MODELING OF SEA BOTTOM USING UNIFORM RECTANGULAR BÉZIER PIECES

ARTUR MAKAR

Polish Naval Academy
69, Śmidowicza St., 81-103 Gdynia, Poland
A.Makar@amw.gdynia.pl

In the article the essence of uniform rectangular Bézier pieces have been shown. These issues are essential during creation the digital terrain model DTM. For creation the bottom model bathymetric surveys of Motlawa River have been used. The visualization of the bottom using uniform rectangular Bézier pieces has been presented.

INTRODUCTION

Modeling of surfaces in hydroacoustics and hydrography have many applications, e.g. for modeling of sea bottom and surface of constant sound speed in water [2, 10, 12]. There are used well known methods and developed new algorithms, which are used in computer graphics [1, 3, 4, 5, 8, 9, 11, 13, 14, 15].

1. DETERMINATION OF THE PIECE

Rectangular Bézier pieces (aka tensor Bézier pieces) of n – degree in relation to u variable and m – degree in relation to v variable ((m, n) – degree) are described by the equation [1]:

$$\boldsymbol{p}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} \boldsymbol{p}_{i,j} \boldsymbol{B}_i^n(u) \boldsymbol{B}_j^m(v) \tag{1}$$

For determination the piece of (m, n) – degree, there is necessary to give (n+1)(m+1) control points of $\boldsymbol{p}_{i,j}$. The set of segments connecting control points, which only one index differs from 1, is called control frame of the piece. In this control frame we distinguish rows, i.e. broken lines with $\boldsymbol{p}_{0j},...,\boldsymbol{p}_{nj}$ points for established j, and columns, i.e. broken lines with $\boldsymbol{p}_{i0},...,\boldsymbol{p}_{im}$ points for established i.

The method for determination the piece – using tensor base created from functions used for determination Bézier curves – makes possible to use for rectangular Bézier pieces all of theorems and algorithms connected with curves. We can observe, that:

– $\boldsymbol{B}_i^n(u)\boldsymbol{B}_j^m(v)$ functions for $i = 0,\ldots,n$, $j = 0,\ldots,m$ on the basis of relationship $\sum_{i=0}^{n}\boldsymbol{B}_i^n(t)=1$ determine distribution of one. The picture of control frame in any affine transformation determines the picture of the piece in this transformation, because determination them is independent from control points' selection of the piece,
– because on the section [0,1] Bernstein polynomials are non-negative, so functions of tensor base are non-negative in the rectangle [0,1]×[0,1], therefore point $\boldsymbol{p}(u,v)$ for $u,v\in[0,1]$ is in convex border of control points set,
– because $\boldsymbol{B}_i^n(0)\boldsymbol{B}_j^m(0)=0$ for $i\neq 0$ or $j\neq 0$, so $\boldsymbol{p}(0,0)=\boldsymbol{p}_{00}$. Similarly, in remaining corners: $\boldsymbol{p}(1,0)=\boldsymbol{p}_{n0}$, $\boldsymbol{p}(0,1)=\boldsymbol{p}_{0m}$, $\boldsymbol{p}(1,1)=\boldsymbol{p}_{nm}$. Extreme rows and columns of the control frame determine border curves of the piece.

2. RATIONAL BÉZIER PIECE

Definition of the Bézier piece is generalization of qualification of polynomial Bézier piece. Relationship between them is the analogy to relationship between rational and polynomial Bézier curves. To control points $\boldsymbol{p}_{i,j}$, which represents polynomial Bézier piece, assigned weight coefficients (or weights) $w_{i,j}$. Polynomial Bezier piece of (*n*, *m*) degree, described by equation:

$$\boldsymbol{p}(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m} w_{i,j}\boldsymbol{p}_{i,j}B_i^n(u)B_j^m(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} w_{i,j}B_i^n(u)B_j^m(v)} \tag{2}$$

can be modeled by selection control points and weights.

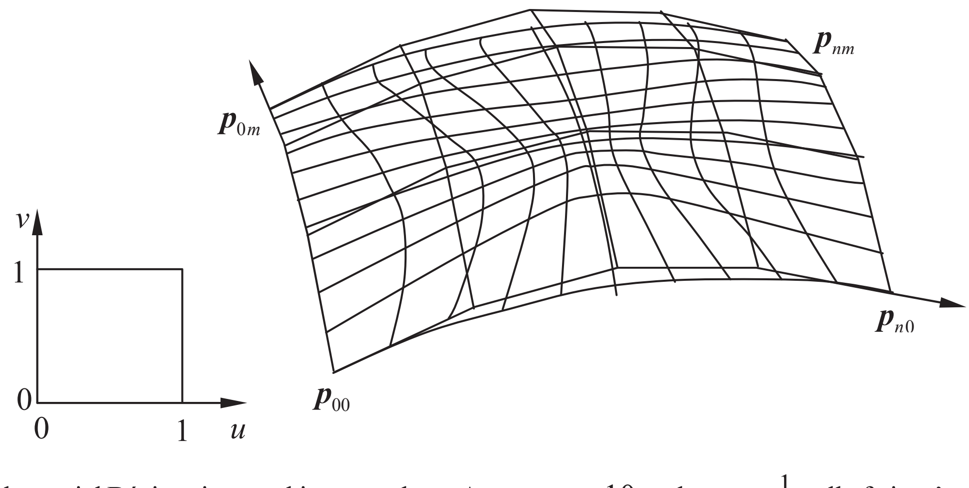

Fig. 1. Polynomial Bézier piece and its control net. Apart $w_{2,1}=10$ and $w_{2,2}=\frac{1}{100}$ all of piece's weights are 1

If $w_{i,j}=0$, then point $\boldsymbol{p}_{i,j}$ has not influence on the piece. Apart control points with zero weights can be used control vectors $v_{i,j}$ and equation:

$$\boldsymbol{p}(u,v)=\frac{\sum_{i=0,\ldots,n,j=0,\ldots,m,w_{i,j}\neq 0} w_{i,j}\boldsymbol{p}_{i,j}B_i^n(u)B_j^m(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} w_{i,j}B_i^n(u)B_j^m(v)}+\frac{\sum_{i=0,\ldots,n,j=0,\ldots,m,w_{i,j}\neq 0} v_{i,j}B_i^n(u)B_j^m(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} w_{i,j}B_i^n(u)B_j^m(v)} \tag{3}$$

Polynomial piece in 3D space has the uniform representation, which determines the polynomial plate in the space of homogenous co-ordinations $\Re^4$. The point $\boldsymbol{p} \in E^3$, with co-ordinations *x*, *y*, *z*, is represented by any vector $\boldsymbol{P} = [X, Y, Z]^T \in \Re^4$ such as $x = \frac{X}{W}$x, $y = \frac{Y}{W}$, $z = \frac{Z}{W}$. So polynomial Bézier piece

$$\boldsymbol{P}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} \boldsymbol{P}_{i,j} B_i^n(u) B_j^m(v) \tag{4}$$

with control points

$$\boldsymbol{P}_{i,j} = \begin{bmatrix} w_{i,j}\boldsymbol{p}_{i,j} \\ w_{i,j} \end{bmatrix} \text{ for } w_{i,j} \neq 0 \text{ or } \boldsymbol{P}_{i,j} = \begin{bmatrix} v_{i,j} \\ 0 \end{bmatrix} \text{ for } w_{i,j} = 0 \tag{5}$$

is one of many rational pieces representing piece $\boldsymbol{p}$.

3. BASIC CHARACTERISTICS OF RATIONAL PIECES

Characteristics of rational pieces can be easy justified on the basis of characteristics of polynomial and rational Bézier curves and polynomial rectangular pieces.

Relationship between rational piece and its control net is affine invariable.

Characteristics of convex surroundings: If weights are positive, then for $(u,v) \in [0,1]\times[0,1]$ point $\boldsymbol{p}(u,v)$ is located inside of convex surroundings of the points' set $\boldsymbol{p}_{i,j}$.

Interpolation of corners of the net and edge curves: Corner points of control net are corners of the rational piece. Points in extreme rows and columns of the net with suitable weights represent rational Bézier curves, which are placed on the edge of the piece.

Non hodograph characteristic: Vectors of particle derivatives of rational Bézier piece usually are not linear combination of vectors $\Delta_1\boldsymbol{p}_{i,j} = (\boldsymbol{p}_{i+1,j} - \boldsymbol{p}_{i,j})$ and $\Delta_2\boldsymbol{p}_{i,j} = (\boldsymbol{p}_{i,j+1} - \boldsymbol{p}_{i,j})$ with positive coefficients, because in general case rational Bézier piece is not tensor piece, i.e. there are not functions $f_i(u)$ and $g_j(v)$, that $\boldsymbol{p}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} \boldsymbol{p}_{i,j} f_i(u) g_j(v)$. If exist numbers $a_0,...a_n$, $b_0,...b_m$, that $w_{i,j} = a_i b_j$, then rational Bézier piece can be written in the form:

$$\boldsymbol{p}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} \boldsymbol{p}_{i,j} \left(\frac{a_i B_i^n(u)}{\sum_{k=0}^{n} a_k B_k^n(u)} \right) \left(\frac{b_j B_j^m(v)}{\sum_{k=0}^{m} b_k B_k^m(v)} \right) \tag{6}$$

If a_i and b_j are positive, then on the segment [0,1] functions $f_i(u) = \left(\frac{a_i B_i^n(u)}{\sum_{k=0}^{n} a_k B_k^n(u)} \right)$ and $g_j(v) = \left(\frac{b_j B_j^m(v)}{\sum_{k=0}^{m} b_k B_k^m(v)} \right)$ are negative. On the basis of the characteristics of the hodograph of rational Bézier curves with all positive weights, can be proved that rational Bézier pieces have characteristic of hodograph – e.g. for $u, v \in [0,1]$ vector $\boldsymbol{p}_u(u,v)$ is linear combination of vectors $\Delta_1 \boldsymbol{p}_{i,j}$ with nonnegative coefficients.

Determination of points and division the rational piece can be reduced to determination of points and division the uniform rational piece. Generally, there rational de Casteljau algorithm can not be used for columns and rows of control net. It is possible if the piece can be presented in the tensor form, i.e. when exist numbers a_i and b_j, that $w_{i,j} = a_i b_j$.

Increasing of the degree can be realized by presentation the uniform piece in the base of Bernstein polynomials higher degree or multiplying it by any polynomial.

Particle derivatives of k-th degree along the edge curve depend of k+1 extreme rows or columns of the net. They can be calculated using rational representation of the piece.

4. RESULTS

Hydrographic surveys have been realized using singlebeam echosounder Simrad EA400 with frequencies: 50kHz and 200kHz on Motlawa River in Gdansk.

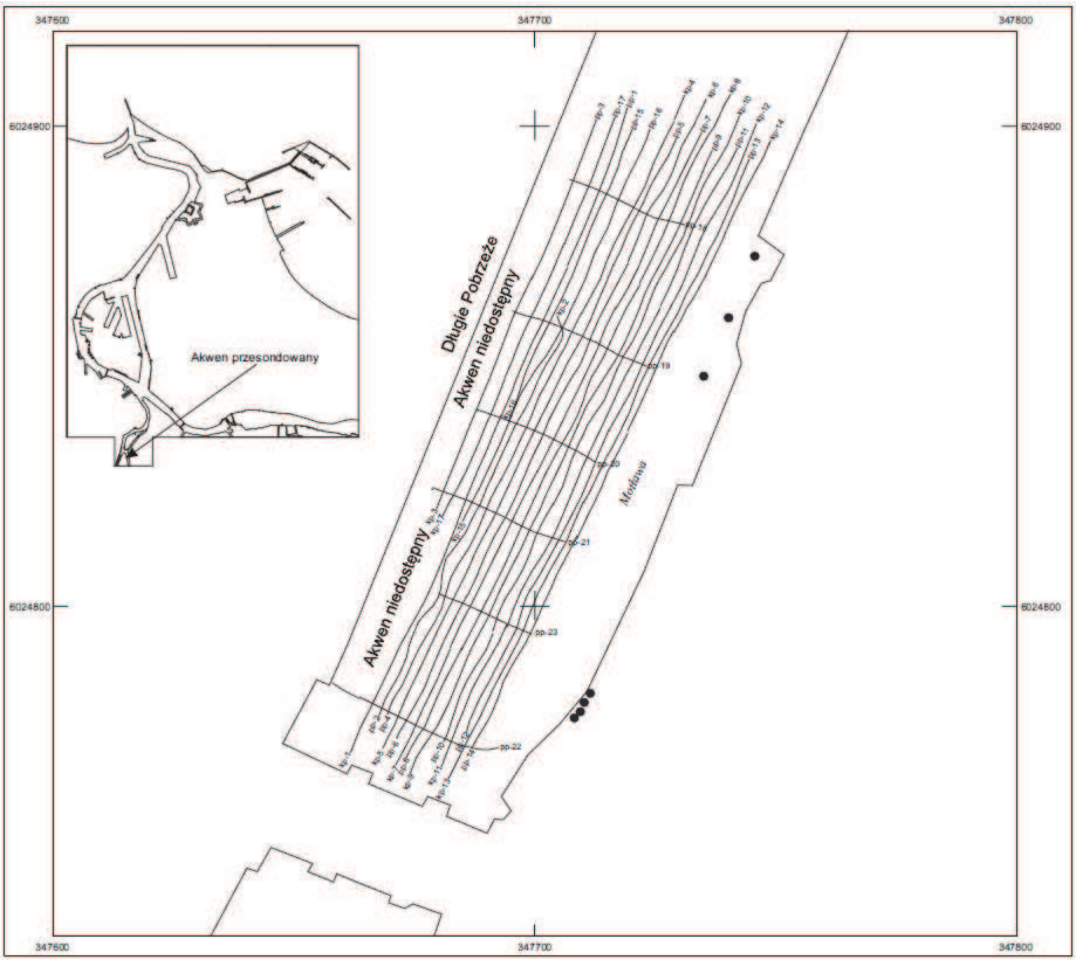

Fig. 2. Profile sheet

The area of surveys with profiles have been presented in Fig. 2, bathymetric sheet has been presented in Fig. 4 and spatial presentation of the bottom has been presented in Fig. 4.

Fig. 3. Bathymetric sheet

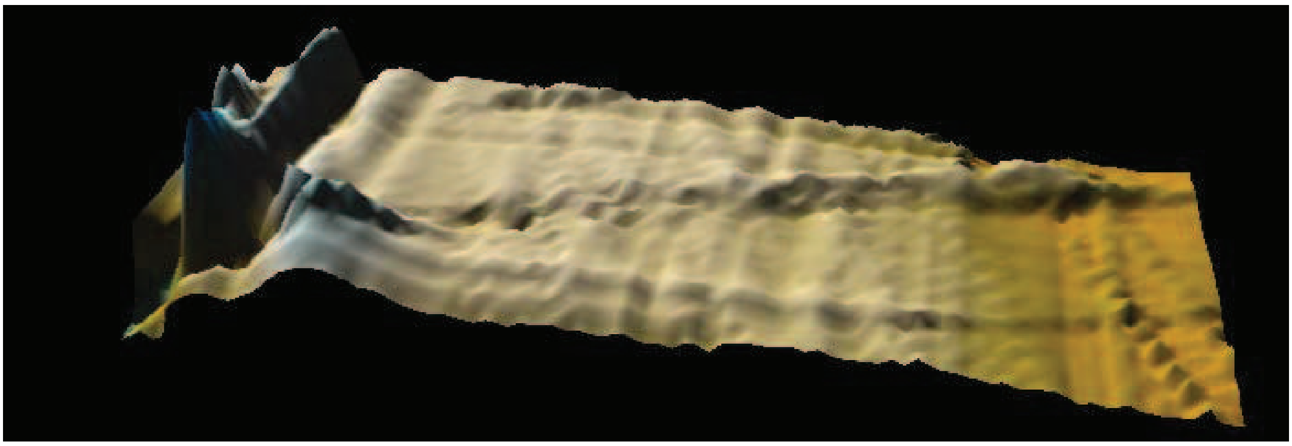

Fig. 4. 3D visualization of the sea bottom

5. CONCLUSIONS

Rational rectangular Bézier pieces are another method used in modeling of surfaces and it is generalization of polynomial Bézier piece. Relationship between them is similar to rational and polynomial Bézier curves.

Presented method has been successfully used for presentation of the bottom on the basis of hydrographic surveys using singlebeam echosounder.

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