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REGULAR AND CHAOTIC VIBRATIONS OF A PRELOADED PENDULUM SYSTEM

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Vibrations of a single preloaded pendulum, with the impact phenomenon with a stop taken into account, are analysed in the paper. The model, under some simplification, represents a simple one-degree-of-freedom element of a vehicle latch system. The response of the system has been tested numerically for a wide range of parameters and with various excitation amplitudes and frequencies. Elastic and rigid impacts have been studied, and results for both impact models and for regular and chaotic motions have been compared. To find all possible solutions to the system, basins of attraction have been calculated as well. The final verification of the model response has been presented for the experimental signals recorded during vehicle side impact tests. It has been shown that the response of the latch element under study may lead to door opening, and that is why it has to be replaced with another design.

Key words: vibration, chaotic motion, impact, pendulum system

1. Introduction

Vehicle door latch systems belong to those elements that should be designed in such a way that required safety conditions are satisfied. These systems should not allow the door opening when subjected to high amplitude/high frequency acceleration signals. They should work properly even in the case of vehicle crash that can lead to accelerations acting over a short time but with very large amplitudes. However, as pointed by Thomsen (2002), low amplitude but high frequency excitations may change apparent characteristics of mechanical systems. The excitation may result in loss of contact between latch elements that are otherwise biased by springs, and next to an impact phenomenon. Systems with discontinuities are very sensitive to external inputs, and under some conditions tend to respond with irregular motions (Wiercigroch and Kraker, 2000). Moreover, prediction of their response with analytical methods is limited to a specific class of models. By these reasons, analyses of models with impacts are mainly carried out numerically.

The purpose of this paper is to study the behaviour of a simple model of a latch element represented by a single physical pendulum pushed against the support with a torque produced by a preloaded spring. The value of the torque should be large enough to keep the door locked, but it should not make it difficult to open the door. The model is tested with respect to a range of external excitations consisting of harmonic acceleration signals with large amplitudes and high frequencies, and also accelerations recorded during a real vehicle crash test.

2. System model

Let us consider a model of a latch element that is commonly used in vehicle door latch systems. After some simplifications, the latch element (or the door handle) can be represented by a one-degree-of-freedom pendulum-like model represented in Fig. 1.

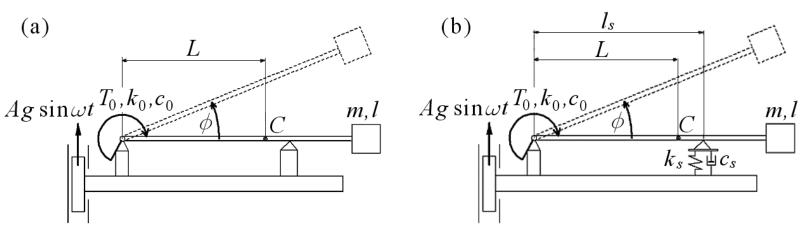

Fig. 1. Physical model of the latch element: (a) rigid impact model, (b) soft impact model

The latch lever, represented by the physical pendulum, is pushed against the stop by a spring with the stiffness k_0 and the preload torque T_0. The pendulum mass, its moment of inertia, the centre of mass location and damping at the pivot are denoted by m, I_0, L, c_0, respectively. The element should not rotate above a pre-specified threshold even if its base is subjected to an external acceleration. In the presented model, the acceleration is assumed as a harmonic function of time $a = Ag\sin\omega t$, where g denotes the acceleration of gravity, A is the amplitude of dimensionless acceleration expressed with respect to g, ω denotes the frequency of excitation and t is time. It is worth to note that the displacement of the base is expressed by $y_0 = -(Ag/\omega^2)\sin\omega t$.

The ideal behaviour of the latch system would be if the pendulum stays in contact with the support, independently of the excitation level. However, one can expect that under some conditions the loss of contact between the pendulum and the stop will occur. In such a case, the rotation angle of the pendulum may increase over the safety limit. The threshold angle that should not be exceeded is assumed to be $\phi = 0.1\,\mathrm{rad}$.

The rigid impact model is governed by the classical Newton impact law (Fig. 1a). Because, in the real system, the support is not ideally stiff but rather a deformable body, an elastic impact, which seems to be closer to the real system, is also proposed (Fig. 1b). Both types of impacts were studied in the literature (Thomsen, 2000; Wiercigroch and Kraker, 2000; Pavlovskaia and Wiercigroch, 2000). A soft impact with a deformable stop is represented by a mass-less, viscous-elastic element with the stiffness k_s, damping coefficient c_s and location l_s from the pivot. The viscous-elastic support, represented by a spring and a damper, allows for modelling the dissipated energy, deformation of the stop, and the time duration of the contact between both impacting bodies. These quantities depend on the force impulse during the impact and they are not constant in time as in the classical Newton approach. Therefore, this model is considered as the main contact model in the subsequent analysis. Nevertheless, for comparison and validation of certain results, the rigid impact model will also be tested in parallel.

Differential equations of motion of the systems represented in Fig. 1 take the form:

— For the rigid-impact model

$$I_0\ddot{\phi} + c_0\dot{\phi} + k_0\phi = -T_0 - mAgL\sin\omega t\cos\phi \tag{2.1}$$

where, with the Newton impact law, if $\phi = 0$ then $\phi_+ = \phi_-$ and $\dot{\phi}_+ = -R\dot{\phi}_-$, R designates the coefficient of restitution, and ϕ_- and ϕ_+ are the angles before and after the impact, respectively.

— For the elastic-impact model

$$I_0\ddot{\phi} + c_R\dot{\phi} + k_R\phi = -T_0 - mAgL\sin\omega t\cos\phi \tag{2.2}$$

where

$$c_R = \begin{cases} c_0 & \text{for} \quad \phi \geqslant 0 \\ c_0 + c_s l_s^2 & \text{for} \quad \phi < 0 \end{cases} \qquad k_R = \begin{cases} k_0 & \text{for} \quad \phi \geqslant 0 \\ k_0 + k_s l_s^2 & \text{for} \quad \phi < 0 \end{cases}$$

and c_R and k_R represent the damping and stiffness for both cases: outside of the contact area and during the contact. The natural frequency of the system takes two values: for $\phi \geqslant 0$, $\omega_0 = \sqrt{k_0/I_0}$ (outside of contact), and for $\phi < 0$, $\omega_R = \sqrt{k_R/I_0}$ (during contact). It is worth to mention that the proposed soft-impact model does not take into account the dynamics of the deformable stop. This assumption can be justified by the fact that after the loss of contact the natural frequency of the stop is much higher than that of the pendulum, and due to damping of the stop, the force quickly declines to zero.

To be able to compare both impact models we can assume that the change of velocity during the elastic impact should relate to the coefficient of restitution R as follows

$$\dot{\phi}(t_{impact}) = -R\dot{\phi}(0) \tag{2.3}$$

what means that over the impact time t_{impact} the velocity is reduced R times. The time duration of the impact equals to $t_{impact} = \pi/\omega_d$, where $\omega_d = \omega_R\sqrt{1-\xi^2}$ represents the natural frequency of underdamped vibrations, and $\xi = c_R/(I_0\omega_R^2)$ denotes the dimensionless damping coefficient.

Taking into account (2.3) and the solution for free damped vibrations, one can write

$$R = \exp\Big(-\frac{\xi\pi}{\sqrt{1-\xi^2}}\Big) \tag{2.4}$$

Solving (2.4), we can determine the equivalent dimensionless damping coefficient with respect to the given coefficient of restitution R of the rigid impact

$$\xi = -\frac{\ln R}{\sqrt{\pi^2 + (\ln R)^2}} \tag{2.5}$$

Relationship (2.5) is presented in Fig. 2, and is used in the paper for the determination of the damping coefficient of the viscous-elastic impact model.

Because mathematical models (2.1) and (2.2) include discontinuous functions, it is difficult, or even impossible, to solve the problem analytically. Therefore, Eqs. (2.1) and (2.2) are solved numerically using the Runge-Kutta method of the fourth order. Before starting the main analysis, the proper integration step size has to be tested. Because the system under consideration is discontinuous in time, this point is crucial for further numerical analysis.

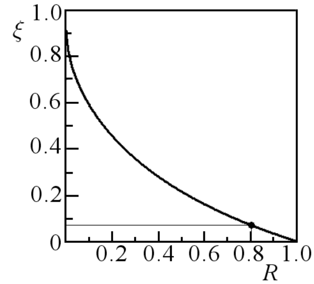

Fig. 2. Equivalent damping versus restitution coefficient

Figure 3a presents a phase space diagram received for the soft-impact model defined by Eq. (2.2). This result was produced after several tests with an integration step $\Delta t = 2\pi/(N\omega)$, where N is the number of steps per the oscillation period $T = 2\pi/\omega$. The trajectory presented in Fig. 3a is calculated for $N = 10\,000$ and $\omega = \omega_0$, which corresponds to $\Delta t = 3.1416 \cdot 10^{-5}$ s. This result is in full agreement for soft (2.2) and rigid-impact (2.1) model. For comparison, the solution plotted in Fig. 3b was received with $N = 1000$ ($\Delta t = 3.1416 \cdot 10^{-4}$) and the same set of parameters. As we can see, this result is far from the correct one presented in Fig. 3a. On the basis of several integration step tests carried out for both models, it has been determined that the integration step should not be larger than $1/10\,000$ of the vibration period.

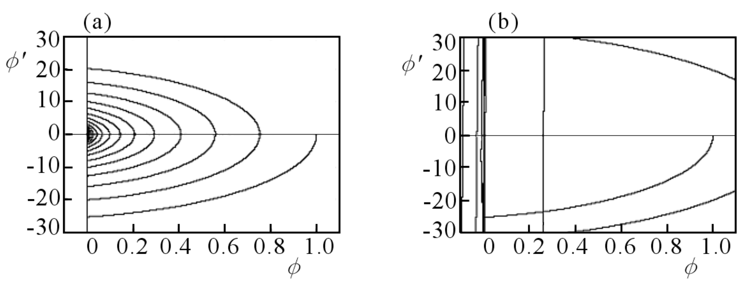

Fig. 3. Phase diagrams of free impacts with different integration steps: (a) $N = 10\,000$, (b) $N = 1000$

First, the free vibrations of both considered models are analysed. Assuming that the torque $T_0 = 0$ and the impact duration time is very short, the period of the pendulum response can be approximated as $T \approx \pi/\omega_0$. It means that without the preload torque ($T_0 = 0$), two impacts per period of free vibrations take place. Comparisons of time histories of the system with and without the impact are presented in Fig. 4a.

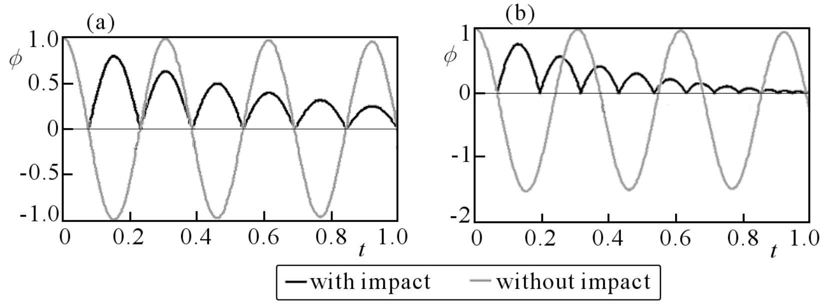

Fig. 4. Time histories of free vibrations with and without the impact for the preload torque $T_0 = 0\,\mathrm{Nm}$ (a) and $T_0 = 0.1\,\mathrm{Nm}$ (b)

Free vibrations change their nature if the preload torque is greater than zero. The influence of the torque T_0 on the free vibrations of the system is presented in Fig. 4b. If the initial torque $T_0 = 0$ (Fig. 4a), the period of vibrations with the impact is close to half of the period of free vibrations of the pendulum and it is constant. For the torque $T_0 = 0.1\,\mathrm{Nm}$ (greater than zero) the period of vibrations with the impact is different, and moreover, it is decreasing with the decreasing amplitude (Fig. 4b).

Figure 5 presents a spectrogram of free vibrations shown in Fig. 4b with the preload torque $T_0 = 0.1\,\mathrm{Nm}$. This graph illustrates an evident increase of the vibration frequency with the stiffening effect in the time domain. The character of this phenomenon is highlighted by the black solid line seen in Fig. 5.

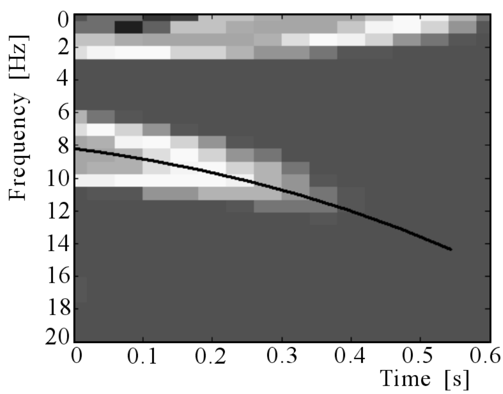

Fig. 5. Spectrogram of free impact oscillations with the preload torque $T_0 = 0.1\,\mathrm{Nm}$

3. Dynamics of the system under harmonic excitation

Systems with discontinuities subjected to external excitations usually have high tendency to produce complex and irregular responses. Therefore, their dynamics has been tested for different accelerations of the base, with various amplitudes A and frequencies ω of the excitation. Numerical calculations were carried out for the following exemplary parameters of the latch element

$$
\begin{array}{lll}
I_0 = 8.888 \cdot 10^{-4}\ \mathrm{kgm}^2 & c_0 = 9.3 \cdot 10^{-5}\ \dfrac{\mathrm{Nms}}{\mathrm{rad}} & k_0 = 0.370\ \dfrac{\mathrm{Nm}}{\mathrm{rad}} \\
T_0 = 0.1\ \mathrm{Nm} & m = 0.1475\ \mathrm{kg} & L = 0.065\ \mathrm{m} \\
R = 0.8 & k_s = 10^7\ \dfrac{\mathrm{N}}{\mathrm{m}} & c_s = 133.59\ \dfrac{\mathrm{Ns}}{\mathrm{m}} \\
\omega = 0 - 1000\ \dfrac{\mathrm{rad}}{\mathrm{s}} & A = 0 - 100 & l_s = 0.1\ \mathrm{m}
\end{array}
$$

According to (3.1), the range of amplitude of acceleration has been assumed up to 100 g, and frequency up to about 160 Hz. These are typical values; however, in more critical situations, these values may reach 500 g and 300 Hz.

The natural frequency of the single pendulum system without impact with parameters listed above has values of either $\omega_0 \approx 20.4\,\mathrm{rad/s}$ or $\omega_R \approx 10\,607\ \mathrm{rad/s}$ (when the pendulum is in contact with the support for the elastic impact model).

Let us assume that the system is excited with the constant frequency $\omega = 20\,\mathrm{rad/s}$, close to the natural frequency ω_0. The bifurcation analysis is performed for both models with the amplitude of the excitation A as the bifurcation parameter. Bifurcation diagrams for rigid and elastic impacts are presented in Figs. 6a and 6b, respectively.

The results obtained for both models exhibit similar phenomena. Dark areas that correspond to non-periodic motion occur for both models in the same place. However, pure periodic motion is only observed for the rigid impact model (solid line in Fig. 6a). For the soft impact model, due to deformation of the support, the corresponding motion is "almost" periodic and the line is broader. Dark areas in Fig. 6a and Fig. 6b represent more complex motion: quasi-periodic or chaotic. An exemplary chaotic attractor for $A = 40$ is presented in Fig. 7.

Chaotic motion evidently takes place for both impact models, however, the soft impact (Fig. 7b) leads to a more complex fractal structure of the chaotic attractor.

An increase of the excitation frequency results in growth of chaotic motion zones, as it can be seen in the bifurcation diagrams plotted for $\omega = 100\,\mathrm{rad/s}$

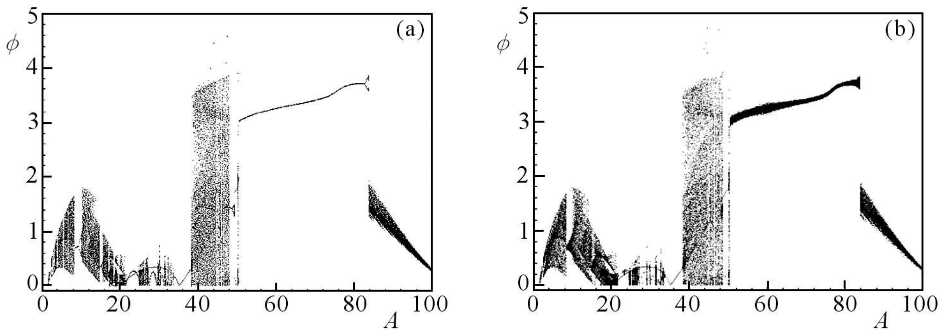

Fig. 6. Bifurcation diagrams versus excitation amplitude A ($\omega = 20\,\mathrm{rad/s}$): (a) rigid impact model, (b) elastic impact model

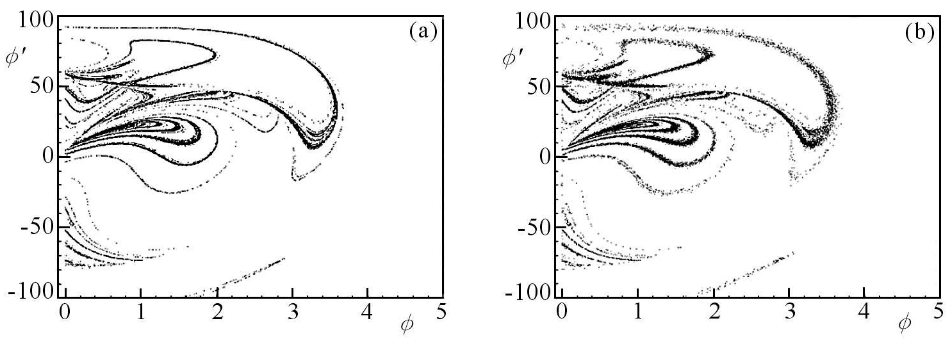

Fig. 7. Chaotic attractor for $A = 40$ and $\omega = 20\,\mathrm{rad/s}$: (a) rigid impact, (b) elastic impact

(Fig. 8a). From the practical point of view, the critical situation arises when the pendulum loses the contact with the support. This may lead to a cascade of impacts and high amplitude oscillations. Fig. 6 and Fig. 8a show that at the beginning of the diagrams ($A < 1$), the response of the pendulum is close to zero, but after crossing the critical point, the system begins to oscillate.

A zoomed portion of the beginning of the bifurcation diagram in Fig. 8a is presented in Fig. 8b. It can be seen that for $A <\sim 1$ the system does not oscillate, but after that it jumps to periodic motion, and later on the cascade of period doubling bifurcations goes to chaos. Because the solution presented in Fig. 8 is very sensitive to initial conditions, an additional analysis of the basins of attraction has been carried out.

Depending on the initial conditions, the solution can tend either to zero (trivial solution No. I, see Fig. 9) or to permanent oscillations (non-trivial solutions No. II and No. III). These three different possible solutions are pre-

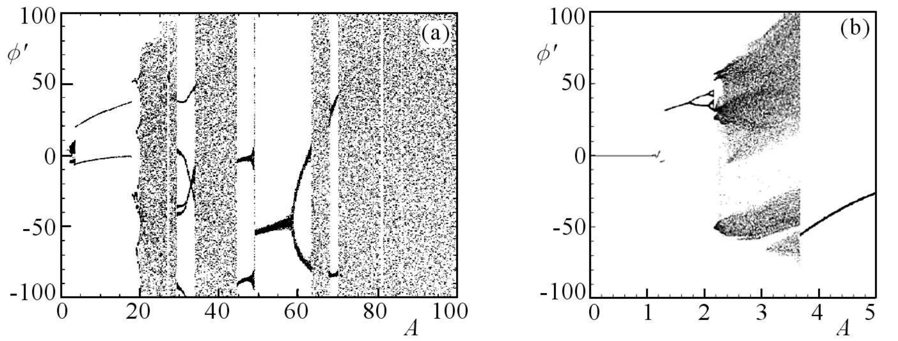

Fig. 8. Bifurcation diagrams of the soft impact model: (a) angular velocity of the pendulum versus amplitude A ($\omega = 100\,\mathrm{rad/s}$), (b) zoom

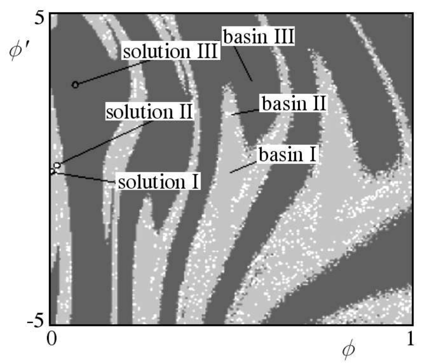

Fig. 9. Basins of attractions for trivial and non-trivial solutions, $\omega = 100\,\mathrm{rad/s}$, $A = 1$

sented as time histories in Fig. 10. Summarising, the response of the system for certain initial conditions leads to oscillations even for a small amplitude of the excitation.

4. System response under vehicle impact excitation

The final test of the response of the latch element has been carried out for an excitation in form of a short time shock. The lateral acceleration signal was recorded during a real vehicle crash test by one of OEM customers of Magna Closures of America. The acceleration of the base of the latch can reach very large values, with amplitudes up to 500 g and frequency in the range of

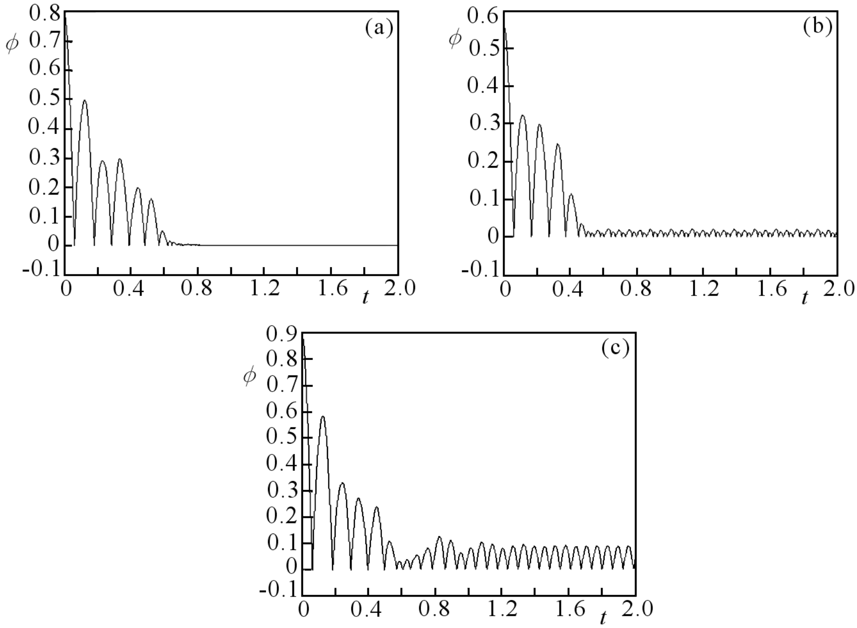

Fig. 10. Time histories of the pendulum angular displacement rotation angle for various initial conditions ($\omega = 100\,\mathrm{rad/s}$, $A = 1$): (a) $\phi = 0.8$, $\dot{\phi} = 0$, (b) $\phi = 0.56$, $\dot{\phi} = 0$, (c) $\phi = 0.9$, $\dot{\phi} = 0$

0-3000 rad/s. Recalling the results with harmonic excitation, it has been noticed that even for a low excitation amplitude, the pendulum swings out of the allowed range (which for the real mechanism corresponds to angles of about 0.1 rad/s).

Therefore, one can assume that for the model under consideration, during the side impact excitation will be sufficiently produced to lead to door opening. In Fig. 11a, the time history of the lateral acceleration recorded during the vehicle side crash test is presented. During the short time event (35 ms duration), the amplitude of the acceleration reached almost $A = 1\,000$ of the gravity acceleration g. Figure 11b presents the response of the latch element model to this input. It can be seen that the pendulum angle (in radians in Fig. 11b) reached high values. This test showed that during the side impact with given severity the latch subsystem consisting of the single pendulum would not work properly and the door would open.

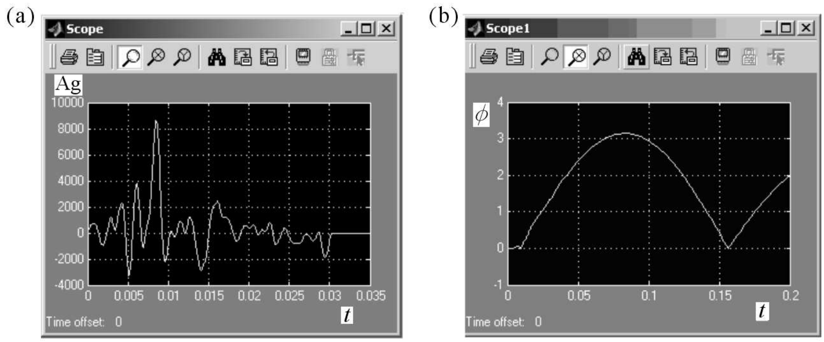

Fig. 11. Vibration of the system under shock excitation: (a) latch lateral acceleration measured during the crash test (in m/s^2), (b) pendulum response (angular displacement in radians)

5. Conclusions

The results obtained for a single pendulum model of a vehicle latch element show that the response of the system may lead to the loss of the contact between the pendulum and its support, and to the impact phenomenon. Harmonic excitations transit the system to high-level regular or chaotic motions. The analysis of basins of attraction shows three different solutions for low acceleration amplitudes ($A < 1$), one trivial and two nontrivial. An increase of the latch base excitation amplitude causes transition to chaos of the pendulum after period doubling bifurcation (as shown in Fig. 8b). Two different approaches for the impact phenomenon gave equivalent results, however the soft (elastic) impact exhibits attractors with a more fractal nature. Numerical simulations performed for various physical parameters revealed that the amplitude of the system response cannot be reduced. The acceleration signals obtained from the real vehicle side crash test and applied to the model also showed that the behaviour of the single pendulum latch element did not satisfy the imposed safety conditions. Therefore, a modification of the latch subsystem will be developed in forthcoming investigations, based on adding a second pendulum that will play the role of a counterweight.

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Drgania regularne i chaotyczne napiętego wstępnie układu mechanicznego z wahadłem

Streszczenie

W niniejszej pracy przedstawiono drgania pojedyńczego wahadła fizycznego napiętego wstępnie sprężyną, z uwzględnieniem możliwości zjawiska uderzenia w ogranicznik ruchu. Rozważany układ reprezentuje prosty model jednego z elementów mechanizmu zamykania drzwi pojazdu samochodowego. Zbadano numerycznie odpowiedź układu dla sztywnego i lepko-sprężystego modelu uderzenia w szerokim zakresie wymuszeń harmonicznych. Porównano dla obu modeli uderzenia obszary występowania ruchu regularnego i chaotycznego. Przeprowadzono również analizę obszarów przyciągania rozwiązań, określając wszystkie możliwe rozwiązania systemu. Końcową weryfikację odpowiedzi układu przeprowadzono dla sygnału eksperymentalnego, zarejestrowanego podczas próby bocznego zderzenia pojazdów. Pokazano, ze odpowiedź mechanizmu może doprowadzić do otwarcia drzwi pojazdu i dlatego inne rozwiązanie techniczne powinno zostać użyte.

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