CONTINUOUS WAVELET TRANSFORM IN HYDROACOUSTICS SIGNALS ANALYSIS

ANDRZEJ ZAK

Polish Naval Academy
69, J. Śmidowicza St., 81-103 Gdynia, Poland
a.zak@amw.gdynia.pl

Paper presents method of Continuous Wavelet Transform (CWT) used in process of hydroacoustics signals analysis. The continuous wavelet transform can be used to produce spectrograms which show the frequency content of signals as a function of time. Such representation is very useful during analysis of non-stationary signals which can be acquired during passive measurements of hydroacoustics field of moving ships. In paper there is described the continuous wavelet transform analysis method by presenting the mathematical background of CWT and its properties. At the end some examples of hydroacoustics signals analysis and its brief descriptions was shown.

INTRODUCTION

It is known from Fourier theory that any signal can be expressed as the sum of a series of sines. This sum is also referred to as a Fourier expansion. The big disadvantage of a Fourier expansion is that it has only frequency resolution and no time resolution. This means that we might be able to determine all the frequencies present in a signal but we do not know when they appear. To overcome this problem several solutions have been developed which are more or less able to represent a signal in the time and frequency domain at the same time [5].

The idea behind these time-frequency joint representations is to cut the signal of interest into several parts and then analyze the parts separately. It must be noticed that analyzing a signal this way will give more information about the when and where different frequency components appear, but it leads to a fundamental problem as well: how to cut the signals? The problem here is that cutting the signal into short block cause that we have results with hi-resolution in time domain but in very low-resolution in frequency domain and cutting signal into long block cause that we have results with low-resolution in time domain and hi-resolution in frequency domain.

The underlying principle of the phenomena just described is due to Heisenberg's uncertainty principle, which, in signal processing terms, states that it is impossible to know the exact frequency and the exact time of occurrence of this frequency in a signal. In other words, a signal can simply not be represented as a point in the time-frequency space. The uncertainty principle shows that it is very important how one cuts the signal [5].

The wavelet transform or wavelet analysis is probably the most recent solution to overcome the shortcomings of the Fourier transform. In wavelet analysis the use of a fully scalable modulated window solves the signal-cutting problem. The window is shifted along the signal and for every position the spectrum is calculated. Then this process is repeated many times with a slightly shorter (or longer) window for every new cycle. In the end the result will be a collection of time-frequency representations of the signal, all with different resolutions. Because of this collection of representations we can speak of a multiresolution analysis. In the case of wavelets we normally do not speak about time-frequency representations but about time-scale representations, scale being in a way the opposite of frequency, because the term frequency is reserved for the Fourier transform [5]. Because the frequency is much more intuitive for people to understand, there is method to recalculate scale into such called pseudo-frequency which corresponds to the center frequency of wavelet represented in given scale.

1. ANALYSIS METHOD

Like the Fourier transform, the continuous wavelet transform uses inner products to measure the similarity between a signal and an analyzing function. In the Fourier transform, the analyzing functions are complex exponentials, $e^{j\omega t}$. The resulting transform is a function of a single variable, ω. In the short-time Fourier transform, the analyzing functions are windowed complex exponentials, $w(t)e^{j\omega t}$, and the result in a function of two variables. The STFT coefficients, $F(\omega,\tau)$ represent the match between the signal and a sinusoid with angular frequency ω in an interval of a specified length centered at τ [5, 6].

In the CWT, the analyzing function is a wavelet, ψ. The CWT compares the signal to shifted and compressed or stretched versions of a wavelet. By comparing the signal to the wavelet at various scales and positions, we can obtain a function of two variables.

The continuous wavelet transform is defined as follows [2, 6]:

$$CWT_x^{\psi}(\tau,s) = \frac{1}{\sqrt{|s|}}\int x(t)\psi^*(\frac{t-\tau}{s})dt \tag{1}$$

As seen in the above equation, the transformed signal is a function of two variables, τ and s, the translation and scale parameters, respectively. Factor $s^{-0.5}$ is for energy normalization across the different scales. $\psi(t)$ is the transforming function, and it is called the mother wavelet.

The term wavelet means a small wave. The smallness refers to the condition that this (window) function is of finite length (compactly supported). The wave refers to the condition that this function is oscillatory. The term mother implies that the functions with different region of support that are used in the transformation process are derived from one main function, or the mother wavelet. In other words, the mother wavelet is a prototype for generating the other window functions [1, 2].

The term translation is used in the same sense as it was used in the STFT; it is related to the location of the window, as the window is shifted through the signal. This term, obviously, corresponds to time information in the transform domain. However, we do not have a frequency parameter, as we had before for the STFT but instead, we have scale parameter.

The parameter scale in the wavelet analysis is similar to the scale used in maps. As in the case of maps, high scales correspond to a non-detailed global view (of the signal), and low scales correspond to a detailed view. Similarly, in terms of frequency, low frequencies (high scales) correspond to a global information of a signal (that usually spans the entire signal), whereas high frequencies (low scales) correspond to a detailed information of a hidden pattern in the signal (that usually lasts a relatively short time).

There is clearly a relationship between scale and frequency. Recall that higher scales correspond to the most 'stretched' wavelets. The more stretched the wavelet, the longer the portion of the signal with which it is being compared, and therefore the coarser the signal features measured by the wavelet coefficients (fig. 1).

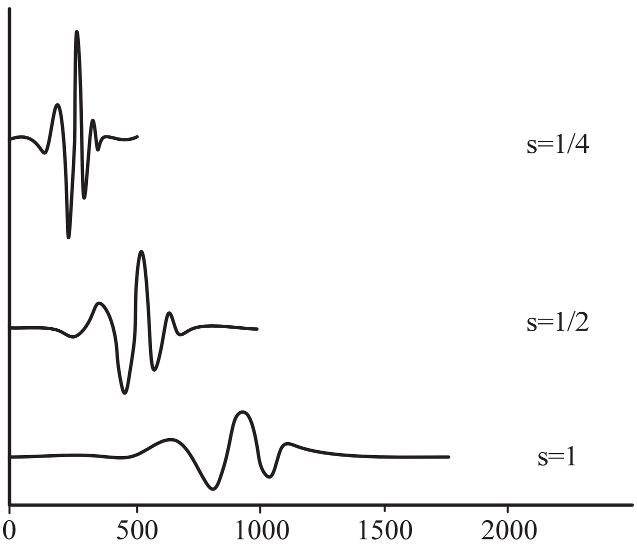

Fig. 1. Meaning of scale in wavelet transform [1, 6]

To summarize, the general correspondence between scale and frequency is [6, 4]:

– Low scale s $\Rightarrow$ Compressed wavelet $\Rightarrow$ Rapidly changing details $\Rightarrow$ High frequency ω,

– High scale s $\Rightarrow$ Stretched wavelet $\Rightarrow$ Slowly changing, coarse features $\Rightarrow$ Low frequency ω.

While there is a general relationship between scale and frequency, no precise relationship exists. Users familiar with Fourier analysis often want to define a mapping between a wavelet at a given scale with a specified sampling period to a frequency in hertz. A way to do it is to compute the center frequency, F_c, of the wavelet and to use the following relationship [4].

$$F_a = \frac{F_c}{sT_s} \tag{2}$$

where: s is a scale, T_s is the sampling period, F_c is the center frequency of a wavelet in Hz, F_a is the pseudo-frequency corresponding to the scale a, in Hz.

The idea is to associate with a given wavelet a purely periodic signal of frequency F_c. Shifting (translation) a wavelet simply means delaying (or advancing) its onset. Mathematically, delaying a function $f(t)$ by k is represented by $f(t-k)$ (fig. 2) [3, 6].

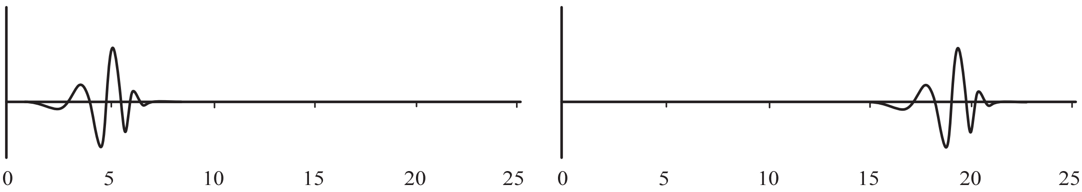

Fig. 2. Meaning of translation in wavelet transform [6]

Lets look at CWT as a window transform. In Short-Time Fourier Transform, the STFT is described as a windowing of the signal to create a local frequency analysis. A shortcoming of the STFT approach is that the window size is constant. There is a trade off in the choice of window size. A longer time window improves frequency resolution while resulting in poorer time resolution because the Fourier transform loses all time resolution over the duration of the window. Conversely, a shorter time window improves time localization while resulting in poorer frequency resolution [6].

Wavelet analysis represents the next logical step: a windowing technique with variable-sized regions. Wavelet analysis allows the use of long time intervals where you want more precise low-frequency information, and shorter regions where you want high-frequency information (fig. 3) [1, 6].

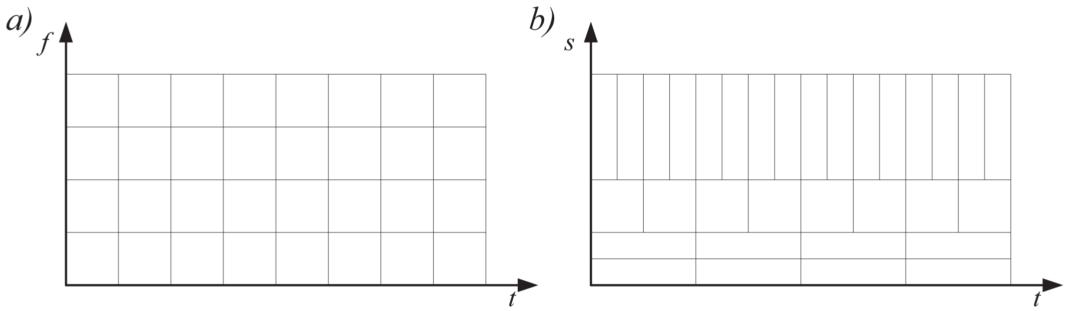

Fig. 3. Partition of time-frequency space in a) STFT b) CWT

The most important properties of wavelets are the admissibility and the regularity conditions and these are the properties which gave wavelets their name. It can be shown that square integrable functions $\psi(t)$ satisfying the admissibility condition [5]:

$$\int \frac{\left|\Psi(\omega)\right|^2}{|\omega|} d\omega < +\infty \tag{3}$$

which can be used to first analyze and then reconstruct a signal without loss of information. In (3) $\Psi(\omega)$ stands for the Fourier transform of $\psi(t)$. The admissibility condition implies that the Fourier transform of $\psi(t)$ vanishes at the zero frequency, i.e. [5]:

$$\left|\Psi(\omega)\right|^2 \Big|_{\omega=0} = 0 \tag{4}$$

This means that wavelets must have a band-pass like spectrum. This is a very important observation, which we will use later on to build an efficient wavelet transform.

A zero at the zero frequency also means that the average value of the wavelet in the time domain must be zero [5]:

$$\int \psi(t)dt = 0 \tag{5}$$

and therefore it must be oscillatory. In other words, $\psi(t)$ must be a wave.

2. RESULTS OF RESEARCH

Below there are presented results of calculation continuous wavelet transform for ship of radio-electronic reconnaissance. Measurements were conducted on Polish Naval Acoustics Measurement Range. Hydrophones were positioned on the depth of 20 m and signals after analog to digital conversion were transmitted to the recording system. Continuous wavelet transform was calculated using biorthogonal wavelets in application Matlab®. Fig. 4 Shows time series of analyzed signal and results of calculation CWT in scale from 1 to 256. To shows how is looking CWT in selected band of frequency during research the scale was changed. According to above it is possible to receive CWT in frequency band:

- of about 5Hz to 30Hz (scale: 132 to 550 with signal down sampling coefficient 10),
- of about 30Hz to 100Hz (scale: 63 to 260 with signal down sampling coefficient 5),
- of about 100Hz to 1kHz (scale: 15 to 210 with signal down sampling coefficient 2).

Results of this calculation are shown on figures 5 and 6. Because the measured ship was going through measurement range with working depth echosounder it was interesting to calculate CWT for moment where echosounder is well heard. Result is presented on fig. 7 where the moment of sending ping by echosounder is very well visible. Because during analysis of hydroacoustic signals from such measurement it is interesting the moment when main diesel engine are above the hydrophones the same calculation were made in this research (fig. 8).

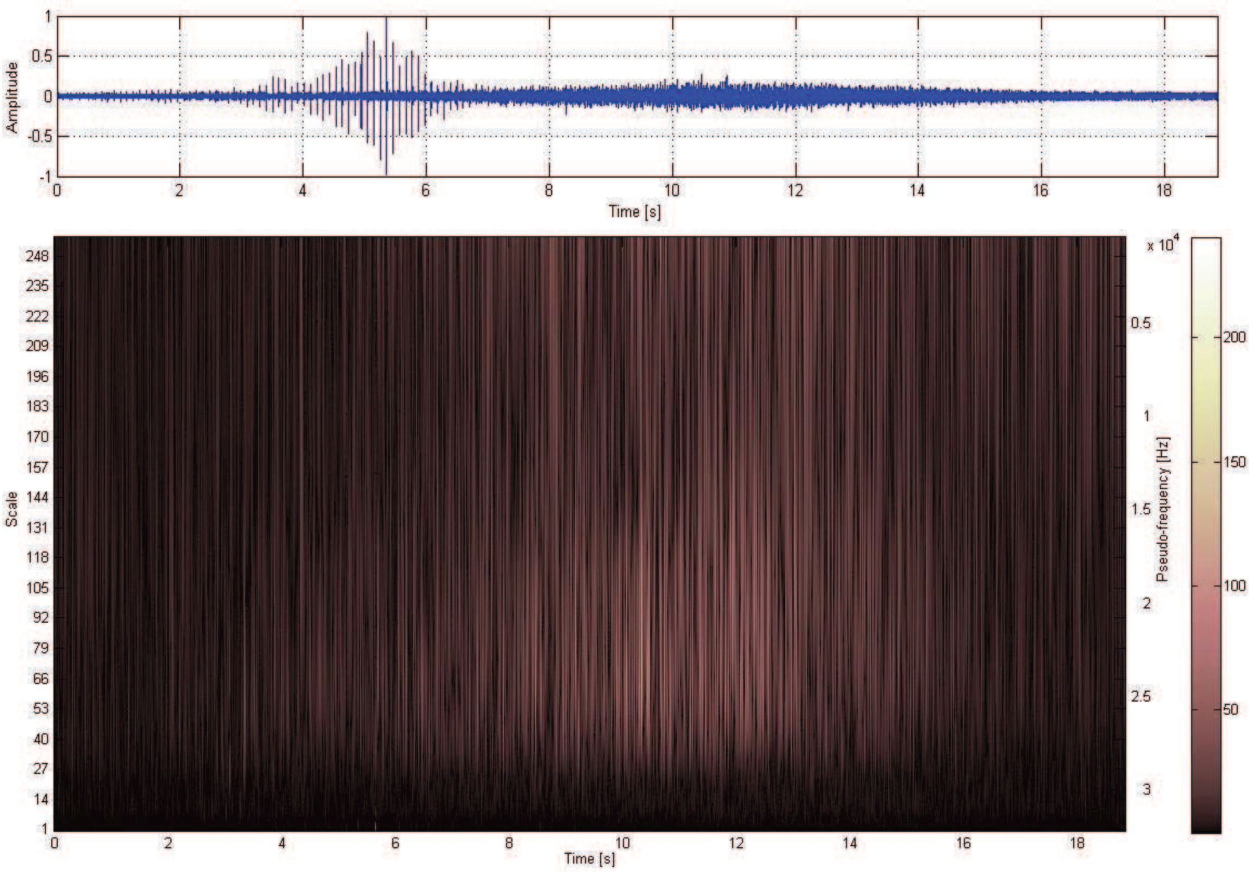

Fig. 4. Time series and results of CWT calculation

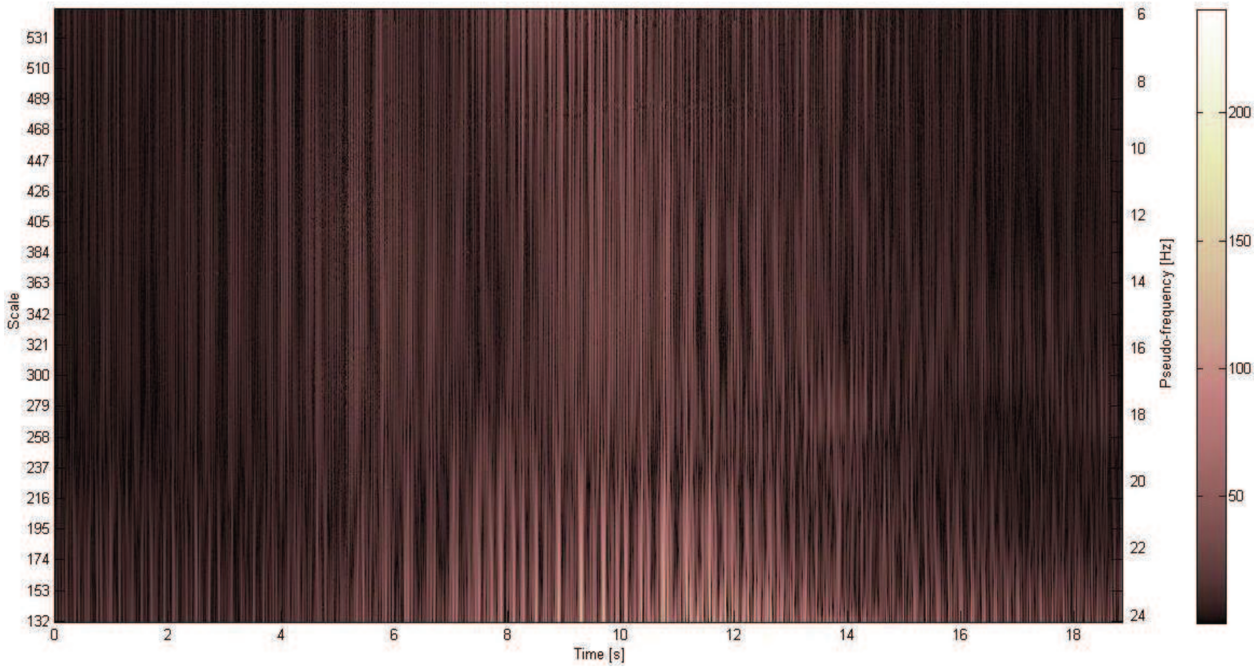

Fig. 5. Results of CWT calculation for frequency band about 5Hz to 30Hz

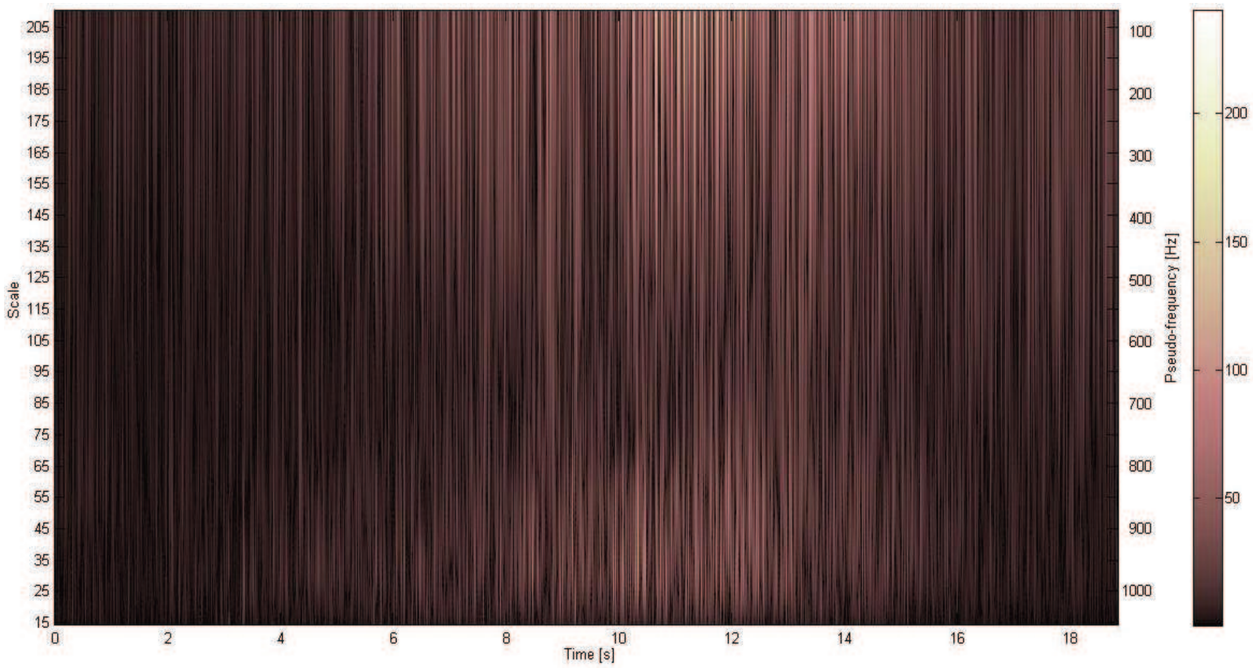

Fig. 6. Results of CWT calculation for frequency band about 100Hz to 1kHz

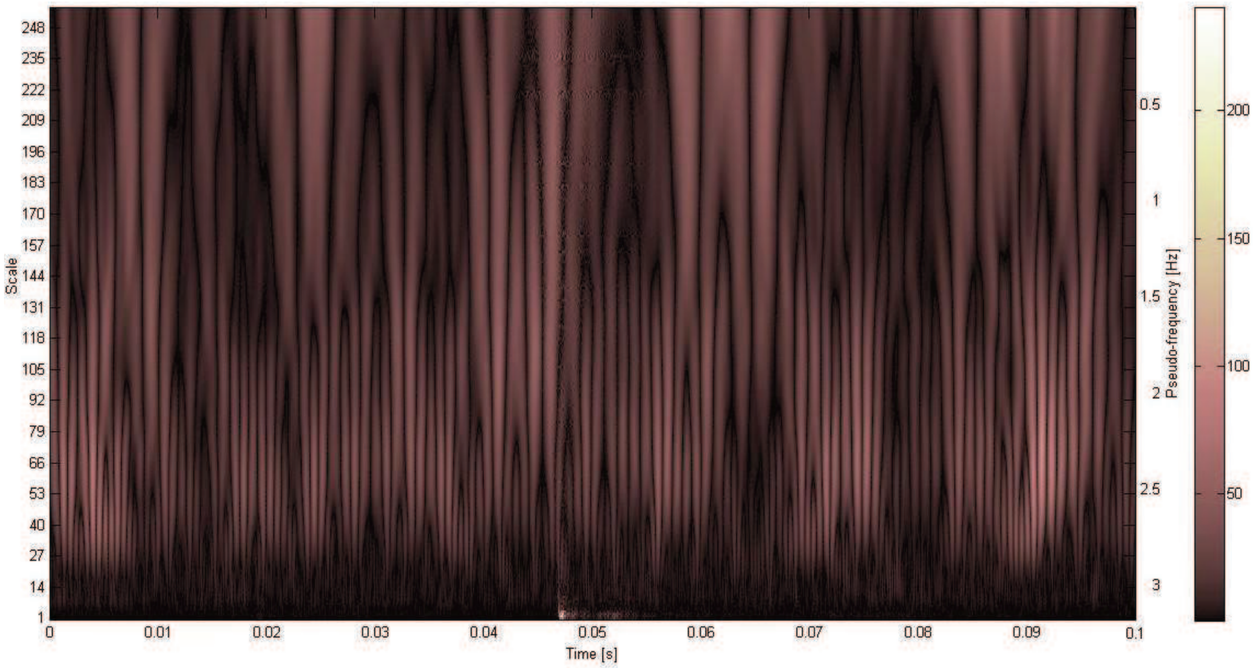

Fig. 7. Results of CWT calculation in moment of depth echosounder working

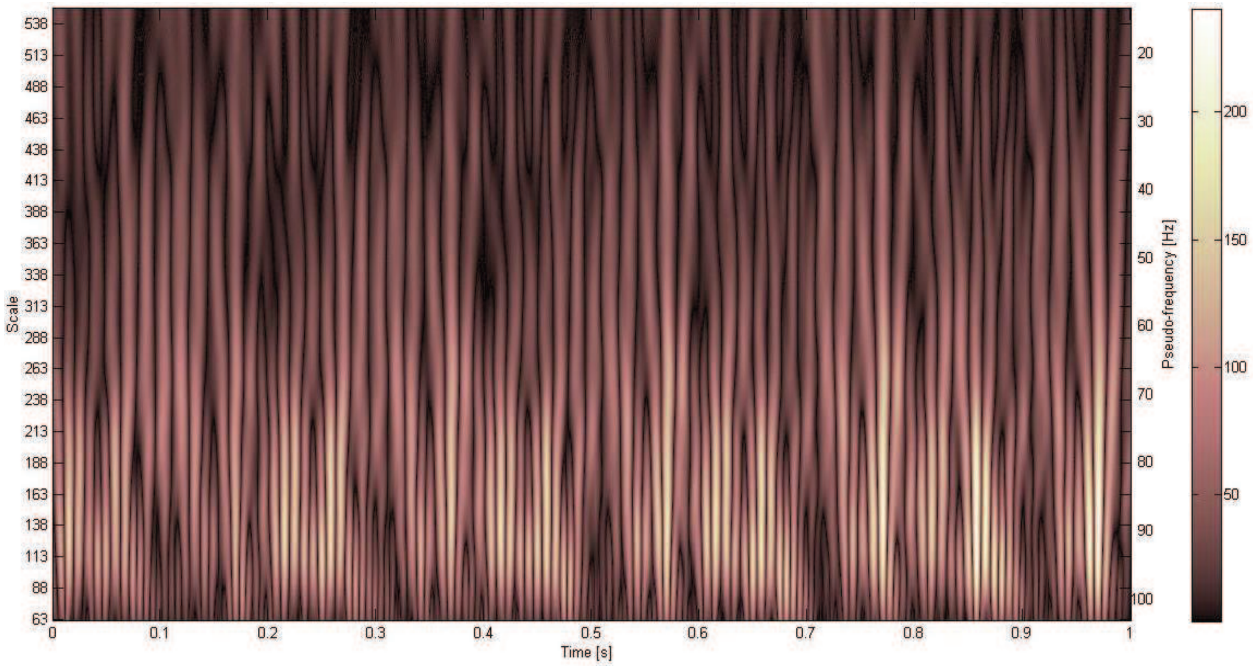

Fig. 8. Results of CWT calculation in moment of diesel engines working

3. SUMMARY

Paper presents application of continuous wavelet transform to analysis of hydroacoustics signal acquired during passive measurements. The wavelet transform is a relatively new concept (it is about 10 years old), but yet there are quite a few application of them but till now there is now information about using them in hydroacoustics. Theoretically continuous wavelet transform is perfect in application of analyzing hydroacoustics signals which mostly are non-stationery signals. This paper shows that its usefulness is not only theoretical but also practical. As it was shown results of calculations and possibilities of methods gives to hydroacoustics new tools in processing and analyzing signals. However it must be noticed that we must still learn how to properly interpreted the received results of calculations.

ACKNOWLEDGMENTS

The investigation was supported by the Ministry of Science and Higher Education (Grant No OR00 0047 08).

REFERENCES

[1] Białasiewicz J. T., Wavelets and approximation (in Polish), WNT, Warsaw, 2004.
[2] Polikar R., The Engineer's Ultimate Guide To Wavelet Analysis, The Wavelet Tutorial, website: http://users.rowan.edu/~polikar/WAVELETS/WTtutorial.html.
[3] Hennel J., Olejniczak Z., How to understand wavelets. Fundamentals of wavelet analysis of signals (in Polish), ZamKor, Krakow 2010.
[4] Wojtaszczyk P., Teoria falek, PWN, Warsaw 2000.
[5] Valens C., A Really Friendly Guide to Wavelets, website: http://polyvalens.pagesperso-orange.fr/clemens/wavelets/wavelets.html.
[6] help of Matlab® application.