MODELING OF VERTICAL DISTRIBUTION OF SOUND SPEED IN WATER USING RATIONAL BÉZIER COURVES

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Modeling of one variable and two variables functions can be used in hydroacoustics and hydrography i.e. for modeling the vertical distribution of the sound speed in water (one variable functions) or sea bottom (two variable functions). There are many mathematics methods of modeling one variable functions.

In the paper modeling of the one variable function for the vertical distribution of the sound speed in water using rational Bézier functions have been shown.

INTRODUCTION

Polynomial Bézier curves have many advantages in modeling. They are modification of curves described in [10,11], which were used for description the sound speed in water. Knowledge about vertical distribution of the sound speed in water is essential issue in theory of acoustic wave's propagation [1, 3], determination of the depth using acoustic methods, determination of measurement's accuracy [10] and determination of acoustic wave reflection points in bathymetric surveys [3].

Many methods were been used for describing the vertical distribution of the sound speed in water [4, 5, 6, 7, 10, 11, 12, 13, 14], e.g. Uniform B-Splines, NonUniform Rational B-Splines NURBS, Bézier curves and other well known interpolation methods [2].

1. BÉZIER CURVES AND POLYNOMIALS

One of Bezier curve definition described it as *p* curve, when each point of *p(t)* can be constructed using adequate *t*:

- let's choose in free method a sequence of *n*+1 points $p_0,..., p_n$ and let's into consideration a broken line with these points,
- now, we divide all *n* segments of his broken line in established proportion,
- this proportion can be described by one number parameter *t*: each section is divided in proportion: $t:1-t$,

– next, we receive n points, which are points of another broken line, which consists of $n-1$ sections. This process is repeated for obtaining one point.

Described algorithm is called Casteljau algorithm, when for $t \in [0,1]$ corners are cutted. As a result of this process, the broken line makes the curve. The iteration step can be written in the form [2, 10]:

$$p_i^{(j)} := (1-t)p_i^{(j-1)} + tp_{i+1}^{(j-1)} \quad \text{for} \quad i = 0,...,n \quad \text{and} \quad j = 1,...,n \tag{1}$$

Start points are called control points, the output broken lines called Bezier control line. Looking for de Casteljau algorithm we can observe:

– Bezier curve is polynomial one: if there are n+1 control points, curve's coordinates are described by polynomials of t variable of the degree not higher than n: so Bezier curve term is specific for individual polynomial curve representation,
– the curve has the convex property: for $t \in [0,1]$ a point $p(t)$ lies on convex line of $p_0,..., p_n$ points,
– construction of the curve is affine constant: the picture of $p_0,..., p_n$ points in free affine transformation determines the picture of the p curve in this transformation,
– occurs the interpolation of final points of the broken line: $p(0) = p_0$, $p(1) = p_n$.

Bernstein polynomials of n – degree are defined by equation [2, 10]:

$$B_i^n(t) \overset{def}{=} \binom{n}{i} t^i (1-t)^{n-i} \quad \text{for } i = 0,\dots,\text{n} \tag{2}$$

These polynomials are linear independent. They determine the space base of polynomials of degree not more than n, because they are n+1

$$B_i^n(t) \overset{def}{=} 0 \quad \text{for } i < 0 \text{ or } i > 1 \tag{3}$$

Bernstein polynomials meet recurrent relationship [1]:

$$B_i^n(t) = (1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t) \tag{4}$$

Polynomial $B_0^0(t)$ is equal to 1. For each n we also have $\binom{n}{0} = \binom{n}{n} = 1$, so for i = 0 and $i = n$ foregoing equation results from an agreement:

For $n > 1$, $i = 1,\dots,n$-1

$$(1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t) = B_i^n(t) \tag{5}$$

Polynomials of higher and higher degrees can be obtained using the pattern, which is the generalization of Pascal triangle.

Turned out, that control points of Bézier curve are coefficients of the curve in Bernstein polynomials space:

$$p(t) = \sum_{i=0}^{n} p_i B_i^n(t) . \tag{6}$$

2. MODELING OF RATIONAL BÉZIER COURVES

Displacement the point $\boldsymbol{p}_i$, like for the polynomial curve, causes displacement of points of the curve in this same direction (Fig. 1). Increasing the weight w_i causes displacement of points of the curve in the $\boldsymbol{p}_i$ direction. Precisely, if the curve before and after changing is adequately marked $\boldsymbol{p}$ i $\boldsymbol{q}$, then for each t points $\boldsymbol{p}(t)$, $\boldsymbol{q}(t)$ and $\boldsymbol{p}_i$ are collinear (Fig. 2). If the weights are increased on the same factor, the the output curve will be obtained. If $w_i = 0$, then changing the vector $\boldsymbol{v}_i$ on $\Delta\boldsymbol{v}_i$ – causes displacement points of the curve in $\Delta\boldsymbol{v}_i$ direction.

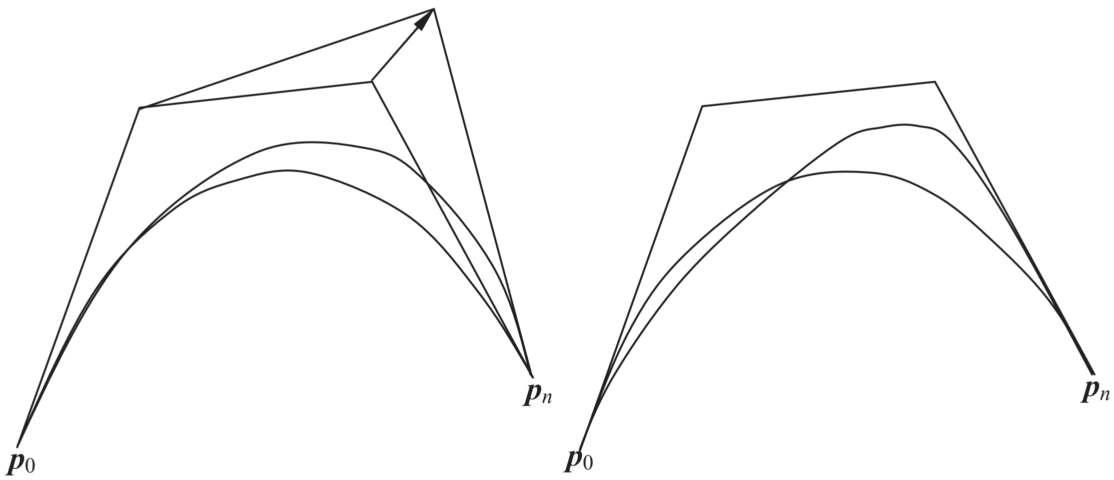

Fig. 1. Effect of displacement of the control point and changing its weight

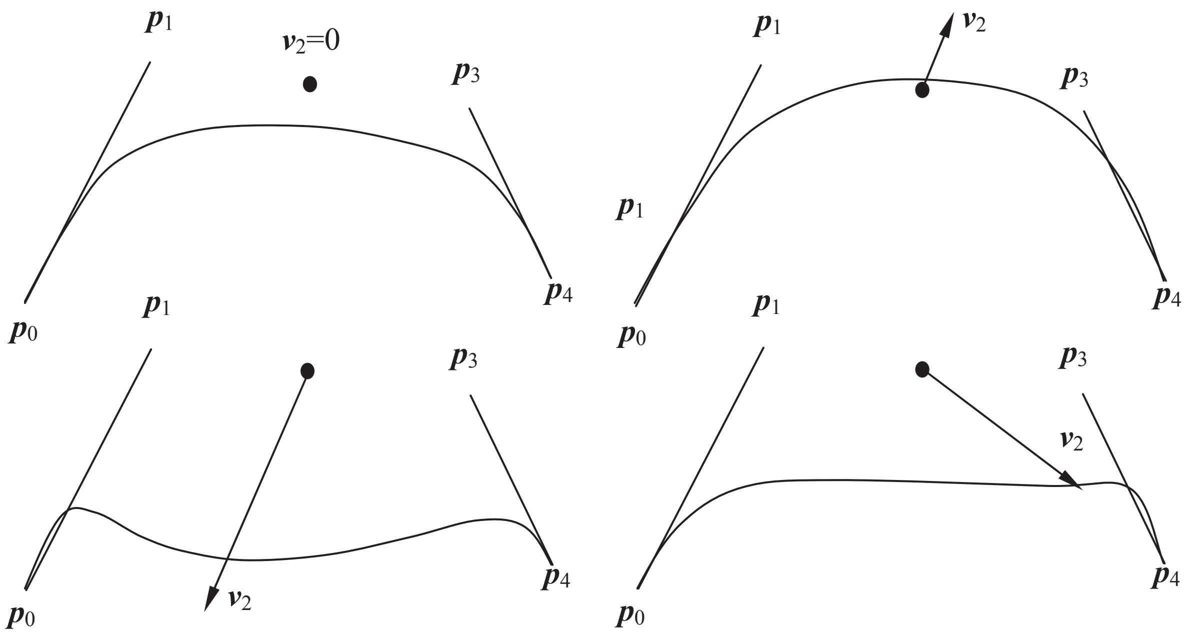

Fig. 2. Modeling of the rational curve by selecting the vector v_i.
Curves have weights $w_0 = w_1 = w_3 = w_4 = 1$, $w_2 = 0$

3. CHARACTERISTICS OF RATIONAL BÉZIER COURVES

From the description of rational representation of Bézier curve and characteristic polynomial one results following characteristics of rational Bézier curves:

Generalization polynomial curves: If all of weights are equal, so denominator in equation

$$\boldsymbol{p}(t) = \frac{\sum_{i=0}^{n} w_i \boldsymbol{p}_i B_i^n(t)}{\sum_{i=0}^{n} w_i B_i^n(t)} \tag{7}$$

have constant function and the curve is polynomial Bézier one.

Uniform set of weights: multiplication of all of weights by this same constant (different from zero) does not change the curve.

Characteristic of convex surroundings: if all of weights are positive (or not negative), so for $t \in [0,1]$ point $\boldsymbol{p}(t)$ is located in convex surroundings of the control points set.

Interpolation of final points: $w_0 \neq 0 \Rightarrow p(0) = p_0$, $w_n \neq 0 \Rightarrow p(1) = p_n$.

Affine invariability: image of the control points set $\boldsymbol{p}_i$ in any affine transformation f represents (in constant weights) the image of the curve $\boldsymbol{p}(t)$ in this transformation; if in representation occurs vectors $\boldsymbol{v}_i$, so there is necessary to transform them by linear part of transformation f.

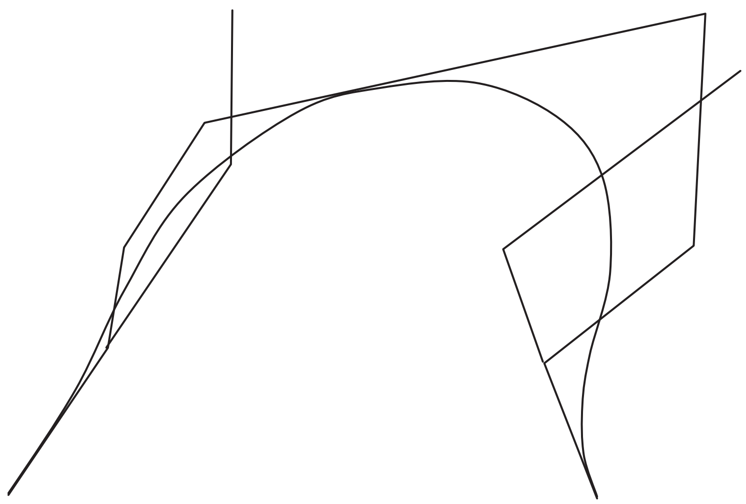

Fig. 3. Broken control line of the curve with weights with different marks and broken lines of parts with weights with constant marks

Division of the arc on parts: rational de Casteljau algorithm, apart the point $\boldsymbol{p}(t)$, finds control points and weights of curve's parts. One of these arcs is determined by points $p_i^{(i)}$ and weights $w_i^{(i)}$, and another by points $p_i^{(n-i)}$ and weights $w_i^{(n-i)}$. Even if initial weight coefficients have different marks, in representation of sufficiently short parts all of weights have this same mark.

The curve can be also divided on parts by dividing the uniform curve. Polynomial de Casteljau algorithm is easier and more convenient then rational one.

4. RESULTS

Sound speed in water have been measured during bathymetric soundings in Gdansk Harbour, October, 2010. The area of the sounding have been presented below. Vertical distribution of the sound speed in water have been measured using CTD probe in one hour intervals: 12:00, 13:00, 14:00.

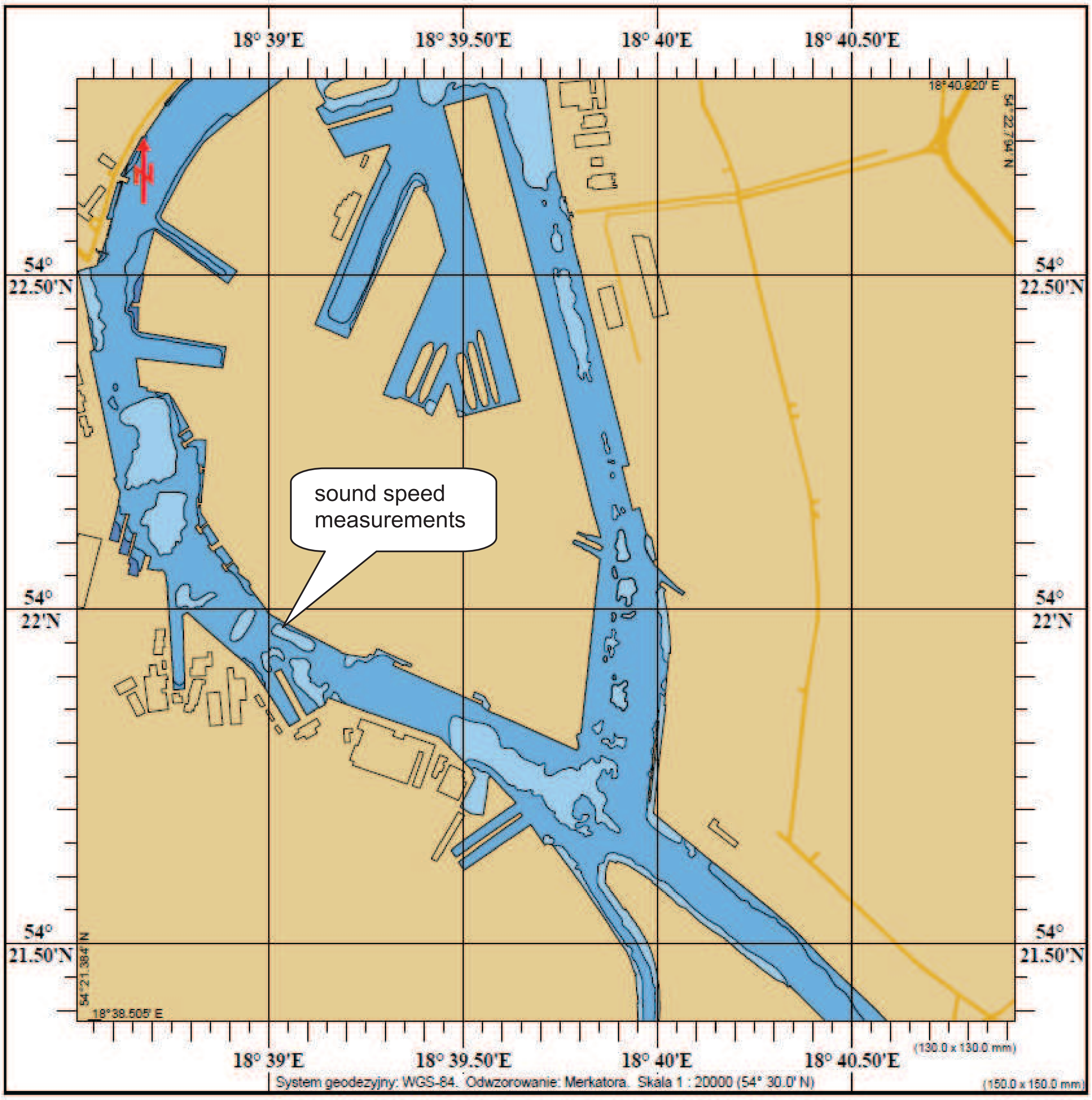

Fig. 4. The area of sound speed measurements

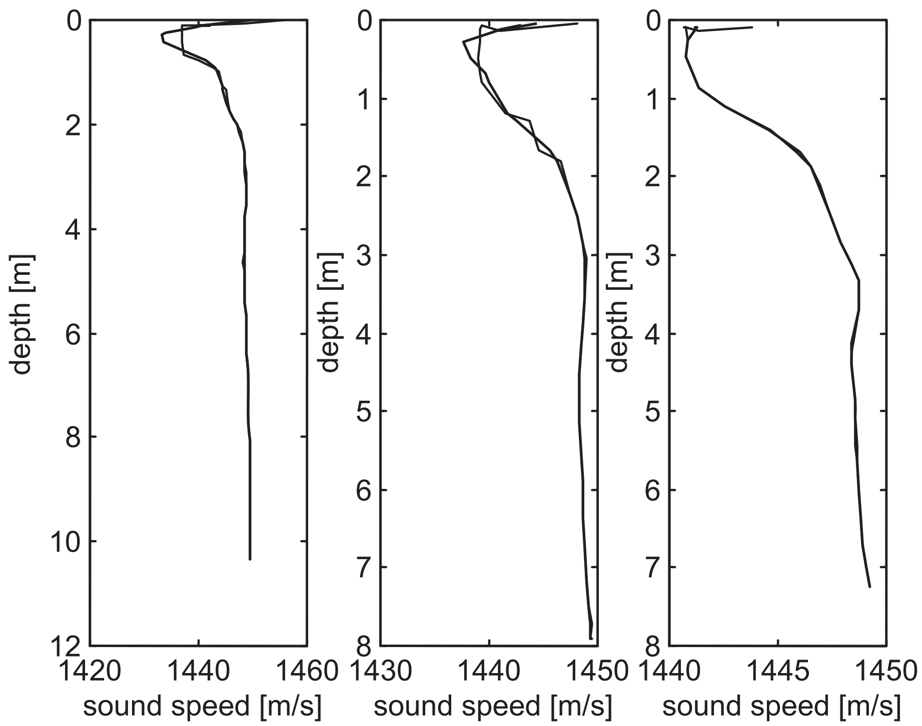

Fig. 5. Real and approximated vertical distributions of the sound speed in water – 31-st of October, 2010: 12:00, 13:00 and 14:00

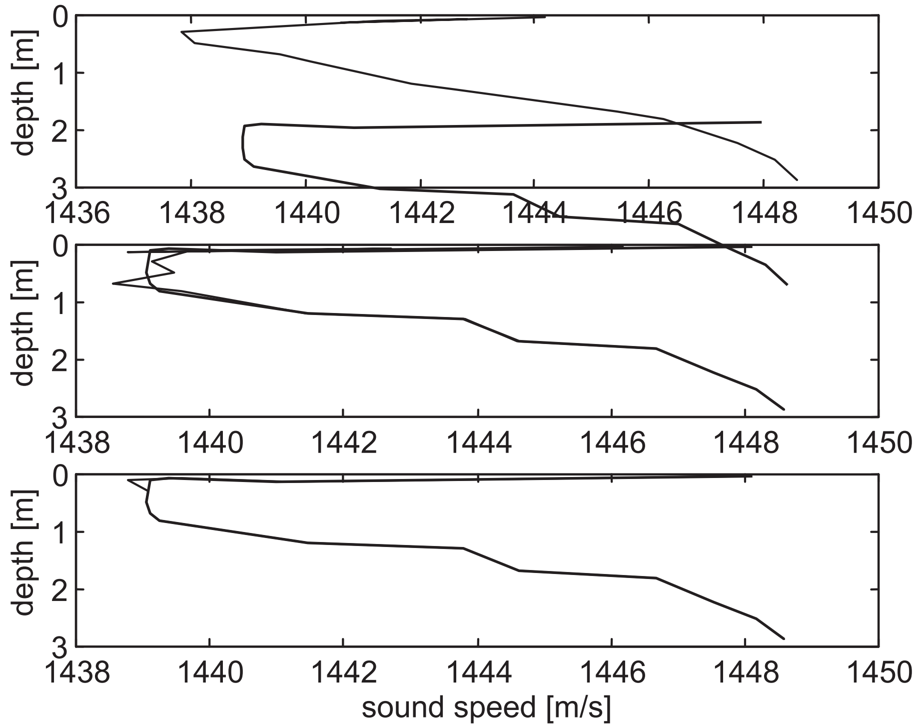

Fig. 6. Short parts of vertical distributions of sound speed in water

5. CONCLUSIONS

Polynomials Bézier curves hale many advantages in modeling but they have also several disadvantages. These disadvantages are in circles and ellipse modeling. Only one of polynomial circle is parabola.

Presented mathematical algorithm of using Bézier curves for presentation the sound speed in water is suitable for hydroacoustis and hydrography.

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