

LOW-THRUST CHAOTIC BASED TRANSFER FROM THE EARTH TO A HALO ORBIT

ELBERT E.N. MACAU

*Laboratório Associado de Computação e Matemática Aplicada – LAC, Instituto Nacional de Pesquisas Espaciais – INPE, São José dos Campos, SP, Brazil
e-mail: elbert@lac.inpe.br*

Halo orbits around the Lagrange points serve as excellent platforms for scientific missions involving the Sun as well as planetary explorations. The satellites ISEE-3, GGS_WIND, SOHO and ACE have exploited these orbits to accomplish their missions. The trajectory design in support of such missions is increasingly challenging as more complex missions, like the case of the Next Generation Space Telescope, are envisioned in the near future. The purpose of this paper is to introduce a new chaotic-based transfer strategy that can be used to transfer a spacecraft from the vicinity of the Earth to a halo orbit around the equilibrium point L_1 of the Earth-Sun system. The strategy exploits the inherent exponential sensitivity of chaotic time evolutions to perturbations to guide a spacecraft from its starting position to an invariant stable manifold of the chosen halo orbit. As just judiciously chosen perturbations are used, the transfer operation requires very small amounts of fuel. As an example, we applied the method in context of the restricted three-body problem, which gives a good insight into the strategies to be used in a real situation.

Key words: chaos, control of chaos, orbital transfer

1. Introduction

In the restricted three-body problem, halo orbits are spatial periodic solutions that are present around the collinear libration points. They have been calculated both numerically (Breakwell and Brown, 1979; Howell, 1984; Howell and Pernicka, 1988) from the equation of motion and analytically (Goudas, 1963; Zagouras and Kazantzis, 1979; Richardson, 1980a,b) by means of the Lindstedt-Poincaré procedure. The analytical approaches show that the linearized motion about the collinear libration points encompass a periodic

orbit in the plane of the primary motion and an uncoupled periodic out-of-plane motion. Increasing the amplitude of these orbits, certain combinations of in-plane and out-of-plane amplitudes exist, such that the corresponding frequencies are equal and a perfectly periodic three-dimensional motion occurs. Robert Farquhar coined the term "halo" for these orbits in his PhD thesis (Farquhar, 1968b). Later on, he proposed placing a communication satellite in such an orbit about the libration point L_2 in the Earth-Moon system (Farquhar, 1968a). This satellite was envisioned in the context of the Apollo 18 mission and would allow a continuous contact with both the far side of the Moon and the Earth, as it would never be blocked from view by the Moon. As the Apollo 18 mission was canceled, his idea was archived.

Farquhar's idea of exploiting halo orbits was resumed, but this time associated with the collinear libration points in the Sun-Earth system. Spacecrafts in halo orbits near these libration points offer valuable opportunities for scientific investigations concerning solar exploration, cosmic background radiation, and solar and heliospheric effects on planetary environments. Thus, in 1978, the International Sun-Earth Explorer-3 (ISEE-3) spacecraft was the first one that was successfully deployed into a halo orbit, specifically about the interior collinear libration point L_1 (Farquhar *et al.*, 1977; Richardson, 1979). A number of other missions succeeded this pioneer mission: WIND, SOHO, ACE, Genesis and others are currently in development.

This increased interest in libration point missions has motivated the research involving a new trajectory design process in the scope of the three-body problem (Gómez *et al.*, 1993; Howell *et al.*, 1994, 1997). In fact, traditional approaches based on conic approximations are not adequate, while standard targeting and optimization strategies that use variational methods do not work properly because of the extreme nonlinearities and high sensitivities in the problem of finding a proper transfer trajectory from the Earth to the desired Halo orbit. Despite the success in guiding the previously cited missions, more efficient and flexible techniques are still needed, especially such which would exploit the richness of behaviour present in a nonlinear system, and also incorporate ideas insights from the theoretical understanding of this type of the multi-body problem. A remarkable step in this direction was introduced by Gómez *et al.* (1993). Their strategy involves computation of a stable manifold of a halo orbit and determination of a transfer trajectory that allows the injection of the spacecraft onto this manifold. In the transfer from a parking orbit about the Earth, their strategy can be easily achieved when the orbits of the stable manifold come close to the parking orbit. However, this happens in general when the halo orbit is large enough. If it is not the case, a strategy

of transfer between halo orbits can be applied to guide the spacecraft from a large halo orbit to the desired small one.

In this paper, we consider another approach to the problem of finding a transfer trajectory from the neighborhood of the Earth to a halo orbit. This approach, named the targeting type of the control problem for a chaotic system, was introduced in the scope of the Theory of Chaotic System to drive trajectories in chaotic systems (Shinbrot *et al.*, 1990). Its main characteristic is to take advantage of the richness of behaviour present in chaotic dynamics to accomplish the goal of finding a proper trajectory from a source point to a target point. In fact, the hallmark of chaotic behaviour is the extremely high sensitive dependence on initial conditions, which prevents long-term prediction of the state of the system from measured data. On the other hand, this inherent exponential sensitivity of chaotic time evolution to perturbations can be exploited in driving trajectories to some desired final state by the use of a carefully chosen sequence of small perturbations to some control parameters. The key point is that in being so small, the perturbations does not change the system dynamics significantly, but make it work properly to accomplish the transfer from the initial state to the desired final state (Macau and Grebogi, 2001). In Celestial Mechanics, this approach has been used before to accomplish both theoretical (Boltt and Meiss, 1995a,b; Macau, 2000) and real mission (Belbruno, 1994). Its main appeal is that it requires small amounts of fuel to attain the desired goal. However, usually a transfer takes a significantly long time if compared with traditional techniques.

The problem of transfer from the Earth to a halo orbit by using the targeting control of chaos presents a high level of complexity if compared with the previously cited problems. For these ones, the problems were solved in the environment of the planar, circular, restricted three-body problem. Because of the presence of a constant of motion, i.e., the Jacobi integral, the Hamiltonian flow is confined to a three-dimensional submanifold. The introduction of a Poincaré section allow the problem to be solved in the scope of a two-dimensional area-preserving map. This is a specific situation in which chaotic targeting methods are well studied and have even been applied with success in experimental situations. On the other hand, the problem that we consider here takes place in the environment of the three-dimensional, circular, restricted three-body problem, which means that the state space is of dimensional six. The Jacobi integral confines the flow to a five-dimensional submanifold. For this case, the problem can not be easily solved with the introduction of a Poincaré section, because even with it we are in the scope of a four-dimensional area preserving map. It means that the proper perturbations to be applied must be determi-

ned in stable and unstable manifolds of high dimension, which, in general, is a hard task. For this specific situation, Kostelich *et al.* (1993) introduced a method on which we based our work. This method must also be adapted to the Celestial Mechanics situation where perturbations can just be applied to the velocity, as a perturbation in a position has no physical meaning.

We proceed as follows. In the next section, we describe the high-dimension chaotic targeting method in detail. Then, we present halo orbits in context of the three-dimensional, circular, restricted three-body problem. In the subsequent section, we show how the presented high-dimension chaotic targeting method can be applied to find a transfer trajectory from the neighbourhood of the Earth to a halo orbit. A general discussion about this approach is provided in the last section.

2. Higher-dimension chaotic targeting

Let us consider a discrete in time dynamical system

$$X_{n+1} = F(X_n) \tag{2.1}$$

where $\mathbf{X}_i \in \mathfrak{R}^n$, and F is a smooth function. Let us suppose that our goal is to target the point Q from the point P , both located inside the chaotic invariant set. Our aim is to find "small" perturbations δ so that if those numbers are adequately applied to the original trajectory that passes through P we have a perturbed trajectory that eventually hits the target point Q . Let us now suppose that in the neighbourhood of P there is a point X_0 that belongs to the trajectory $\{X_i\}$, and in the neighbourhood of Q there is another point Y_N that belongs to the trajectory $\{Y_i\}$ so that those trajectories come close to each other at the point X_j of the trajectory $\{X_i\}$, where $j > 0$, and at the point Y_k of the trajectory $\{Y_i\}$, where $i < N$. Note that the restriction over the indices of the trajectory points indicates that the trajectory $\{X_i\}$ passes first through point X_0 before getting to the point X_j , while the trajectory $\{Y_i\}$ first passes through the point Y_k , before getting to Y_N . It means that if we could follow the trajectory $\{X_i\}$ until the point X_j , and then from this point move to point Y_k of the trajectory $\{Y_i\}$, we would have a transfer trajectory from the neighbourhood of the point P to the neighbourhood of the point Q . The goal of the chaotic targeting method is to find proper perturbations to accomplish it. This can be done in the following way: For a hyperbolic situation, associated to each point on the invariant set, there are stable and unstable

manifolds. The stable manifold of Z is a set of points W with the property $\|F^k(Z) - F^k(W)\| \rightarrow 0$ as $k \rightarrow \infty$, while the unstable manifold of Z is the set of points U with the property $\|F^{-k}(Z) - F^{-k}(U)\| \rightarrow 0$ as $k \rightarrow \infty$. According to Macau and Grebogi (2001), in a hyperbolic situation, if the distance between X_j and Y_k is sufficiently small, then the unstable manifold of X_j and the stable manifold of Y_j intersect each other at a point q . This fact can be exploited to accomplish our goal if a proper perturbation is applied to the sequence of points of the orbit that passes through X_j . In fact, if the point q belongs to the intersection of the unstable manifold of X_j with the stable manifold of Y_j , then forward iterations of q converge to forward iterations of Y_k and backward iterations of q converge to backward iterations of X_j . It is known that in each point p of the trajectory, the local stable and unstable manifolds are respectively tangent to the eigenspaces E_p^s and E_p^u of the linearized system, where E_p^s span the subspaces whose eigenvalues have modulus < 1 , while E_p^u span the subspaces whose eigenvalues have modulus > 1 . Thus, if the proper perturbation $\alpha_{X_{j-m}}$ is applied in the direction of the $E_{X_{j-m}}^u$, it produces a perturbed orbit that passes through q , and converges to the trajectory $\{Y_i\}$ after Y_k . Consequently, this procedure generates the desired path that allows us to smoothly pass from the trajectory $\{X_i\}$ to the trajectory $\{Y_i\}$ and so be transferred to the neighbourhood of the point Q . In addition, that argument indicates that the perturbation $\alpha_{X_{j-m}}$ can be calculated by solving the following equation

$$F^{m+t}(X_{j-m} + \alpha_{X_{j-m}}E_{X_{j-m}}^u) = Y_{k+t} + \beta_{k+t}E_{Y_{k+t}}^s \tag{2.2}$$

In this equation, the values of m and t can be adequately adjusted for each system by an empirical procedure. In applying this procedure, another important simplification can be introduced. If we consider any orbit $\{Z_k\}_{k=1}^n$ that contains Z_i , almost any variation near Z_{i-m} will expand along the unstable manifold of Z_i if the value of m chosen is large enough. A similar statement can be made regarding the stable manifold of Z_i for variations near Z_{i+m} iterated in the backward direction (Macau and Grebogi, 2001). Thus, to calculate the perturbation, we introduce a small perturbation $\hat{\beta}e_\beta$ where e_β is a unit vector in the direction of the perturbation at the position Y_{k+t} and iterate this perturbed point t times backward in time. This will typically generate a nearby trajectory that will deviate progressively from the original trajectory at each backward iteration, expanding away from $\{Y_i\}$ along the direction of the stable manifold at the points on the orbit $\{Y_i\}$ (Macau and Grebogi, 2001) (we assume that the direction of the small perturbation $\hat{\beta}e_\beta$ is not precisely such that it has no component in the stable direction). We also

introduce a small perturbation $\widehat{\delta}e_\delta$ to the orbit $\{X_i\}$ at the iteration $j - m$ where e_δ is a unit vector in the direction of the perturbation, and iterate this perturbed point forward in time m iterates. This will typically generate a nearby trajectory that will deviate progressively from the original trajectory at each forward iteration, expanding away from $\{X_i\}$ along the direction of the unstable manifold at the points on the orbit $\{X_i\}$ (Macau and Grebogi, 2001). Thus, the perturbations can be calculated by solving the following equation

$$F^m(X_{j-m} + \widehat{\delta}e_\delta) = F^{-t}(Y_{k+t} + \widehat{\beta}e_\beta) \quad (2.3)$$

which can be calculated by using the Broyden method (Press *et al.*, 1996).

The use of the previous equation makes the problem more easily to be solved, in particular because it is not necessary to determine local approximations of the stable and unstable manifolds at each step of the trajectories. However, to be effective, it requires a closer approximation of the trajectories than applying Eq. (2.2).

Note that because of the approximations and numerical roundoff errors, this described method must be repeated from time to time in order to keep the new trajectory close to the path leading to the target point Q .

Whether using Eqs. (2.2) or (2.3), this approach can just be applied if the trajectories are sufficiently close to each other. Thus, before using it, the points X_0 in the neighbourhood of P and Y_N in the neighbourhood of Q must be found so that forward iterations of X_0 and backward iterations of Y_N imply in trajectories that come close enough to each other. As a chaotic system has the property of being topologically transitive, a trajectory starting from almost any point of the chaotic invariant set is dense in this set. It means that starting from almost any point P and Q of the chaotic attractor, we can rely on the ergodic wander of the forward orbit from P and backward orbit from Q to bring these orbits sufficiently close to each other so that equations (2.2) or (2.3) can be applied with success. However, for such a process, in especially in the case of high-dimensional system, the transport time can be extremely long. This situation is worst in the case of Hamiltonian systems, because in this case the phase space is divided into layered components which are separated from each other by Cantory (Mackay *et al.*, 1984). Typically, a trajectory initialized in one layer of the chaotic region wanders in that layer for a long time before it crosses the Cantory and wanders in the next region. Thus, if P and Q are located in different layers, the time that is required before the trajectory come close to each other can be prohibitively long.

In particular, in high-dimensional systems and in situations in which the existence of layer is not critical, Kostelich *et al.* (1993) proposed to face this

problem by building a *hierarchy* or a "tree" of trajectories. Thus, we start a trajectory from P , say $\{X_0, X_1, \dots\}$. To each point of this trajectory, we apply a small perturbation and we start another trajectory from the perturbed point. So, if δ is a perturbation and $Z_0O^i = X_i + \delta$, we have a tree of trajectories $\{Z_0^i, Z_1^i, \dots\}$, for $i = 0, 1, \dots$ associated with the trajectory from P . The same procedure is applied to the backward trajectory from Q . By doing so, we increase the probability of finding iterated points from the two trees close to each other and, consequently, the time (number of iterations) needed to find those points is substantially decreased.

3. Equations of motion

The three-dimensional, circular, restricted three-body problem involves two finite masses m_1 and m_2 , assumed to be point masses, moving around their common mass center under gravitational influence of each other. We use a rotating coordinate system with the origin at the barycenter and angular velocity normalized to the unity, as shown in Fig. 1, and we also normalize the sum of the masses to one, i.e., $m_1 + m_2 = 1$. We define the characteristic parameter μ as the mass ratio m_2 to the sum $m_1 + m_2$. The position of the primaries in the rotating frame are fixed at $x_1 = (-\mu, 0, 0)$ and $x_2 = (1 - \mu, 0, 0)$. The xy -plane is the plane of motion of m_1 and m_2 . The third body, m_3 , is assumed massless but may travel in all of the three dimensions of the space. In this frame, the equations of motion for m_3 , which we associated with the spacecraft, are the following

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x} \qquad \ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y} \qquad \ddot{z} = \frac{\partial U}{\partial z} \qquad (3.1)$$

where

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1 - \mu}{d_1} + \frac{\mu}{d_2} \qquad d_1 = \sqrt{(x + \mu)^2 + y^2 + z^2} \qquad (3.2)$$

$$d_2 = \sqrt{(x - 1 + \mu)^2 + y^2 + z^2}$$

This system does admit a constant of integration, the Jacobi constant C , such that

$$C = 2U - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \qquad (3.3)$$

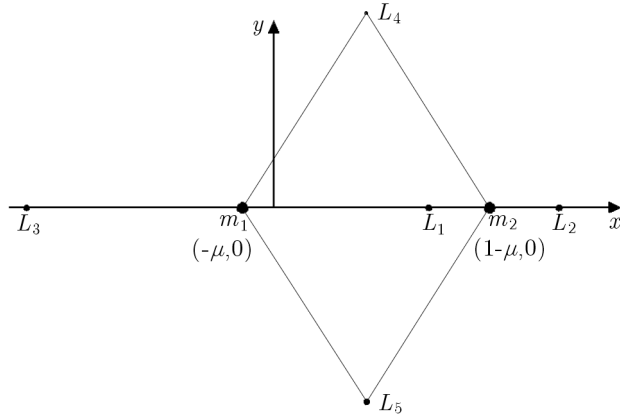


Fig. 1. Libration points in the three-body problem

In this system, there are five equilibrium points, or *libration points*, where the gravitational and centrifugal forces balance each other. In Fig. 1, we represent these points considering that $m_2 < m_1$. As the primaries travel around the barycenter, all five points remain in the same position relative to the masses for a given μ .

As stated before, halo orbits are spatial periodic solutions that are present around the collinear libration points. Those orbits can be calculated both numerically and analytically. For this work, using $\mu = 0.04$ and $C = 2.9307$, we determined about the libration point L_1 a halo orbit that appears in Fig. 2. In the next section, we explain our method of transfer from an orbit in the neighbourhood of the first primary to this halo orbit.

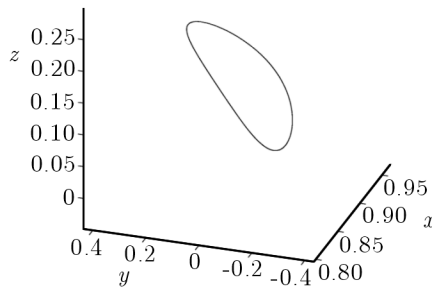


Fig. 2. Halo orbit to be targeted

4. Chaotic targeting to a halo orbit

Let us consider a spacecraft at a point P in the neighbourhood of the primary m_1 , but located inside the chaotic region for $C = 2.9307$. Figure 3 shows together the trajectory of this spacecraft from the point P and the halo orbit for which we aim to find a transfer trajectory from P . Note that, usually, we start the design of a mission considering that the spacecraft is located in a parking orbit around the primary. If this is the case, an initial Δv is required to inject the spacecraft into the chaotic region.

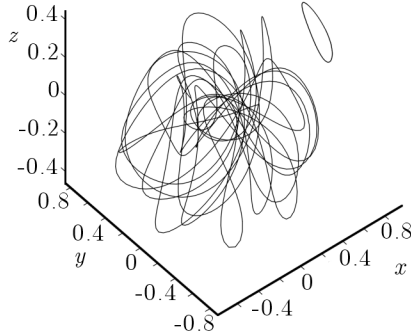


Fig. 3. "Parking" and halo orbits

The first step is to determine two trajectories that come close to each other so that one passes in the neighbourhood of P before it gets the position of the closest proximity, while the other, hereafter called the *reference trajectory*, first passes through this position before attains the halo orbit. Following Section 2, we identify the first trajectory by $\{X_i\}$, and the second one by $\{Y_i\}$ as well as the point of closest approach in each trajectory by, respectively, X_j and Y_0 . To make the task of finding these trajectories easy, we introduce a Poincaré section at $z = 0$, mapping the xy -plane to itself whenever the trajectory traverses this plane with $\dot{z} > 0$. We start it by selecting a large number of points $\{Y^1, Y^2, \dots, Y^M\}$ uniformly distributed on the halo orbit. We choose a small integer number k . To each of those points, we apply a small perturbation δv on the velocities and obtain the points $\{Y_k^1, Y_k^2, \dots, Y_k^M\}$ so that $Y_k^i = Y^i + \delta v$, for $i = 1, \dots, M$. We call the set of these numbers by H_k . Having each of the points Y_k^i as the initial condition, we integrate system (3.1) backward in time to obtain a set of points $H_{k-1} = \{Y_{k-1}^1, Y_{k-1}^2, \dots, Y_{k-1}^M\}$ in which each trajectory crosses the Poincaré section for the first time. We continue this procedure to get a set of points $H_{k-2} = \{Y_{k-2}^1, Y_{k-2}^2, \dots, Y_{k-2}^M\}$ of the second crossing of the Poincaré section, and so one until we obtain a set of points H_0 . In our exemple, we selected $k = 3$. For the halo orbit, this is a "tree" of trajectories mentioned in Section 2.

From the point P , we proceed by generating its "tree" trajectory. To do so, we associate this position with X_0^0 , and having this value as the initial condition, we integrate system (3.1) forward in time until the trajectory crosses the Poincaré section for the first time. We call this point X_1^0 . In the orbit that goes from X_0^0 to X_1^0 , we select a large number of points $\{X^1, X^2, \dots, X^L\}$ uniformly distributed. To each of those points, we apply a small perturbation $\delta\tilde{v}$ on the velocities and obtain a set of points $\{X_0^1, X_0^2, \dots, X_0^L\}$ so that $X_0^i = X^i + \delta\tilde{v}$, for $i = 1, \dots, L$. We call this set of points $\{X_0^0, X_0^1, X_0^2, \dots, X_0^L\}$ P_0 . We choose a small integer number j . Having each of the points X_0^i as the initial condition, we integrate system (3.1) forward in time to obtain a set of points $P_1 = \{X_1^0, X_1^1, X_1^2, \dots, X_1^L\}$ in which each trajectory crosses the Poincaré section for the first time. As before, we continue this procedure to get a set of points $P_2 = \{X_2^0, X_2^1, X_2^2, \dots, X_2^L\}$ of the second crossing of the Poincaré section, and so on until we obtain a set of points P_j . In our example, we selected $j = 3$.

Now, we plot the set of points H_0 and P_j in the Poincaré section as shown in Fig. 4. Using the scenario displayed in the Poincaré section, we select pairs of points (α_i, β_i) , $\alpha_i \in P_j$ and $\beta_i \in H_0$, within at least a distance d_{mx} between one another. Doing so, we obtain the set $PH = \{(\alpha_i, \beta_i), i = 1, 2, \dots\}$. This set belongs to the trajectories from P and to the halo orbit that come spatially close to each other. From this set, we select the pair that comes closest to each other in the full state space, i.e., not just in relation to its position values, but also in relation to the velocity. Let us say that these points are X_j^α and Y_0^β . Note that the Poincaré section is used as a kind of reference to allow both an easy identification of the desired trajectories that come close to each other and the building of the "tree" of trajectories. Thus, we have the trajectory $\{X_i^\alpha\}$ that comes from the point P , and the trajectory $\{Y_i^\beta\}$ that goes to the halo orbit, and these two trajectories come closest to each other in their points X_j^α and Y_0^β , respectively.

As these trajectories are found, we can proceed to the next step, which is to find proper perturbations to "path" the trajectory $\{X_i^\alpha\}$ to $\{Y_i^\beta\}$. In order to accomplish that, we use the method presented in Section 2, which implies solving equation (2.3) for those trajectories about the points X_j^α and Y_0^β . However, this method was originally developed for dynamical systems discrete in time, while in our situation we deal with a continuous dynamical system. For this specific situation, we can associate a system continuous in time to a discrete system by considering the solution of the former system at discrete time intervals, i.e., at $t = 0, 1\tau, 2\tau, \dots$. As we use a numerical integrator to solve the system of differential equations, we employ a multiple of the integra-

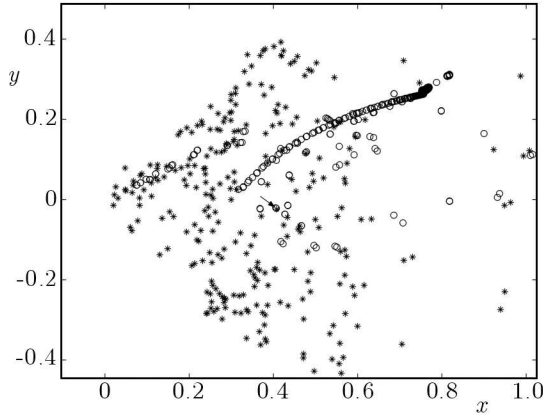


Fig. 4. Reference Poincaré section used to find convenient trajectories

tion step as the discretization interval, i.e., $\tau = nh$, where h is the integration step used. By doing so, we have a six dimensional discrete in time dynamical system for which the method is applicable.

Another consideration is related to the perturbation. For celestial mechanics problems, just Δv makes sense. Thus, the perturbations to be found are suitable Δv that allow a smooth "path" from the trajectory $\{X_i^\alpha\}$ to the trajectory $\{Y_i^\beta\}$.

In our example, we use equation (2.3) to the points X_j^α and Y_0^β . Using a try and error process, the parameters t and m were set to 2 and 8, respectively, which imply the smallest value of Δv . Thus, we have found the following values: $\Delta \dot{x} = 7.4611 \cdot 10^{-3}$; $\Delta \dot{y} = 1.0848 \cdot 10^{-2}$; $\Delta \dot{z} = 1.6492 \cdot 10^{-2}$.

The final transfer trajectory that we have obtained by using our method is shown in Fig. 5. To get this result, besides the Δv that was used to "path" the trajectories $\{X_i^\alpha\}$ and $\{Y_i^\beta\}$, another Δv was required, associated to the "tree" of trajectories from P , in order to direct the trajectory to the point X_j^α . That Δv was equal to $2.3 \cdot 10^{-3}$.

5. Conclusions

The method introduced here uses a chaotic-based strategy to transfer a spacecraft from a point located in the neighbourhood of the first primary, but inside the chaotic region, to a halo orbit. This method requires very a small amount of fuel, because it exploits the extremely high sensitive dependence of the chaotic evolution to a perturbation to accomplish the goal. Although in

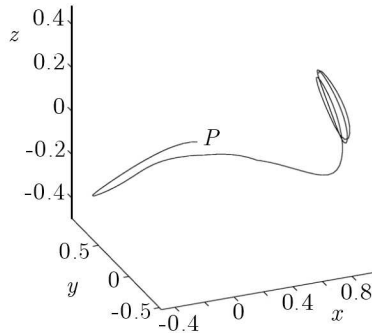


Fig. 5. Transfer trajectory to the halo orbit

this work we have applied the method to transfer a spacecraft to a halo orbit around the equilibrium point L_1 , it can be applied to transfer into halo orbits around other equilibrium points as well.

An important characteristic of this approach is that the issue if the stable manifold of the desired halo orbit comes close to the parking orbit or not is irrelevant. This happens because the search for the position of minimum proximity between the trajectory that comes from the parking orbit to the one that goes to the halo orbit is conducted inside the chaotic region. Because of the topologically transitivity of a chaotic evolution, these trajectories will eventually come sufficiently close to each other to allow the determination of a path trajectory between them by the method explained in the text. However, as in Hamiltonian systems the phase space is divided into layered components which are separated from each other by Cantory (Mackay *et al.*, 1984), we may expect situations in which it is necessary to wait for a long time until the trajectories come close to each other. Despite we have not found any situation like that in the cases that we had considered, the determination of the circumstances in which this situation might happen and how to overcome it are issues that deserve a more thorough better investigation.

In the context of theory of chaos, chaotic targeting methods are regarded as procedures to find the fastest trajectory that allows the transportation between two points in the phase space. Although in this work our goal is to find a low-thrust transfer orbit, we consider that exploitation of other concepts from that theory may conceive a method to be used in Celestial Mechanics to allows not only low-thrust, but also fast low-thrust transfer trajectories.

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Nisko-energetyczny, chaotyczny transfer sztucznego satelity z Ziemi na orbitę typu halo

Streszczenie

Orbitę typu halo wokół punktów Lagrange'a są doskonałymi platformami dla stacji kosmicznych wypełniających misje naukowe w pobliżu Słońca i planet. Przykładem są satelity ISEE-3, GGS-WIND, SOHO i ACE. Projektowanie orbit dla takich misji stanowi ogromne wyzwanie, zwłaszcza że one same stają się coraz bardziej wyszukanyimi i złożonymi zadaniami, tak jak np. w przypadku teleskopu kosmicznego następnej generacji. Konieczność ich eksploracji łatwo przewidzieć w przyszłych badaniach. W tej pracy zajęto się zagadnieniem wynoszenia statku kosmicznego z sąsiedztwa Ziemi na orbitę halo wokół punktu Lagrange'a L_1 w układzie Ziemia-Słońce, opartym na nowej strategii zawierającej ruch chaotyczny statku. Strategia bazuje na wykładniczej wrażliwości trajektorii chaotycznej na zakłócenia warunków początkowych, co ma bezpośrednie przełożenie na sposób przeniesienia statku kosmicznego z orbity wyjściowej na stabilną rozmaitość orbity halo. Przy odpowiednio dobranych perturbacjach, transfer statku wymaga minimalnego zapotrzebowania na paliwo. W pracy rozważono taki transfer w odniesieniu do zagadnienia trzech ciał, co pozwoliło uzyskać bardzo dobry wgląd do problemu projektowania zmiany orbity w realnych warunkach.