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INFLUENCE OF THE NOTCH RADIUS ON CHANGES OF THE ΔJ PARAMETER UNDER FATIGUE CRACK GROWTH RATE

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In the paper, the influence of the notch radius on changes of the ΔJ parameter under low and high-cycle fatigue is discussed. The tests were carried out on plates made of FeP04-UNI 8092 deep-drawing steel. The specimens were characterised by double symmetric lateral notches with the notch root radius ranging from 0.2 mm to 10 mm. The MTS 809 servo-hydraulic device was used for tests performed at the Department of Management and Engineering in Vicenza (Padova University). All fatigue tests were performed under force control, by imposing a constant value of the nominal load ratio, $R = 0$, and a load amplitude $P_a = 6$ and 7 kN (which corresponded to the nominal amplitude of normal stresses $\sigma_a = 100$, 117 MPa before the crack initiation). The test frequency ranged from 13 and 15 Hz.

Key words: fatigue crack growth, number of cycles, ΔJ parameter, load ratio, notch

Notations

a	–	crack length
a_0	–	notch depth
da/dN	–	fatigue crack growth rate
n'	–	cyclic strain hardening exponent
w	–	specimen width
E	–	Young's modulus
K_t	–	theoretical stress concentration factor
K'	–	cyclic strength coefficient
N	–	number of cycles
Pa	–	amplitude of load
R	–	load ratio

Y	–	correction factor
ν	–	Poisson's ratio
ρ	–	notch tip radius
σ_{max}	–	maximum stress
σ_{nom}	–	nominal stress
σ_u	–	ultimate tensile stress
σ_y	–	yield stress
ΔJ	–	energy parameter
$\Delta\varepsilon_p$	–	plastic strain range

1. Introduction

Fatigue crack growth is directly connected with existing stress concentrators in a material. Cracks usually initiate in the notch (micro notch). In many cases, shape of the notch influences life of the tested material (element). That problem was discussed in a paper by Williams (1952). He showed that, according to theory of elasticity, the asymptotic stress state near a re-entrant corner is singular and its degree of singularity just depends on the notch opening angle. The stress field intensity is a function of the overall geometry of the component and the far-field loading. Since Williams's pioneering work, several studies have been undertaken to determine the stress field present at various notch geometry and at singular points where the interface between two elastic solids intersects the traction-free edge. Some recent papers (Lazzarin *et al.*, 1997; Taylor, 1999) suggest that there exists a strong relation between the dimension of this critical zone and El-Haddad *et al.* (1979) length parameter $l_0(\Delta K_{th0})$. Averaging the maximum principal stress in a finite volume ahead of the notch tip could be a valid idea from the engineering point of view, but presents some drawbacks from the theoretical point of view. In fact, the more l_0 increases, the greater the influence of the other principal stresses becomes (values of l_0 equal 3.0 mm have been determined in some cast irons by Taylor *et al.*) (1996)). It is the authors' opinion that an energetic approach is undoubtedly preferable, despite the fact that it necessarily results in some algebraic complications. The approach to fatigue crack growth using the J parameter seems to be the most promising (Tanaka, 1983). The ΔJ parameter allows one to estimate changes of local and global energies while fatigue crack growth in the tested element (Rozumek *et al.*, 2006; Berto and Lazzarin, 2007; Rozumek, 2008).

The aim of this paper is taking into account the influence of the notch radius on changes of the ΔJ parameter under fatigue crack growth rate.

2. Materials and test procedure

The tests were carried out on plates made of FeP04-UNI 8092 deep-drawing steel, weakened by symmetric lateral notches of varying acuity. In the tests, the MTS 809 servo-hydraulic device was used at the Department of Management and Engineering in Vicenza (Padova University). Table 1 contains chemical compositions of the material, and its some mechanical properties are shown in Table 2.

Table 1. Chemical composition (in wt%) of FeP04 steel

C	Mn	Si	P	S	Al	Cu	Fe
0.05	0.30	0.05	0.032	0.02	0.043	0.07	balance

Table 2. Monotonic quasi-static tension properties of FeP04 steel

σ_y [MPa]	σ_u [MPa]	E [GPa]	ν
210	330	191	0.30

Coefficients of the Ramberg-Osgood equation describing the cyclic strain curve under tension-compression with $R = -1$ for FeP04 steel are the following: the cyclic strength coefficient $K' = 838\,\mathrm{MPa}$, the cyclic strain hardening exponent $n' = 0.220$. All fatigue tests were performed under force control, by imposing a constant value of the nominal load ratio, $R = 0$, and a load amplitude $P_a = 6$ and $7\,\mathrm{kN}$ (which corresponded to the nominal amplitude of normal stresses $\sigma_a = 100$, $117\,\mathrm{MPa}$ before crack initiation). The test frequency ranged from 13 and 15 Hz. The specimens had double symmetric lateral notches with notch root radii ranging from 0.2 mm to 10 mm (Fig. 1). The theoretical stress concentration factor in the specimen under tension $K_t = 9.61$, 4.30, 3.23 and 1.85 was estimated by making use of the model (Thum *et al.*, 1960). The surfaces of the specimens, $w = 50\,\mathrm{mm}$ wide, had been accurately polished in order to make the cracks originated from the notch tip easily distinguishable. The fatigue crack increments were measured with a micrometer located in a portable microscope with magnification of 20 times and accuracy 0.01 mm. At the same time, the number of loading cycles N was registered.

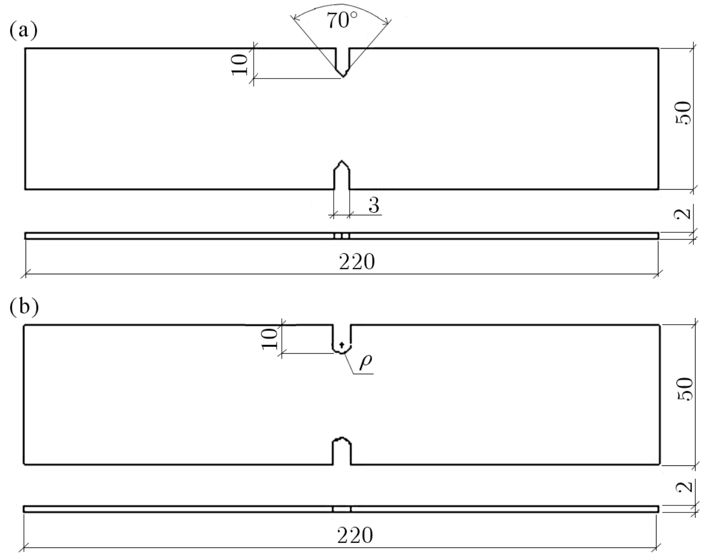

Fig. 1. Geometry of specimens characterised by: (a) sharp and (b) blunt notches; all dimensions in mm

3. Experimental results and discussion

The tests of fatigue crack growth in FeP04 steel subjected to tension were performed in the low and high cycles fatigue under a controlled loading. During the tests, a number of cycles to the crack initiation N_i, (i.e. to the moment of occurrence of a visible crack) was determined, and fatigue crack lengths were measured. The test results were shown as graphs of the crack length a versus the number of cycles N and fatigue crack growth rate da/dN versus the ΔJ parameter. The cracks initiated (minimal observable crack length about 0.1 to 0.2 mm) at the same time on the left and on the right of the slot. From Fig. 2 presenting the crack length a versus number of cycles N it appears that after changing the notch root radii ρ from 0.2 to 10 mm, the fatigue life increases. It is evident that with the highest radii, the initiation phase, which depends on the stress conditions at the notch tip, prevails.

The author proposed to describe the fatigue crack growth range according to the energy approach based on the ΔJ parameter, which could be written as (Rozumek, 2004)

$$\Delta J_1 = \pi Y_1^2 \Big(\frac{\Delta \sigma_{nom}^2}{E} + \frac{\Delta \sigma_{nom} \Delta \varepsilon_p}{\sqrt{n'}} \Big) a \tag{3.1}$$

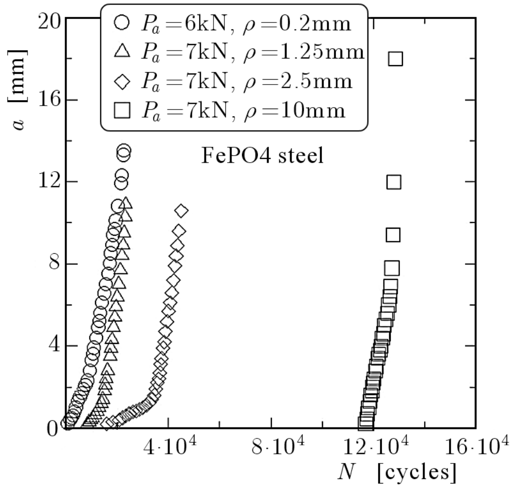

Fig. 2. Fatigue crack length versus number of cycles for different radii of the notch root

If Eq. (3.1) is related to short cracks and includes K_t, it takes the following form

$$\Delta J_2 = \pi Y_1^2 \Big(\frac{\Delta\sigma_{max}^2}{E} + \frac{\Delta\sigma_{max}\Delta\varepsilon_p}{\sqrt{n'}} \Big) a \tag{3.2}$$

where: $\Delta\sigma_{max} = K_t \Delta\sigma_{nom}$ is the maximum stress range, $\Delta\sigma_{nom}$ – nominal stress range, a is the crack length, Y_1 – correction factor dependent on specimen geometry and loading type

$$Y_1 = 1.12 + 0.203 \Big(\frac{2(a+a_0)}{w} \Big) - 1.197 \Big(\frac{2(a+a_0)}{w} \Big)^2 + 1.930 \Big(\frac{2(a+a_0)}{w} \Big)^3$$

a_0 – notch depth, $\Delta\varepsilon_p$ – plastic strain range corresponding to the stress range.

In Eqs. (3.1), (3.2) and (3.3), the exponent n' concerns the functions in which materials slightly harden or are cyclically stable (Rozumek and Pawliczek, 2004). A next modification of Eq. (3.1) including the notch influence by introduction of the correction factor Y_2, allowing its application for short and long cracks, can be expressed as

$$\Delta J_3 = \pi Y_1^2 Y_2^2 \Big(\frac{\Delta\sigma_{nom}^2}{E} + \frac{\Delta\sigma_{nom}\Delta\varepsilon_p}{\sqrt{n'}} \Big) a \tag{3.3}$$

where

$$Y_2 = \sqrt{1 + \mathrm{e}^{-\beta}} \qquad \text{with} \qquad \beta = \frac{3\rho(w - 2a_0)a}{\rho + w - 2a_0}$$

and ρ – notch tip radius.

Figures 3 and 4 show the fatigue crack growth rate da/dN versus ΔJ relations under different notch root radii and load amplitude conditions. Figure 3 presents a comparison of the parameter ΔJ with and without the coefficient K_t, and in Fig. 4 one can see the influence of different correction factors Y_1 and Y_2 on variations of the parameter ΔJ. Equation (3.2) can be applied only for short fatigue cracks growth because local stress concentrations cause high stresses and accompanying strains (see Fig. 3), and in the case of long cracks the values calculated according to Eq. (3.2) are overestimated. From Fig. 3 it also appears that the smaller is the notch root radius, the greater is the influence of stress concentration. That conclusion can be proved by literature. Lazzarin *et al.* (1997) proposed the correction factor Y_2 including occurrence of the notch for the least (theoretical) notch radius $\rho = 0$. Introducing the relationships proposed by Lazzarin *et al.* (1997) into Eq. (3.3), we obtain results in which the values including the notch influence are smaller than the values with no influence of the notch. For the notch radius ρ close to zero, the results described by Eqs. (3.1) and (3.3) coincided, and for $\rho = 10$ mm the greatest differences were obtained. It is inconsistent with the real behaviour of the notch and cracks in the elements (see Rozumek, 2008).

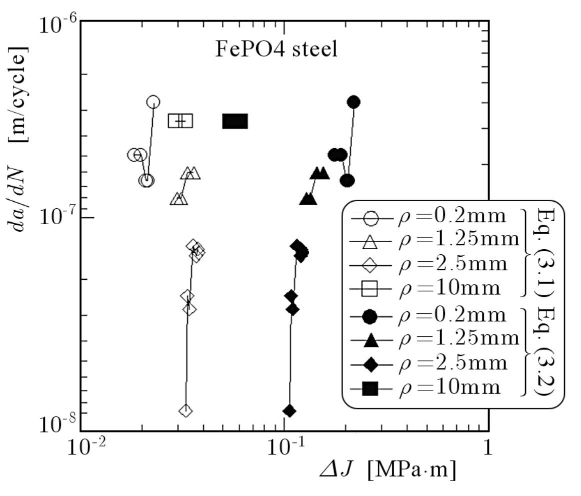

Fig. 3. Fatigue crack growth rate da/dN versus the ΔJ parameter with and without the factor K_t

In order to obtain real results, the notch root radius ρ approaching infinity ($\rho \to \infty$) was assumed and the correction factors Y_2 and β were corrected. Then, for the least notch radius ρ, we obtain the greatest differences between the values obtained from Eqs. (3.1) and (3.3) (see Fig. 4).

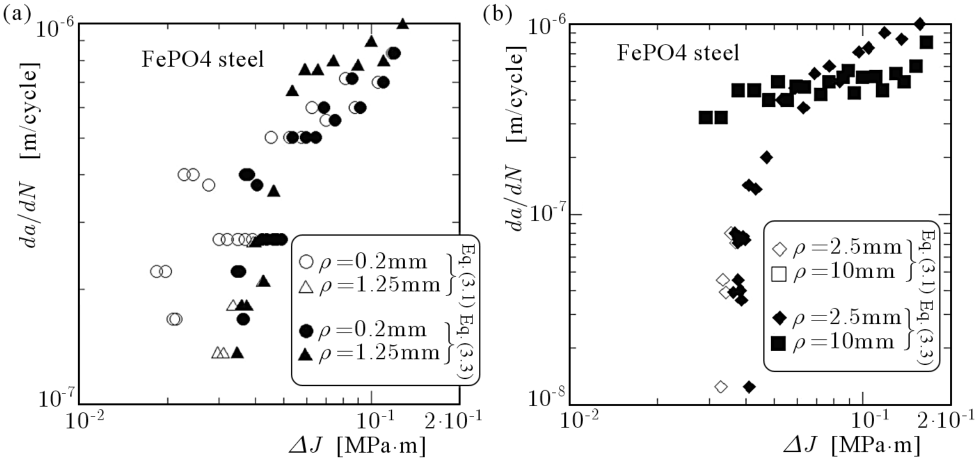

Fig. 4. Fatigue crack growth rate da/dN versus the ΔJ parameter with and without the correction factor Y_2 for (a) $\rho = 0.2$, 1.25 mm and (b) $\rho = 2.5$, 10 mm

From Fig. 4 it appears that the is the greater the notch radius, the smaller is the difference between the results obtained from Eqs. (3.1) and (3.3). The greatest notch influence was observed for $\rho = 0.2$ mm, up to about $a = 6$ mm. Figure 4 was divided into Figs. 4a and 4b. Figure 4a shows how the range of the parameter ΔJ varies versus the fatigue crack growth rate for the notch radius $\rho = 0.2$ and 1.25 mm with and without the correction factor Y_2. Figure 4b presents the influence of the correction factor Y_2 and its lack (Eq. (3.1) and Eq. (3.3)) for the notch radius $\rho = 2.5$ and 10 mm.

As the notch radius increases from $\rho = 0.2$ to 10 mm, the value of ΔJ_3 increases in relation to ΔJ_1. From Fig. 4b it appears that for the notch root radius $\rho = 10$ mm the results coincide. However, for $\rho = 0.2$ to 2.5 mm there is a significant difference between the results including and not including the notch effect. The difference can be seen during crack growth and the notch influence, then it disappears. From Figs. 3 and 4 it appears that the notch strongly influences variation of the ΔJ parameter both globally and locally. In the elastic-plastic range, the stresses and strains were calculated by means of the finite element software FRANC2D. The cyclic stress-strain curve based on the nonlinear material model was introduced into FRANC2D. In that case, the cyclic stress-strain curve for FeP04 steel described by the Ramberg-Osgood relation was used. It was decided that calculations would be based on incremental elastic-plastic analysis including the kinematic model of material hardening. The calculations were performed for two-dimensional geometrical models of notched specimens. The geometrical model of the specimen was simulated in the CASCA software. The finite element mesh was automatically generated and it contained more than 3900 six-nodal triangular elements. The crack growth direction was assumed on the basis of observation of the current crack path obtained from the tests. Such simulation of the crack propagation allowed one to obtain the strain and stress maps for each realised crack increment.

4. Conclusions

The presented results of the fatigue crack growth in the plane notched specimens made of FeP04 steel and subjected to tension loading allow one to formulate the following conclusions:

- Equation (3.2) can be applied only for short fatigue cracks growth because local stress concentrations cause high stresses and accompanying strains.
- Equation (3.3) well describes the results including an additional influence of the notch during fatigue crack growth and it can be applied for determination of length and quality of the notch influence.
- It has been shown that the notch strongly influences variation of the ΔJ parameter, both globally and locally.

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Wpływ promienia karbu na zmiany parametru ΔJ przy prędkości wzrostu pęknięć zmęczeniowych

Streszczenie

W pracy przedstawiono wpływ promienia karbu na zmiany parametru ΔJ podczas badań wykonywanych w zakresie niskiej i wysokiej liczby cykli zmęczeniowych. Badaniom poddano próbki płaskie wykonane ze stali FeP04-UNI 8092. Próbki charakteryzowały się dwustronnymi symetrycznymi zewnętrznymi karbami z promieniami dna karbu w zakresie od 0,2 do 10 mm. Wyniki badań prezentowane w pracy wykonywano na maszynie hydraulicznej MTS 809 w Department of Management and Engineering w Vicenzy (Universytet w Padwie). Wszystkie badanie zmęczeniowe były wykonywane przy obciążeniu z kontrolowaną siłą przy stałej wartości współczynnika asymetrii cyklu $R = 0$ i amplitudzie obciążenia $P_a = 6$ i 7 kN (która korespondowała z nominalną amplitudą naprężenia normalnego $\sigma_a = 100$, 117 MPa przed inicjacją pęknięcia). Badania wykonywano w zakresie częstotliwości obciążenia 13 i 15 Hz.

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