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THE CONTROL LAWS HAVING A FORM OF KINEMATIC RELATIONS BETWEEN DEVIATIONS IN THE AUTOMATIC CONTROL OF A FLYING OBJECT

Edyta Ładyżyńska-Kozdraś

Warsaw University of Technology, Faculty of Mechatronics, Warsaw, Poland

e-mail: e.ladyzynska@mech.pw.edu.pl

The paper presents some practical applications of control laws in dynamics of flying objects. The control laws considered have the form of kinematics and geometrical deviation relations between the actual parameters and those specified that result from guidance of the considered object. The specified parameters were introduced into the control laws as: parameters determined by motion of the target in the guided missile or motion parameters of the beam tracking the target. A general model of dynamical behaviour of the guided object was employed in the considerations.

Key words: automatic control of missiles, homing and beam-riding guidance, numerical simulations

1. Introduction

Before starting the design process of anti-aircraft missile one should solve the problem of control. A proper control should include the flight control systems that ensure the optimal missile guidance onto a target.

The paper aims at presentation of a general model of a flying object with non-holonomic constraints imposed. These constraints take the form of control laws assumed for the examined object motion in terms of kinematical relations between deviations from the preset to current values, respectively, of selected parameters. Sample missiles of different types guided onto maneuvering targets served to illustrate the approach.

It is well known that the crucial element of each control system consists in the guidance algorithm it performs. The algorithm imposes constraints on the object motion; therefore, a proper choice of the control method is essential. When dealing with missile guidance, one can apply two or three-point methods, respectively (Ben-Asher and Yaesh, 1998; Blakelock, 1991; Dziopa, 2006; Menon *et al.*, 2003; Zarchan, 2001). In the two-point method, constraints are imposed on the missile-target motion, while in the three-point method the guidance station should be considered as well.

To implement automatic control and navigation systems of advanced objects, especially those computer-aided, one should assume proper physical models and develop the mathematical ones capable of representing dynamical properties of the object. Then, proper control laws and kinematical relations of guidance and navigation should be formulated. The kinematical and dynamical behaviour of the executing system should be assumed as well and the signaling way of current position parameters. Motion of the object and the program of predetermined trajectory with specific limits imposed should be accepted as well. When developing a model, a kind of compromise should be agreed on the influence of different aspects of the problem; i.e., accuracy of theoretical analysis of the physical problem, complexity of mathematical equations, availability of technical means and personal knowledge – skills intuition and experience should be properly balanced.

A new, relatively uncomplicated approach to the problem of automatic control of an object has been presented in the paper, basing on a general mathematical model of a flying object under control with the control laws introduced. The method efficiency can be improved when the control laws are considered as non-holonomic constraints imposed upon the object motion.

2. General form of control laws

The paper presents sample applications of control laws to stabilisation, control, guidance and navigation, respectively, of flying objects under automatic control. The preset flight parameters (bearing the index z) may appear in the control laws in different ways; i.e., as parameters of a steady flight, as parameters resulting from the accepted guidance method, flight program or way of reaching the preset target as well as those from the tracking of terrain obstacles, and finally as parameters ensuring the required flight state.

The control laws for flying objects given below have the form of kinematical equations of deviations between the preset and current values of flight parameters observed in the roll, pitch, yaw and velocity channels, respectively. The differences appearing between the current parameters and the preset ones, respectively, determine the deflections. As a result, the forces acting on control surfaces change so that the object returns onto its predetermined trajectory.

The accepted control laws have the form (Ładyżyńska-Kozdraś, 2006; Ładyżyńska-Kozdraś and Maryniak, 2003; Maryniak, 1987):
— in the pitch channel

$$\begin{aligned} T_3^H\dot{\delta}_H + T_2^H\delta_H &= K_{x_1}^H(x_1 - x_{1z}) + K_{z_1}^H(z_1 - z_{1z}) + K_U^H(U - U_z) + \\ &+ K_W^H(W - W_z) + K_Q^H(Q - Q_z) + K_\theta^H(\theta - \theta_z) + \delta_{H0} \end{aligned} \tag{2.1}$$

— in the yaw channel

$$\begin{aligned} T_3^V\dot{\delta}_v + T_2^V\delta_V &= K_{y_{11}}^V(y_1 - y_{1z}) + K_V^V(V - V_z) + K_P^V(P - P_z) + \\ &+ K_\phi^V(\phi - \phi_z) + K_R^V(R - R_z) + K_\psi^V(\psi - \psi_z) + \delta_{V0} \end{aligned} \tag{2.2}$$

— in the roll channel

$$\begin{aligned} T_3^L\dot{\delta}_L + T_2^L\delta_L &= K_V^L(V - V_{1z}) + K_W^L(W - W_z) + K_P^L(P - P_z) + \\ &+ K_\phi^L(\phi - \phi_z) + K_Q^V(Q - Q_z) + K_R^V(R - R_z) + \delta_{L0} \end{aligned} \tag{2.3}$$

— in the velocity channel

$$\begin{aligned} T_3^T\dot{\delta}_T + T_2^T\delta_T &= K_{x_{11}}^T(x_1 - x_{1z}) + K_{z_{11}}^T(z_1 - z_{1z}) + K_U^T(U - U_z) + \\ &+ K_W^T(W - W_z) + K_\theta^T(\theta - \theta_z) + K_Q^H(Q - Q_z) + K_\phi^T(\phi - \phi_z) + \\ &+ K_\psi^T(\psi - \psi_z) + \delta_{T0} \end{aligned} \tag{2.4}$$

where

T_{ij}	–	time constants
K_{ij}	–	amplification coefficients
$\delta_H, \delta_V, \delta_L, \delta_T$	–	deflections of: elevator δ_H, rudder δ_V, ailerons δ_L and throttle lever δ_T, respectively
ϕ, θ, ψ	–	the angles of roll ϕ, pitch θ and yaw ψ, respectively (Fig. 1)
x_1, y_1, z_1	–	components of the position vector of the object relative to the fixed gravitational frame of reference (Fig. 1)
U, V, W	–	components of the velocity vector (Fig. 1)
P, Q, R	–	angular velocities of: roll P, pitch Q and yaw R, respectively (Fig. 1).

Since they are non-intergrable and impose limitations on the system motion, these control laws define two equations of non-holonomic constraints.

The control laws together with the equations of motion determine the object trajectory and its behaviour along it.

The square coefficient of control quality was applied to the control process quality assessment (Ładyżyńska-Kozdraś *et al.*, 2005), with all control channels considered $(n = 4)$, i.e.

$$J = \sum_{i=1}^{4} \int_{0}^{t_k} [y_i(t) - y_{zi}(t)]^2 \, dt \tag{2.5}$$

where

$y_{zi}(t)$ – denotes the predetermined course of the variable

$y_i(t)$ – stands for the actual course of the variable.

Since the coefficient given in Eq. (2.5) has not been normalised, it cannot be applied to the analysis of transient processes in which the quantities under control reveal different orders of magnitude. One should, therefore, normalise it using e.g. the formulae for relative deviations (Ładyżyńska-Kozdraś, 2006; Ładyżyńska-Kozdraś and Maryniak, 2003)

$$J = \sum_{i=1}^{4} \int_{0}^{t_k} \Big[\frac{y_i(t) - y_{zi}(t)}{y_{i\,max}}\Big]^2 \, dt \tag{2.6}$$

where $y_{i\,max}$ is the maximum preset range of the ith state variable or the preset value y_{zi} of the ith state variable if it takes a non-zero value.

It should be noted, however, that depending on the task to be executed and the type of flying object the control laws can be reduced and adapted adequately. This will be shown in simulation examples in the next part of this paper.

The kinematical relations representing the object motion are the following functions of its linear and angular velocities

$$\dot{\boldsymbol{r}} = [\dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{\phi}, \dot{\theta}, \dot{\psi}]^{-1} = \boldsymbol{F}(U, V, W, P, Q, R, \phi, \theta, \psi) \tag{2.7}$$

The kinematical equations of the flying object (bearing the index R) guidance onto the target (bearing the index C) can be written as follows

$$\begin{aligned} \dot{\boldsymbol{r}}_{RP} &= \boldsymbol{f}_1(V_R, V_C, \varepsilon_{RC}, \nu_{RC}, \phi_C, \theta_C, \psi_C, \phi_R, \theta_R, \psi_R) \\ \dot{\boldsymbol{r}}_{1R} &= \boldsymbol{f}_2(V_R, V_C, \phi_C, \psi_C, \theta_C, \phi_R, \theta_R, \psi_R) \end{aligned} \tag{2.8}$$

In the case when the flying object moves on the predetermined trajectory, the program constraints should be imposed

$$\boldsymbol{r}_1 = \boldsymbol{f}_3(x_1, y_1, z_1, \phi_z, \theta_z, \psi_z) \tag{2.9}$$

To ensure the proper course of automatic control of the flying object, one should introduce into the control laws some selected flight parameters that result from the predetermined flight trajectory together with flight parameters of the enemy or parameters of the laser terrain penetration.

One should also consider dynamical behaviour of the control executing system represented by the following equations given in a general form (Ładyżyńska-Kozdraś and Maryniak, 2003; Maryniak, 2007):

— kinematical equation of control in the pitch channel

$$T_1^H \dot{\delta}_H + T_2^H \delta_H = -(M_{z0}^H + K_z^{\alpha_H} \alpha_H + K_z^{\delta_H} \delta_H + K_z^H \dot{\delta}_H) \tag{2.10}$$

— kinematical equation of control in the yaw channel

$$T_1^V \dot{\delta}_V + T_2^V \delta_V = -(M_{z0}^V + K_z^{\beta_V} \beta_V + K_z^{\delta_V} \delta_V + K_z^V \dot{\delta}_V) \tag{2.11}$$

— kinematical equation of control in the roll channel

$$T_1^L \dot{\delta}_L + T_2^L \delta_L = -(M_{z0}^L + K_z^{\alpha_L} \alpha_L + K_z^{\delta_L} \delta_L + K_z^L \dot{\delta}_L) \tag{2.12}$$

where the values of particular coefficients depend on the design and type of the control system applied (mechanical, electrical or electronic) as well as on its performance quality and

$M_{z0}^H, M_{z0}^V, M_{z0}^L$	–	moments of forces necessary for automatic control of: elevator, rudder and aileron displacements, respectively
$K_z^{\alpha_H}, K_z^{\beta_V}, K_z^{\alpha_L}$	–	coefficients of the stiffness forces due to changes in: angles of attack of the elevator unit, sideslip angle and aileron displacement, respectively
$K_z^{\delta_H}, K_z^{\delta_V}, K_z^{\delta_L}$	–	drag coefficients in the control due to its stiffness, depending us on the angles of control surface deflections
K_z^H, K_z^V, K_z^L	–	drag coefficients in the control system due to the velocity of control surface deflections.

In the case of beam-riding missile guidance, the dynamical equations of missile motion were combined with kinematical equations of constraints through the Maggi equations for non-holonomic systems (Ładyżyńska-Kozdraś *et al.*, 2005; Menon *et al.*, 2003). In the case of a missile homing onto a maneuvering target, the Bolztmann-Hamel equations for non-holonomic systems were used (Ładyżyńska-Kozdraś, 2008; Menon *et al.*, 2003). As a result, a system of automatic stabilisations in the roll, pitch and yaw channels, respectively, is obtained.

The approach to the problem of flying object control presented above may be used for a broad range of flying objects; i.e., missile, torpedo, plane or helicopter. Advantages of the approach are particularly visible in the case of a flying object with non-holonomic constraints imposed.

3. Real flight parameters for a sample missile

The current parameters represent the real way the flying object behaves on its trajectory during the guidance process. Over the whole flight, the parameters are automatically registered and read out by the control system and depend only on the real behaviour of the object on its trajectory.

In the three-point methods (e.g. beam-riding guidance) the missile flight is tracked from the earth, therefore, in this case the earth-fixed frame of reference $O_1x_1y_1z_1$ (Fig. 1) is considered as the main one, relative to which all kinematical relations true for the current phase of missile flight are determined.

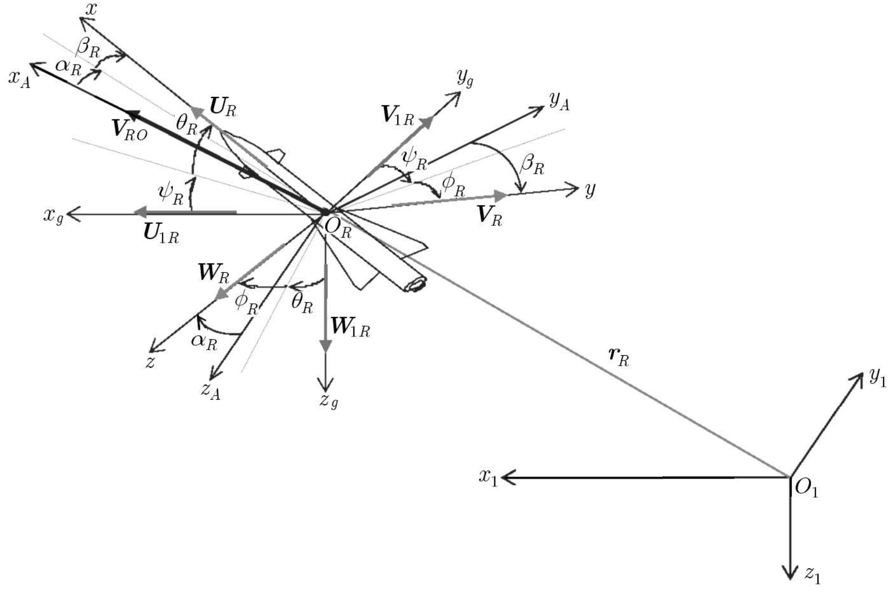

Fig. 1. Real parameters of the missile in the course of guidance

When one employs the two-point method (homing onto a target), the missile is homing basing on the information gathered without a delay from its own on-board equipment. In that case the main frame of reference O_Rxyz is fixed to the moving missile.

It can be seen from Fig.1 that the position of missile body-fixed coordinate system O_Rxyz relative to the gravitational missile-fixed system $O_Rx_gy_gz_g$ is determined unambiguously by the angles of roll ϕ_R, pitch θ_R and yaw ψ_R of the missile, respectively. At the same time the position of moving gravitational system relative to the earth-fixed one $O_1x_1y_1z_1$ is determined by the vector of actual missile position $\boldsymbol{r}_R$.

The real linear velocity of the missile in the system $O_1x_1y_1z_1$ can be written as follows (Fig. 1)

$$\boldsymbol{V}_{RO} = U_{1R}\boldsymbol{i}_1 + V_{1R}\boldsymbol{j}_1 + W_{1R}\boldsymbol{k}_1 \tag{3.1}$$

where

$$U_{1R} = \dot{x}_{1R} \qquad\qquad V_{1R} = \dot{y}_{1R} \qquad\qquad W_{1R} = \dot{z}_{1R} \tag{3.2}$$

while in the missile body-fixed system O_Rxyz (Fig. 1 and Fig. 4) the velocity has the following components

$$\boldsymbol{V}_{RO} = U_R\boldsymbol{i} + V_R\boldsymbol{j} + W_R\boldsymbol{k} \qquad\qquad \begin{bmatrix} U_R \\ V_R \\ W_R \end{bmatrix} = \boldsymbol{\Lambda}_R \begin{bmatrix} \dot{x}_{1R} \\ \dot{y}_{1R} \\ \dot{z}_{1R} \end{bmatrix} \tag{3.3}$$

where the transformation matrix has the following form

$$\boldsymbol{\Lambda}_R = \begin{bmatrix} \cos\psi_R\cos\theta_R & \sin\psi_R\cos\theta_R & -\sin\theta_R \\ l_{21} & l_{22} & \sin\phi_R\cos\theta_R \\ l_{31} & l_{32} & \cos\phi_R\cos\theta_R \end{bmatrix} \tag{3.4}$$

where

$$\begin{aligned} l_{21} &= \sin\phi_R\cos\psi_R\sin\theta_R - \sin\psi_R\sin\phi_R \\ l_{22} &= \sin\phi_R\sin\psi_R\sin\theta_R + \cos\psi_R\cos\phi_R \\ l_{31} &= \cos\phi_R\cos\psi_R\sin\theta_R + \sin\psi_R\sin\phi_R \\ l_{32} &= \cos\phi_R\sin\psi_R\sin\theta_R - \cos\psi_R\sin\phi_R \end{aligned}$$

The angular velocity vector can be written as follows (Fig. 1)

$$\boldsymbol{\Omega}_{RO} = P_R\boldsymbol{i} + Q_R\boldsymbol{j} + R_R\boldsymbol{k} \tag{3.5}$$

where P_R, Q_R, R_R are angular velocities of roll, pitch and yaw, respectively.

The components P_R, Q_R, R_R of instantaneous angular velocity are linear functions of the generalised velocities $\dot{\phi}_R$, $\dot{\theta}_R$, $\dot{\psi}_R$ with the coefficients depending on the generalised coordinates ϕ_R, θ_R, ψ_R

$$\begin{bmatrix} P_R \\ Q_R \\ R_R \end{bmatrix} = \boldsymbol{\Lambda}_{\Omega_R} \begin{bmatrix} \dot{\phi}_R \\ \dot{\theta}_R \\ \dot{\psi}_R \end{bmatrix} \tag{3.6}$$

with the transformation matrix of the following form

$$\mathbf{\Lambda}_{\Omega_R} = \begin{bmatrix} 1 & 0 & -\sin\theta_R \\ 0 & \cos\phi_R & \sin\phi_R\cos\theta_R \\ 0 & -\sin\phi_R & \cos\phi_R\cos\theta_R \end{bmatrix} \tag{3.7}$$

In the case of homing, the angle of attack can be written as follows

$$\alpha_R = \arctan\frac{W_R}{U_R} \tag{3.8}$$

and the sideslip angle is represented by

$$\beta_R = \arcsin\frac{V_R}{V_{RO}} \tag{3.9}$$

In the case of beam-riding guidance, the angle of attack equals

$$\alpha_R = \arctan\frac{W_{1R}}{U_{1R}} - \theta_R \tag{3.10}$$

while the sideslip angle reads

$$\beta_R = \arcsin\frac{V_{1R}}{V_{RO}} - \psi_R \tag{3.11}$$

4. The preset parameters – kinematical missile-beam-target relations

When applying the three-point method (e.g. beam-riding guidance of a missile), the missile reaches the target provided that it is always illuminated by the missile-emitted beam, which means that the radar station (point O_1), missile (point O_R) and target (point O_C) should be situated on the line of sight (Fig. 2) (Dziopa, 2006; Etkin and Reid, 1996; Ładyżyńska-Kozdraś, 2006; Ładyżyńska-Kozdraś and Maryniak, 2003; Ładyżyńska-Kozdraś *et al.*, 2005; Menon *et al.*, 2003).

The condition for target reaching (Fig. 2) is

$$V_R > V_C\frac{\cos\gamma_{Cw}}{\cos\gamma_{Rw}} \tag{4.1}$$

where γ_{Cw}, γ_{Rw} stand for the angles defining the positions of target and missile, respectively, relative to the beam.

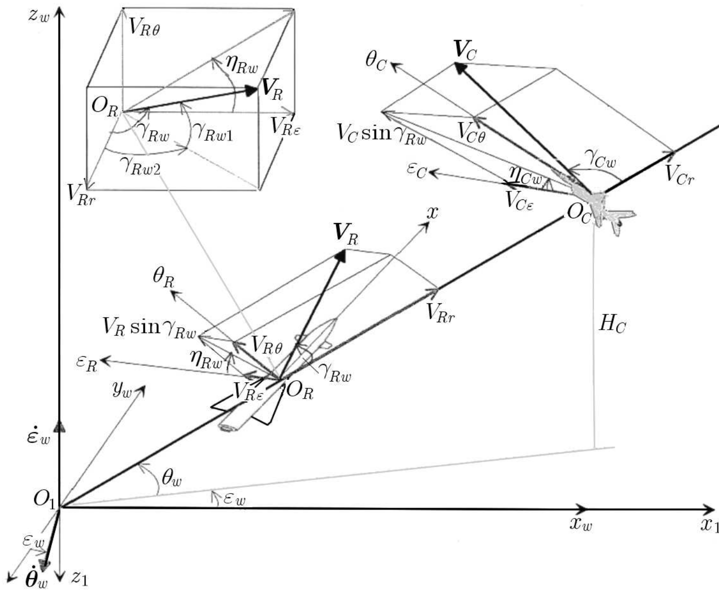

Fig. 2. Missile and target trajectories within the guiding beam

In the course of missile guidance its motion relative to the origin of guiding beam can be determined by the angular velocity equal to the beam angular velocity. Therefore, the angular velocities of pitch and yaw, respectively, of the beam determine the required flight corrections of the missile under control

$$\dot{\varepsilon}_w = \frac{V_C}{r_C} \frac{\sin\gamma_{Cw} \cos\eta_{Cw}}{\cos\theta_w} \qquad \dot{\theta}_w = \frac{V_C}{r_C} \sin\gamma_{Cw} \sin\eta_{Cw} \tag{4.2}$$

The equations of beam position can be easily determined from the trigonometric relations (Fig. 2), depending on the instantaneous target position

$$\varepsilon_w = \arctan\frac{y_{1C}}{x_{1C}} \qquad \theta_w = \arcsin\frac{-z_{1C}}{r_C} \tag{4.3}$$

The values of parameters preset in the control laws (Eqs. (2.1)-(2.4)) result in this case from the kinematical behaviour of the guiding beam, which rotates about a fixed point depending on the target maneuvers relative to the earth-fixed frame of reference $O_1x_1y_1z_1$. Thus:

- the vector of the preset missile position within the beam relative to the earth-fixed system $O_1x_1y_1z_1$ (Fig. 2) can be written as follows

$$r_{Rz} = \sqrt{x_{1Rz}^2 + y_{1Rz}^2 + z_{1Rz}^2} \tag{4.4}$$

where

$$x_{1Rz} = r_R \cos\varepsilon_w \cos\theta_w \qquad y_{1Rz} = -r_R \sin\varepsilon_w \cos\theta_w$$
$$z_{1Rz} = -r_R \sin\theta_w \tag{4.5}$$

- the vector of the preset missile linear velocity within the beam under the ideal guidance reads

$$\boldsymbol{V}_{Rz} = \frac{\partial r_{Rz}}{\partial t} + \begin{vmatrix} \boldsymbol{i}_1 & \boldsymbol{j}_1 & \boldsymbol{k}_1 \\ \dot{\theta}_w \sin\varepsilon_w & \dot{\theta}_w \cos\varepsilon_w & -\dot{\varepsilon}_w \\ r_R \cos\varepsilon_w \cos\theta_w & -r_R \sin\varepsilon_w \cos\theta_w & -r_R \sin\theta_w \end{vmatrix} =$$
$$= U_{1Rz}\boldsymbol{i}_1 + V_{1Rz}\boldsymbol{j}_1 + W_{1Rz}\boldsymbol{k}_1 \tag{4.6}$$

where

$$\begin{aligned}
U_{1Rz} &= \dot{r}_R \cos\varepsilon_w \cos\theta_w - 2r_R\dot{\varepsilon}_w \sin\varepsilon_w \cos\theta_w - 2r_R\dot{\theta}_w \cos\varepsilon_w \sin\theta_w \\
V_{1Rz} &= -\dot{r}_R \sin\varepsilon_w \cos\theta_w - 2r_R\dot{\varepsilon}_w \cos\varepsilon_w \cos\theta_w + 2r_R\dot{\theta}_w \sin\varepsilon_w \sin\theta_w \\
W_{1Rz} &= -\dot{r}_R \sin\theta_w - 2r_R\dot{\theta}_w \cos\theta_w
\end{aligned} \tag{4.7}$$

- the vector of the preset missile angular velocity can be written as follows

$$\boldsymbol{\Omega}_{Rz} = \boldsymbol{\Lambda}_{Rz} \begin{bmatrix} \dot{\theta}_w \sin\varepsilon_w \\ \dot{\theta}_w \cos\varepsilon_w \\ -\dot{\varepsilon}_w \end{bmatrix} = \begin{bmatrix} P_{Rz} \\ Q_{Rz} \\ R_{Rz} \end{bmatrix} \tag{4.8}$$

  where $\boldsymbol{\Lambda}_{Rz}$ is the transformation matrix one can arrive at after replacing the index R with Rz in $\boldsymbol{\Lambda}_R$ (Eq. (3.4)).

The formulae for the preset angles of attack and sideslip during the beam guidance can be derived on the equilibrium condition for the forces acting upon the missile in the horizontal and vertical planes (Fig. 3).

Equation of equilibrium along the axis $O_R z_A$

$$P_{zz} + T_R \sin\alpha_{Rz} = mg\cos(\theta_w + \gamma_{Rw1})$$

Equation of equilibrium along the axis $O_R y_A$

$$P_{yz} = T_R \sin\beta_{Rz}$$

While (Fig. 2)

$$\gamma_{Rw1} = \arcsin\frac{r_R\dot{\theta}_w}{V_R} \qquad \gamma_{Rw2} = \arcsin\frac{r_R\dot{\varepsilon}_w \cos\theta_w}{V_R} \tag{4.9}$$

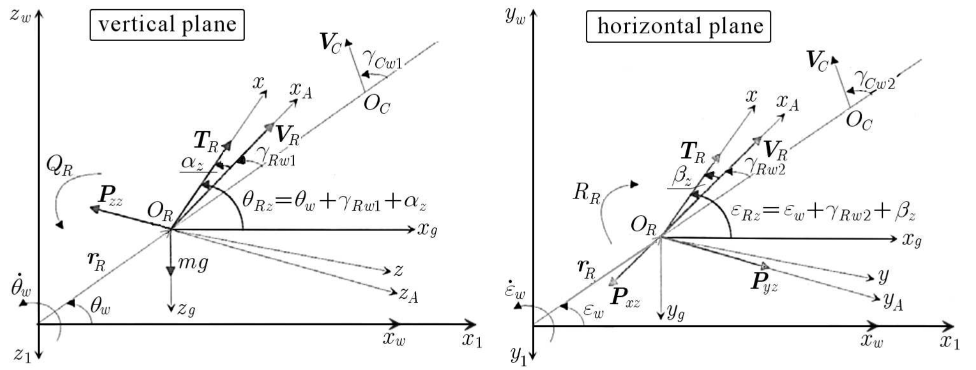

Fig. 3. Missile trajectories in the vertical and horizontal planes – the preset parameters

Then:

— angle of attack

$$\alpha_{Rz} = \arcsin \frac{mg\cos\Big(\theta_w + \arcsin \frac{r_R\dot{\theta}_w}{V_R}\Big) - \frac{1}{2}\rho S_R V_R^2 C_z}{T_R} \tag{4.10}$$

— angle of sideslip

$$\beta_{Rz} = \arcsin \frac{\frac{1}{2}\rho S_R V_R^2 C_y}{T_R} \tag{4.11}$$

— pitch angle (Fig. 3)

$$\begin{aligned} \theta_{Rz} &= \theta_w + \gamma_{Rw1} + \alpha_{Rz} = \\ &= \theta_w + \arcsin \frac{r_R\dot{\theta}_w}{V_R} + \arcsin \frac{mg\cos\Big(\theta_w + \arcsin \frac{r_R\dot{\theta}_w}{V_R}\Big) - \frac{1}{2}\rho S_R V_R^2 C_z}{T_R} \end{aligned} \tag{4.12}$$

— yaw angle (Fig. 3)

$$\psi_{Rz} = \varepsilon_w + \gamma_{Rw2} + \beta_{Rz} = \varepsilon_w + \arcsin \frac{r_R\dot{\varepsilon}_w \cos\theta_w}{V_R} + \arcsin \frac{\frac{1}{2}\rho S_R V_R^2 C_y}{T_R} \tag{4.13}$$

where ρ is the air density at a given altitude $H = -z_{1R}$, $(0 \leqslant H \leqslant 11000\,\mathrm{m})$

$$\rho = \rho_0\Big(1 - \frac{H}{44300}\Big)^{4.256}$$

ρ_0 – air density at the sea level
S_R – missile reference surface (maximum cross-section of its body)

T_R	–	missile engine thrust
P_{yz}	–	resisting force
P_{zz}	–	aerodynamic lift.

5. The preset parameters – kinematical relations between the missile and target when homing

In the two-point missile guidance (homing onto a target), the constraints are directly imposed on the missile-target motion. Let us assume that we deal with passive homing performed along a "curve of pursuit" (Dziopa, 2006; Ładyżyńska-Kozdraś, 2006, 2008; Ładyżyńska-Kozdraś and Maryniak, 2002, 2003). In this method, the arrow of missile velocity is always directed at the target position (Fig. 4).

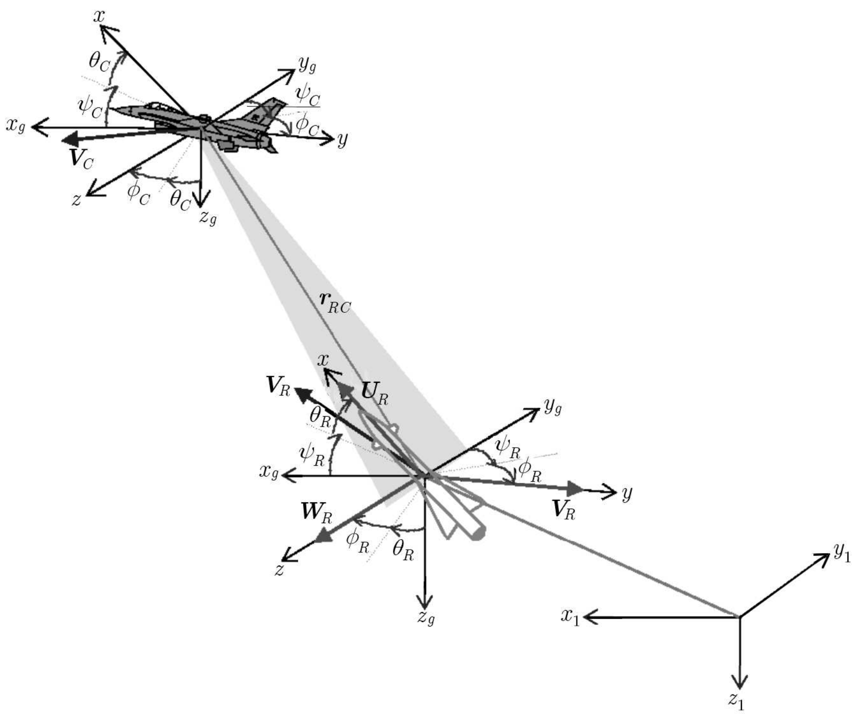

Fig. 4. Flight parameters of the homing missile

The changes in the missile-target distance $\boldsymbol{r}_{RC}$ and the sight angle $\nu = \theta_R$ can be written as functions of parameters of the missile-target motion in the following way

$$\dot{r}_{RC} = V_C\cos(\theta_R - \theta_C) - V_R \qquad\qquad \dot{\nu} = \frac{1}{r_{RC}}V_R\sin(\theta_R - \theta_C) \tag{5.1}$$

V_C – target velocity
θ_C – target pitch angle
ψ_C – target yaw angle.

When the missile follows the target using the curve-of-pursuit homing method, the preset parameters are those of the target, thus:

- the preset angles of roll, pitch and yaw for the missile are equal to those for the target

$$\phi_{Rz} = \phi_C \qquad\qquad \theta_{Rz} = \theta_C \qquad\qquad \psi_{Rz} = \psi_C \tag{5.2}$$

- the components of the preset missile position relative to the gravitational system $O_Rx_gy_gz_g$ are determined by the target position

$$\begin{aligned} x_{1z} &= r_{RC}\cos\psi_C\cos\theta_C \qquad\qquad y_{1z} = -r_{RC}\sin\psi_C\cos\theta_C \\ z_{1z} &= -r_{RC}\sin\theta_C \end{aligned} \tag{5.3}$$

- the components of the preset linear velocity of the missile relative to its body-fixed system read, where $\mathbf{\Lambda}_C$ is the transformation matrix, can be found after replacing the index R with C in $\mathbf{\Lambda}_R$ (Eq. (3.4))

$$\begin{bmatrix} U_{Rz} \\ V_{Rz} \\ W_{Rz} \end{bmatrix} = \mathbf{\Lambda}_C \begin{bmatrix} \dot{x}_{1C} \\ \dot{y}_{1C} \\ \dot{z}_{1C} \end{bmatrix} \tag{5.4}$$

- the components of the preset angular velocity of the missile relative to its body-fixed system can be written as follows, where $\mathbf{\Lambda}_{\Omega_C}$ is the transformation matrix, can be determined by replacing the index R with C in $\mathbf{\Lambda}_{\Omega_R}$ (Eq. (3.7))

$$\begin{bmatrix} P_{Rz} \\ Q_{Rz} \\ R_{Rz} \end{bmatrix} = \mathbf{\Lambda}_{\Omega_C} \begin{bmatrix} \dot{\phi}_C \\ \dot{\theta}_C \\ \dot{\psi}_C \end{bmatrix} \tag{5.5}$$

6. Beam-riding guidance of an earth-to-air missile

A simplified sample case of the beam-riding guidance of a Roland-class earth-to-air missile has been analysed (Ładyżyńska-Kozdraś *et al.*, 2005; Menon *et al.*, 2003; Zarchan, 2001). The preset parameters of the control laws are determined by the beam kinematical behaviour, the rotation of which about a fixed point depends on target maneuvers.

During the missile flight, the current parameters of its flight are registered and compared to those preset, which have been determined by the beam tracking the target. Therefore, the constraints are imposed by means of combing the motion of the line passing through the control point and the missile with motion of the guide beam.

Since the missile control is preformed in the ψ yaw and θ pitch channels in terms of the control surface deflections δ_H and δ_V, the control laws given by Eqs. (2.1) and (2.2) should be transformed to assume the following form (assuming a prompt deflection of the control surfaces – no delay in the control system):

— in the pitch channel

$$\begin{aligned}\delta_H = K_z^H(z_{1R} - z_{1Rz}) + K_U^H(\dot{x}_{1R} - \dot{x}_{1Rz}) + K_W^H(\dot{z}_{1R} - \dot{z}_{1Rz}) + \\ +K_Q^H(Q_R - Q_{Rz}) + K_\theta^H(\theta_R - \theta_{Rz})\end{aligned} \tag{6.1}$$

— in the yaw channel

$$\begin{aligned}\delta_V = K_y^V(y_{1R} - y_{1Rz}) + K_V^V(\dot{y}_{1R} - \dot{y}_{1Rz}) + K_P^V(P_R - P_{Rz}) + \\ +K_R^V(R_R - R_{Rz}) + K_\psi^V(\psi_R - \psi_{Rz})\end{aligned} \tag{6.2}$$

In the roll channel ϕ, the missile is automatically stabilised through ailerons, while there is no control in the velocity channel since the control laws (Eqs. (2.3) and (2.4)) are neglected. Kinematical and geometrical parameters appearing in the control laws (Eqs. (6.1) and (6.2)) are shown in Fig. 1 and described by Eqs. (4.1)-(4.8).

Numerical simulation of a missile guidance onto to flying plane was performed. The equations of missile motions were derived from the Maggi equations for non-holonomic systems (Ben-Asher and Yaesh, 1998; Etkin and Reid, 1996; Greenwood, 2003; Ładyżyńska-Kozdraś *et al.*, 2005; Nizioł and Maryniak, 2005). The coefficients of amplification resulting from the integral criterion employed before (Eqs (2.5) and (2.6)) took the following values

$$\begin{array}{lll} K_z^H = -0.00029 & K_U^H = 0.0007 & K_W^H = 0.00011 \\ K_Q^H = -1.36 & K_\theta^H = -4.3 & K_y^V = 0.00007 \\ K_V^V = -0.00054 & K_P^V = 0.0231 & K_R^V = 1.1 \\ K_\psi^V = -0.074 & & \end{array}$$

Sample simulation results shown in Fig. 5, Fig. 6 prove the efficiency of the missile guidance procedure based on the three-point-guidance method.

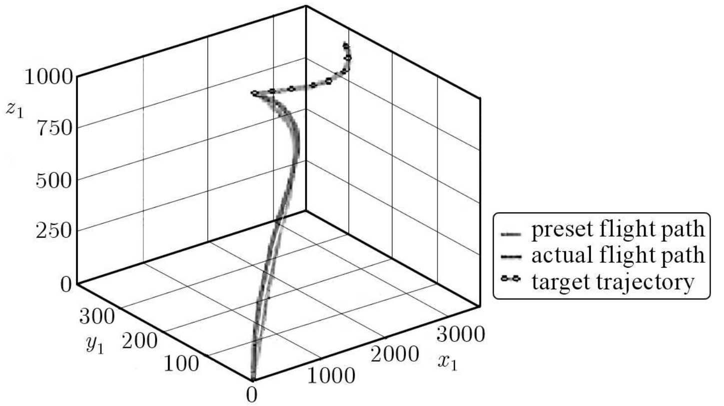

Fig. 5. Flight path – the actual and preset ones, respectively

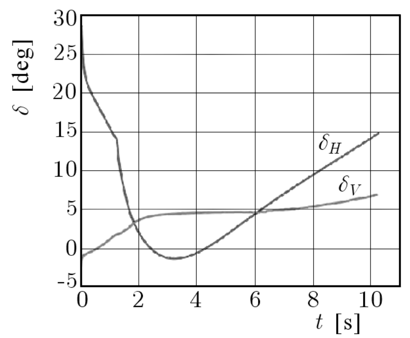

Fig. 6. Histories of the missile elevator and rudder deflections

7. Curve-of-pursuit missile homing onto a manoeuvring target

A flight was analysed of an air-to-air Sidewinder-class missile under the curve-of-pursuit homing onto a maneuvering target (Ładyżyńska-Kozdraś, 2008; Menon *et al.*, 2003; Zarchan, 2001).

In this case, the missile control is preformed in the ψ yaw and θ pitch channels in terms of the control surface deflections δ_H and δ_V, assuming a prompt deflection of the control surfaces - no delay in the control system.

After some adaptation, the control laws (Eqs (2.1) and (2.2)) assumed the following form:
— in the pitch channel

$$\begin{aligned}\delta_H = K_z^H(H_R - H_z) + K_W^H(W_R - W_z) + K_Q^H(Q_R - Q_z) + \\ +K_\theta^H(\theta_R - \theta_z) + \delta_{H0}\end{aligned} \tag{7.1}$$

— in the yaw channel

$$\begin{aligned}\delta_V = K_y^V(y_{1R} - y_{1z}) + K_W^V(W - W_z) + K_R^V(R_R - R_z) + \\ +K_\psi^V(\psi_R - \psi_z) + \delta_{V0}\end{aligned} \tag{7.2}$$

The kinematical and geometrical parameters appearing in the control laws (Eqs (7.1) and (7.2)) are shown in Fig.4 and represented by Eqs (5.1)-(5.5).

The control laws (Eqs (7.1) and (7.2)) were considered as non-holonomic constraints imposed upon the motion of missile under control. The equations of motion were derived using the Boltzmann-Hamell equations for non-holonomic systems (Ben-Asher and Yaesh, 1998; Etkin and Reid, 1996; Greenwood, 2003; Ładyżyńska-Kozdraś, 2008; Nizioł and Maryniak, 2005).

A Sidewinder-class missile was the case-study. Aerodynamical characteristics were determined and verified in terms of a non-controllable missile. Upon application of the square control quality criterion (Eqs. (2.7) and $(2.8)_1$) the coefficients of amplification appearing in the control laws (Eqs. (7.1) and (7.2)) took the following values

$$\begin{aligned} K_\theta^H &= -0.84 & K_W^H &= -0.00005 \\ K_z^H &= 0.00032 & K_Q^H &= 0.0 \\ K_\psi^V &= 0.24 & K_W^V &= -0.0002 \\ K_y^V &= 0.00014 & K_R^V &= 0.0 \end{aligned}$$

Sample simulation results are shown in Fig. 7 and Fig. 8 which also present the trajectories of both the plane and the missile homing onto it. The missile finally reaches the maneuvering target.

8. Conclusions

The paper proves the efficiency of the applied general model of a flying object under control. The control laws assume form of kinematical relations between

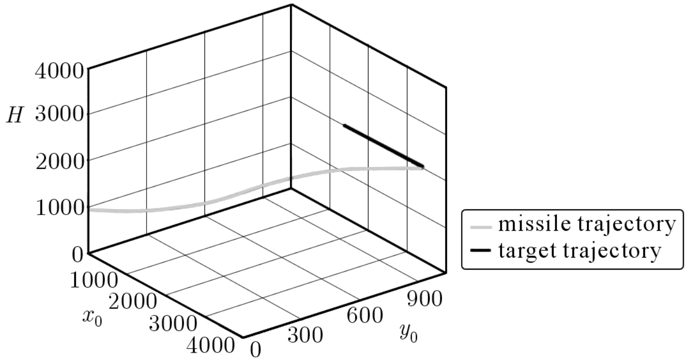

Fig. 7. Guiding performance of the missile onto a maneuvering target

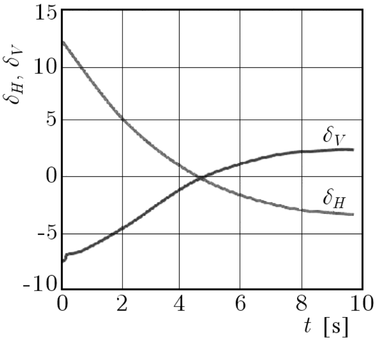

Fig. 8. Histories of the missile elevator and rudder deflections

deviations, i.e. differences between the preset and current values of selected parameters. The control laws formulated in that way may be successfully applied to investigations of motion of different types of flying objects; both unmanned; like missile or torpedos and those with crew; like aircraft or helicopters.

Depending on the problem to be solved and the type of flying object, the control laws may be reduced and adapted adequately.

The advantages of the presented approach are particularly visible when dealing with systems with non-holonomic constraints. High efficiency of the method is revealed when applied to different case studies, which should be emphasised as well.

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Prawa sterowania traktowane jako kinematyczne związki uchybów w automatycznym sterowaniu obiektów latających

Streszczenie

W pracy przedstawiono zastosowania praktyczne praw sterowania w dynamice obiektów latających. Rozpatrywane prawa sterowania stanowią kinematyczne i geometryczne związki uchybów parametrów realizowanych i zadanych wynikających z systemu naprowadzania badanego obiektu. Zadane parametry lotu wprowadzone zostały do praw sterowania jako parametry wynikające z lotu celu przy sterowaniu rakiet samonaprowadzających się, albo jako parametry ruchu wiązki śledzącej cel. Rozważania przeprowadzono dla ogólnego modelu dynamiki obiektu sterowanego.

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