $\mathbf{M}^{(i)}$ were described in details by Kruszewski *et al.* (1999) and by Wittbrodt *et al.* (2006).

The relations on the basis of which the elements of matrices $\mathbf{M}^{(i)}$, $\mathbf{K}_{Y}^{(k)}$, $\mathbf{K}_{\varphi}^{(k)}$ are determined for RFE and SDE (spring-damping elements) modelling the constant part of the structure (the reinforced concrete slab and the steel I-bar), are throughly discussed in the literature on the subject (Kruszewski *et al.*, 1999; Wittbrodt *et al.*, 2006). These relations require knowing both the applied materials and parameters describing the cross-section of an element. Table 4 contains a list of all parameters which were possible to be determined on the basis of the beam inventory or reference data. The table does not contain values of the substitute modulus of elasticity of the reinforced concrete slab E_c.

Table 4. Parameters of the beam model

Reinforced concrete slab	
t_c [m]	6.386E − 02
A_c [m^2]	3.832E − 02
I_c [m^4]	1.302E − 05
ρ_c [kg/m^3]	2.447E + 03
e_c [m]	3.193E − 02
Steel I-section	
A_s [m^2]	2.010E − 03
I_s [m^4]	8.690E − 06
ρ_s [kg/m^3]	7.850E + 03
E_s [N/m^2]	2.050E + 11
e_s [m]	8.000E − 02

The modulus E_c is a substitute modulus which takes into account the longitudinal reinforcement of the concrete slab. It can be easily calculated when the moduli of elasticity for both steel and concrete are known. White the obtaining of the steel modulus is relatively easy, the concrete modulus is much more difficult to be found. Lee *et al.* (1987) as well as Memory *et al.* (1995) during their theoretical analyses aiming at determining dynamic characteristics of reinforced concrete and composite structures maintained that it was important to take into consideration the dynamic modulus of elasticity of concrete E_d. According to Neville (1995), the dynamic modulus E_d can be determined by means of vibrations of a concrete specimen, with only a negligible stress being applied. Owing to a low stress level, there are no micro-cracks and there is no creep. This is the reason that the dynamic modulus of elasticity is considered to be roughly equal to the initial tangent modulus defined in a static test. The dynamic modulus of elasticity is therefore much higher than the secant elasticity modulus E_{cm}, which in a standard procedure is determined during application of a static load onto a sample.

The matrix elements $\mathbf{K}_Y^{(k)}$ for SDEs modelling the connection reflect the stiffness of the connection. If we investigate a flat system, it is necessary to know the stiffness of connecting elements in the vertical direction (perpendicular to the connection plane) K_v and in the horizontal direction (parallel to the connection plane) K_h. The reference literature does not provide any information on this problem.

The three missing parameters defining the model stiffness, i.e. E_c, K_v, K_h were determined on the basis of parametric identification. The best possible fit of natural vibration frequencies obtained in experimentally and numerically was used as the identification criterion. Both flexural and axial vibration modes were analysed. Detailed data of the procedure can be found in Wróblewski (2006). Table 5 contains a comparison of frequencies obtained during the investigations and on the basis of the RFE method model. It also contains values of the identified parameters of the model.

Table 5. Parameters of the beam model

i	Model	RFE	
	$f_{i\,exp}$ [Hz]	$f_{i\,num}$ [Hz]	Δ [%]
1_{flex}	77.75	77.75	0.0
2_{flex}	186.65	187.07	0.2
3_{flex}	304.72	307.10	0.8
4_{flex}	417.57	419.81	0.5
5_{flex}	531.13	526.70	−0.8
K_h [N/m]		2.012E + 09	
K_v [N/m]		2.745E + 08	
E_c [N/m^2]		3.392E + 10	

The damping properties of SDEs are described by means of two matrices: the matrix of translational damping coefficients $\mathbf{C}_Y^{(k)}$ and the matrix of rotational damping coefficients $\mathbf{C}_\varphi^{(k)}$. In a general case, both matrices are 3×3. For flat systems they reduce their dimensions, similarly to $\mathbf{K}_Y^{(k)}$, $\mathbf{K}_\varphi^{(k)}$

$$\mathbf{C}_Y^{(k)} = \operatorname{diag}[c_{Y,1}^{(k)}, c_{Y,2}^{(k)}, c_{Y,3}^{(k)}] \qquad \mathbf{C}_\varphi^{(k)} = \operatorname{diag}[c_{\varphi,1}^{(k)}, c_{\varphi,2}^{(k)}, c_{\varphi,3}^{(k)}] \tag{5.3}$$

For SDEs which replace the constant part of the structure made of a material with properties defined by the Kelvin-Voigt model, the relation between the respective damping and stiffness ratios of the matrix elements can be given by Kruszewski *et al.* (1999)

$$c_{Y,i}^{(k)} = \frac{Q^{-1}}{\omega} k_{Y,i}^{(k)} \qquad c_{\varphi,i}^{(k)} = \frac{Q^{-1}}{\omega} k_{\varphi,i}^{(k)} \qquad i = 1,2,3 \tag{5.4}$$

where Q^{-1} is the loss factor and ,ω is the vibration frequency. The loss factor Q^{-1}, just like the logarithmic damping decrement δ or the damping ratio ξ, is a parameter used for defining the damping. Assumping that the damping is minimal, between the above given parameters, the following relations are to be observed (Marchelek, 1991)

$$Q^{-1} = \frac{\delta}{\pi} \qquad \xi = \frac{\delta}{2\pi} \qquad \delta = 2\pi\xi \tag{5.5}$$

Values of the loss factor depend on the frequency, temperature and other factors. The higher value of the loss factor, the better damping properties of a material. According to Rao (2004), the loss factor for steel amounted to $Q_s^{-1} \in \langle 6 \cdot 10^{-4}, 2 \cdot 10^{-4} \rangle$. Concrete has much better damping properties than steel and owing to this its loss factor values vary in the following range $Q_c^{-1} \in \langle 0.06, 0.02 \rangle$ (De Silva, 2000).

For interfaces of elements and connections, through analogy to loss factors, it is possible to derive a connection loss factor (Marchelek, 1991). In the investigated case, the connection loss factor was introduced and it was defined as Q_{con}^{-1}. Matrices of damping ratios for modelling connections of SDEs were defined according to relation (5.4) by substituting coefficient Q_{con}^{-1}.

It was decided that the loss factors Q_s^{-1}, Q_c^{-1}, Q_{con}^{-1} should be determined by means of fitting the frequency characteristics obtained from the RFE model to the characteristics obtained experimentally. The characteristic obtained on the assumption that the system input is force and its output is acceleration is called inertance. Such characteristics are fitted with each other. One of the optimization procedures of Optimization Toolbox/Matlab software package was used during the analysis. Loss factor values for which the frequency amplitudes for some selected resonance frequencies obtained in the experimental and numerical investigations overlap were sought. An example of such values obtained both experimentally and numerically can be seen in Fig. 5. For comparison purposes, some measuring points located at the ends of the reinforced concrete slab (i.e. points 2 and 34, response direction y, point C excited) were chosen. The loss factors determined in the analysis are presented in Table 6. More detailed information on the loss factor procedure can be found elsewhere (Wróblewski, 2006).

Table 6. Values of loss factors

Q_s^{-1}	0.0003
Q_c^{-1}	0.0222
Q_{con}^{-1}	0.0058

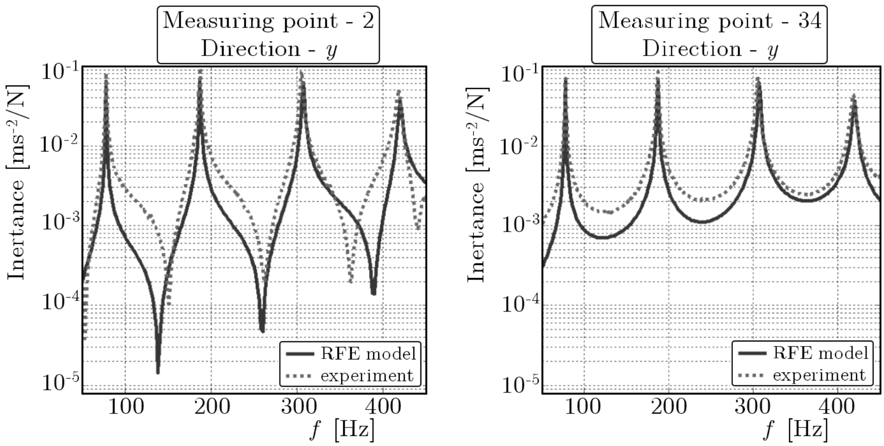

Fig. 5. Comparison of experimental and numerical amplitude and frequency characteristics

The developed model of the composite beam has very low requirements for processing capacity. This fact is due to a limited number of degrees of freedom. The software used in the analysis had been prepared in Matlab environment. The fit of experimental and numerical results is very high, which can be seen in Table 6 and Fig. 5. The thus determined model was used for numerical simulations of damage occurring in the beam connections.

6. Numerical simulation of damage

The carried out numerical simulation of damage aimed at investigating the sensitivity of ETR to the beam damage. Damage simulation in the connection was conducted by removing successive SDEs connections 1, 4, and 7, respectively (compare Fig. 4) at one end of the beam. The removal of SDE number 1 simulated the damage on a 50 mm long segment. The removal of two adjacent SDEs number 1 and 4 simulated the damage on a 150 mm long segment. In a similar way, subsequent segments from which SDEs were removed, can be determined (compare Fig. 4). The modal parameters, i.e. frequency of natural vibrations for a system without damping or with proportional damping f_i and for a system with non-proportional damping f_{ni}, and the damping ratio together with ETR were determined for the first five modes of flexural vibrations. Changes of these parameters are presented in Tables 7, 8 and 9. Because the values of f_{ni} and f_i were almost identical, only one frequency table is presented below – that for f_i.

Table 7. Changes in frequencies of natural vibrations f_i

i	Damaged SDEs	1		1, 4		1, 4, 7	
	$f_{i\,num}$ [Hz]	$f_{i\,num,d}$ [Hz]	Δ [%]	$f_{i\,num,d}$ [Hz]	Δ [%]	$f_{i\,num,d}$ [Hz]	Δ [%]
1_{flex}	77.75	77.69	−0.1	77.37	−0.5	76.49	−1.6
2_{flex}	187.08	186.36	−0.4	182.53	−2.4	172.02	−8.0
3_{flex}	307.10	304.28	−0.9	289.51	−5.7	261.38	−14.9
4_{flex}	419.82	413.77	−1.4	389.09	−7.3	365.67	−12.9
5_{flex}	526.71	518.10	−1.6	493.44	−6.3	477.42	−9.4

Table 8. Changes in damping ratio ξ_i

i	Damaged SDEs	1		1, 4		1, 4, 7	
	$\xi_{i\,num}$ [%]	$\xi_{i\,num,d}$ [%]	Δ [%]	$\xi_{i\,num,d}$ [%]	Δ [%]	$\xi_{i\,num,d}$ [%]	Δ [%]
1_{flex}	0.26	0.26	0.0	0.26	1.1	0.28	5.8
2_{flex}	0.28	0.28	0.5	0.30	7.9	0.39	40.8
3_{flex}	0.33	0.33	1.6	0.40	21.0	0.50	53.4
4_{flex}	0.41	0.42	2.7	0.49	18.5	0.46	12.5
5_{flex}	0.52	0.54	2.6	0.55	6.1	0.49	−7.0

As can be seen from the presented data, the ETR has the highest sensitivity to damage, and the frequency of natural vibrations f_i and f_{ni} have the lowest sensitivity. However, there is no unambiguous relation between the changes in ETR values and frequencies or damping ratios. Having analysed the values of damping ratio presented in Tables 3 and 8, their high consistency is to be stressed. It also confirms the correctness of the loss factor estimation.

Table 9. Chnges of ETR ζ_i

i	Damaged SDEs	1		1, 4		1, 4, 7	
	$\zeta_{i\,num}$ [‰]$\cdot 10^{-2}$	$\zeta_{i\,num,d}$ [‰]$\cdot 10^{-2}$	Δ [%]	$\zeta_{i\,num,d}$ [‰]$\cdot 10^{-2}$	Δ [%]	$\zeta_{i\,num,d}$ [‰]$\cdot 10^{-2}$	Δ [%]
1_{flex}	2.55	2.54	−0.1	2.55	0.2	2.66	4.3
2_{flex}	1.92	1.91	−0.5	2.06	7.2	3.02	57.3
3_{flex}	1.39	1.38	−0.9	1.93	38.7	2.18	56.5
4_{flex}	0.97	0.98	0.6	1.36	39.4	1.16	19.5
5_{flex}	0.76	0.73	−3.9	0.76	−0.3	1.54	103.5

The obtained results do not confirm the relation derived by Liang and Lee (1999) and by Wang and Zong (2002, 2003), which claims that ETR is 1000 times more sensitive than the frequency of natural vibrations f_i. The assumptions made by the researchers are presented in detail below. For these assumptions, the relation in question was obtained. Let "0" denote undamaged structure and "j" stand for changed conditions of the structure after introduction of the damage. Given the above, relation (3.8) can be expressed as

$$\omega_{i0} = \omega_{ni0} \exp(\zeta_{i0}) \qquad \omega_{ij} = \omega_{nij} \exp(\zeta_{ij}) \tag{6.1}$$

The authors assumed that after damage the changes in stiffness of the structure are negligibly small, and therefore

$$\omega_{ni0} = \omega_{nij} = \omega_{ni} \tag{6.2}$$

Formulas (6.1) take the following form

$$\omega_{i0} = \omega_{ni} \exp(\zeta_{i0}) \qquad \omega_{ij} = \omega_{ni} \exp(\zeta_{ij}) \tag{6.3}$$

It can be observed that for $\zeta_{i0} = 0.001$ and for changes in natural vibration frequencies amounting to 0.1%, we obtain

$$\begin{aligned} &\frac{\omega_{ij} - \omega_{i0}}{\omega_{i0}} = 0.1\% \;\rightarrow\; \frac{\exp(\zeta_{ij}) - \exp(\zeta_{i0})}{\exp(\zeta_{i0})} = 0.001 \\ &\frac{\zeta_{ij} - \zeta_{i0}}{\zeta_{i0}} = 1(100\%) \end{aligned} \tag{6.4}$$

For $\zeta_{i0} = 0.001$ and $\Delta\omega_i = 0.1\%$ ETR changes 100%, from which it can be inferred that it is 1000 times more sensitive to changes in the structure than the frequencies ω_i. However, it should be also noted that this result was obtained for the following concrete figures: $\zeta_{i0} = 0.001$ and $\Delta\omega_i = 0.1\%$. How does ETR coefficient change for other values of ζ_{i0} and $\Delta\omega_i$? For low damping observed in steel and in concrete structures, values of ETR amount to 0.00-0.01 (Wang and Zong, 2002). Consequently, the analysis focused on changes of ETR for various values of ζ_{i0} in the range of 0.001-0.005 for frequency variations $\Delta\omega_i = 0.1\%$ and $\Delta\omega_i = 0.2\%$. The results are presented in Tables 10 and 11.

Table 10. Changes of ETR depending on ζ_{i0} for $\Delta\omega_i = 0.1\%$

ζ_{i0}	0.001	0.002	0.003	0.004	0.005
Change of ETR	100.0%	50.0%	33.3%	25.0%	20.0%

Table 11. Changes of ETR depending on ζ_{i0} for $\Delta\omega_i = 0.2\%$

ζ_{i0}	0.001	0.002	0.003	0.004	0.005
Change of ETR	200.0%	100.0%	66.7%	50.0%	40.0%

As can be seen in the above presented tables, the general statement that ETR is 1000 times more sensitive to damage than frequency of natural vibrations is not entirely correct. The statement is true only for $\Delta\omega_i = 0.1\%$ and $\zeta_{i0} = 0.001$. For other values of $\Delta\omega_i$ and ζ_{i0} other sensitivity values are obtained, e.g. for $\Delta\omega_i = 0.1\%$ and $\zeta_{i0} = 0.005$ ETR change amounts to 20%, so in this case ETR is "only" 200 times more sensitive to changes than the frequency of natural vibrations. We should also remember about assumption (6.2). An assumption was made that changes occurring in the structure are so small that the structure stiffness does not change and, as a result, the stiffness matrix $\mathbf{K}$ and frequency of natural vibrations ω_{ni} do not change either. However, our investigations showed that the frequency of vibrations ω_{ni} does change after the damage is introduced into the system, which is shown in Table 7. As can be seen, the biggest change of vibration ω_{ni} for 3 vibration modes and for damaged 3 SDEs amounts to 14.9%. Therefore, it is not correct to assume that $\omega_{ni} = \text{const}$.

7. Summary

The present study focuses on ETR sensitivity to damage in the connections of steel-concrete composite beams. The obtained results confirmed that this coefficient is markedly more sensitive to damage than frequencies of natural vibrations. Although this difference does not amount to 1000 times, as Liang and Lee (1999) as well as Wang and Zong (2002, 2003) claimed, it is nevertheless a significant one. It was also observed that ETR was more sensitive to damage than the damping ratio. It should be noted that the simulation in the connection was conducted for one beam only, for one kind of damage located at one end of the beam. Further studies, both experimental and numerical, are planned. They will focus on a wider range of damage. It should also be stressed that the ETR analysis was conducted on the whole beam. However, detailed studies show that this coefficient can be determined locally, only for a part of the beam. Perhaps this way of determination ETR might prove to be a more effective method of damage detection. This issue is certain to be the focus of future studies.

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Wykorzystanie ETR do diagnostyki uszkodzeń w stalowo-betonowych belkach zespolonych

Streszczenie

Artykuł przedstawia, jak zmienia się współczynnik transferu energii ETR wyznaczany dla stalowo-betonowych belek zespolonych w zależności od stopnia ich uszkodzenia. Model numeryczny belki opracowany został z wykorzystaniem wyników badań doświadczalnych przeprowadzonych na rzeczywistych belkach zespolonych. Artykuł

prezentuje, jak zmieniają się częstotliwości drgań, współczynniki tłumienia oraz ETR w zależności od zmian w strukturze belki. Analizy wykazały, że ETR jest parametrem wykazującym największą wrażliwość na symulowane uszkodzenia. Parametr ten może być użyty do detekcji uszkodzeń w tego typu belkach.

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