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LIMITATIONS IN APPLICATION OF BASIC FREQUENCY SIMPLEST LOWER ESTIMATORS IN INVESTIGATION OF NATURAL VIBRATIONS OF CIRCULAR PLATES WITH VARIABLE THICKNESS AND CLAMPED EDGES

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In this paper, we discussed the effects of application the Bernstein-Keropian simplest lower estimators for calculation of basic natural vibration frequencies of variable-thickness circular plates. Following a thorough analysis, we showed a significant role of the operators of the 4th order Euler equations of motion describing vibrations of circular plates with exponentially variable thickness. The dependence of solutions on values of the variable thickness power index of the plate and Poisson's ratio of the plate material was investigated. The variable power thickness diaphragm model was verified by FEM which confirmed the usefulness of the simplest estimator for following materials: titanium, steel, zinc. Comparison of the results obtained after applying the simplest lower estimators for the basic frequency with the results of exact solutions for particular cases found in the literature, confirmed high accuracy of the applied method.

Key words: circular plates, variable thickness, boundary-value problem, Cauchy function method

1. Introduction

In his paper, Conway (1958b) analyzed the basic natural frequency of axisymmetric vibrations of clamped edge circular plates when the flexural rigidity D varied with the radius r according to the law: $D = D_0 r^m$, where D_0 and m are constants. The author obtained characteristic equations using special Bessel's functions for particular cases when the plate thickness changed lineary and parabolic. He chose a few combinations of variable thickness parameter m and Poisson's ratio ν according to the formula: $\nu = (2m - 3)/9$, which led to exact solutions. Although these solutions have a limited practical value, they are unique and may be used to estimate the accuracy rate of approximate solutions (Vasylenko, 1992; Kovalenko, 1959).

We propose a method of characteristic series using the Cauchy influence function to solve these problems. The method seems to be attractive, because it gives terms of characteristic series in an analytical form. The theory of vibrations of discrete-continuous linearly elastic systems, which has been developed in this work, is useful for constructing and studying universal frequency equations (Jaroszewicz and Zoryj, 2005). This method is based on making use of the Cauchy influence functions and a characteristic series (Jaroszewicz and Zoryj, 1997). Properties of the Cauchy influence function are applied in this method. One property of the Cauchy influence function is especially significant – not only the function itself but also its derivatives in respect to the parameter α always create a basic system for solutions (Haščuk and Zoryj, 1999).

It was shown that the proposed method allows us to analyse the influence of additional mass rings on the frequencies of natural vibrations of clamped circural plates with linearly variable thickness by means of the mass partial discretization methods (Jaroszewicz and Zoryj, 2006; Jaroszewicz *et al.*, 2006). By comparison, the spectral function method, proposed by Bernštein in 1960, was used solely for the analysis of systems characterised by constant parameters with no consideration given to friction (Bernštein and Kieropian, 1960). In the paper by by Jaroszewicz and Zoryj (2000) the characteristic series method was successfully applied to solve the boundary-value problem of free transversal vibrations of an axially loaded vertical cantilever with variable parameters. Main frequencies of axi-symmetric vibrations of thin plates with variable distribution of parameters were analyzed by Jaroszewicz *et al.* (2004) by means of the characteristic series method. The influence of Young's modulus, Poisson's ratio and mass density of the material on the base frequency of circular plates of the diaphragm type with variable thickness was discussed by Domoradzki *et al.* (2005). The paper addresses the problems concerning properties of the fundamental system of Euler's operators as well as limitations and singularities of the above mentioned methods for solving the boundary value problems of variable thickness circular plates.

The authors examine circular plates with variable thickness by means of the influence function method. It is worth pointing out that Conway did not apply the exact solution for the considered case (Conway, 1958a,b).

2. Formulation of the problem

We consider an R-radius circular plate having a clamped edge. Its thickness h and flexural rigidity D change in the following way

$$h = h_0\Big(\frac{r}{R}\Big)^{\frac{m}{3}} \qquad\qquad D = D_0\Big(\frac{r}{R}\Big)^m \qquad\qquad 0 < r \leqslant R \tag{2.1}$$

and

$$D_0 = \frac{Eh_0^3}{12(1-\nu^2)}$$

where D_0, h_0, $m \geqslant 0$ are constants, r denotes the radial coordinate, E – Young's modulus, and ν – Poisson's ratio.

Investigation of free, axi-symmetric vibrations of such a plate is reduced to analysis of the boundary problem (Conway, 1958b, Hondkiewič, 1964; Zoryj and Jaroszewicz, 2005)

$$L_0[u] - pr^{-\frac{2}{3}m}u = 0 \qquad\qquad u(R) = 0 \qquad\qquad u'(R) = 0 \tag{2.2}$$

where

$$\begin{aligned} L_0[u] &\equiv u^{IV} + \frac{2}{r}(m+1)u^{III} + \frac{1}{r^2}(m^2+m+\nu m-1)u^{II} + \\ &\quad + \frac{1}{r^3}(m-1)(\nu m-1)u^I \\ p &= \frac{\rho h_0}{D_0}R^{\frac{2}{3}m}\omega^2 \end{aligned} \tag{2.3}$$

$u = u(r)$ denotes the amplitude of flexural vibrations, ρ – density of the material of the plate, ω – parameter of frequency (angular velocity). Boundary value conditions corresponding to clamped edge $(2.2)_{2,3}$ are defined as zero deflection and zero angle of deflection for $r = R$. Additional conditions pertaining to the centre of symmetry of the plate $(r = 0)$ have limited values of deflection $u(0) < \infty$ and zero values of the angle of deflection $u'(0) = 0$.

The value $m = 0$ refers to the plate with constant thickness; $m > 0$ – to plates of the diaphragm type with thickness decreasing toward the center; $m < 0$ – to plates of the disc type with thickness increasing toward the center (Hondkiewič, 1964; Jaroszewicz and Zoryj, 2005). We determine the border of variation of the power index $m \geqslant 0$ for which the most simple estimators of the basic frequency ω_1 exist and, therefore, they can be calculated, i.e. we search for the lowest proper value of boundary problem (2.2).

In problem (2.2), a limitation of solutions growing r and their first three derivatives with respect to the independent variable r is required (Conway, 1958b).

3. Derivation of the frequency equation

The necessary limited solution of equation $(2.2)_1$ will be derived according to the known formula (Haščuk and Zoryj, 1999; Jaroszewicz and Zoryj, 2005)

$$S_j = s_{j0} + ps_{j1} + p^2 s_{j2} + \dots \qquad j = 1,2 \tag{3.1}$$

where

$$S_{jk} = \int_0^r K(r,\tau)\tau^{-\frac{2}{3}m} s_{j,k-1}(\tau)\,d\tau \qquad k = 1,2,\dots \tag{3.2}$$

S_{10}, S_{20} are the solutions (limited for $r = 0$) to Euler's equation $L_0[u] = 0$. Therefore, $K(r,\tau)$ – its Cauchy's function, is the solution to the equation $L_0[u] = 0$, which satisfies the conditions

$$K(r,\tau) = K'(r,\tau) = K''(r,\tau) = 0 \qquad K'''(r,\tau) = 1 \tag{3.3}$$

In order to build the above mentioned function $K(r,\tau)$, we need solutions to the given equation, which correspond to operator $(2.3)_1$. Substituting $u = r^s$ for $p = 0$ in $(2.2)_1$, we obtain an appropriate algebraic equation with respect to the parameter s (Hondkiewič, 1964)

$$s\Big\{s^3 + 2(m-2)s^2 + [4 - 5m + m(m+\nu)]s + m(2-m)(1-\nu)\Big\} = 0 \tag{3.4}$$

The roots of this equation and the corresponding Cauchy function were previously determined for a few values of m where $m \leqslant 3$ (Conway, 1958b). That is why in this paper, we examine the case

$$2 < m < \infty \qquad \nu = \frac{1}{m} \tag{3.5}$$

The parameter m is the same which was used in operator $(2.3)_1$. In the case of a constant thickness plate $m = 0$, the coefficient of natural frequency $\gamma(m)$, described by formula (4.3), is not dependent on ν, which was shown by Conway (1958b), Hondkiewič (1964), Jaroszewicz and Zoryj (2005).

Obviously, for operator $(2.3)_1$, the coefficient of u' equals zero and the roots of equation (3.4) are as follows

$$s_1 = 0 \qquad s_2 = 1 \qquad s_3 = 1 - m \qquad s_4 = 2 - m \tag{3.6}$$

where s_3 and s_4 are determined from the square equation, which was obtained from (3.4) for $s_1 = 0$ and $s_2 = 1$.

Hence, corresponding fundamental system (3.6) of solutions to the equation $L_0[u] = 0$ takes the following form

$$u_1 = 1 \qquad u_2 = r \qquad u_3 = r^{1-m} \qquad u_4 = r^{2-m} \tag{3.7}$$

The application of the above system, leads to the formula

$$K(r,\tau) = \frac{1}{1-m}\Big[\frac{1}{m}(-r\tau^2 + r^{1-m}\tau^{2+m}) + \frac{1}{2-m}(r^{2-m}\tau^{1+m} - \tau^3)\Big] \tag{3.8}$$

Now solutions (3.1) can be constructed. After calculating the first two integrals of (3.2) (for $j = 1, 2$ and $k = 1$), we obtain the following relations

$$S_1(r) = 1 + A_1 p r^a \qquad S_2(r) = r + B_1 p r^{a+1} \tag{3.9}$$

and

$$A_1 = \frac{1}{ab(b+1)(a-1)} \qquad B_1 = \frac{1}{a(a+1)(b+1)(b+2)} \tag{3.10}$$

where

$$a = 4 - \frac{2}{3}m \qquad b = 2 + \frac{m}{3} \tag{3.11}$$

After taking into consideration the formulas for u_3 and u_4 (3.7) for conditions (3.5), we can draw a conclusion that the limited solutions to equation $(2.2)_1$ are determined using the formula

$$u(r) = C_1 S_1(r) + C_2 S_2(r) \tag{3.12}$$

where C_1 and C_2 are arbitrary constants. After inserting this solution into condition $(2.2)_{2,3}$, we obtain homogenous system of two algebraic equations for the constants C_1 and C_2, whose determinant is the left part of the frequency equation of problem (2.2). Therefore

$$\begin{vmatrix} S_1(r) & S_2(r) \\ S_1'(r) & S_2'(r) \end{vmatrix} = 0 \tag{3.13}$$

Considering formulas (3.9)-(3.11), we obtain the first approximation (accurate to p, hence to the square of the frequency), which takes the following form

$$1 - a_1 p R^a = 0 \tag{3.14}$$

where

$$a_1 = -[A_1(1-a) + B_1(1+a)] \tag{3.15}$$

Taking into consideration the equation

$$pR^a = \frac{\rho h_0}{D_0} R^{\frac{2}{3}m} R^{4-\frac{2}{3}m} = \frac{\rho h_0}{D_0} R^4 \omega^2 \tag{3.16}$$

we transform (3.16) to the following form

$$1 - a_1 \frac{\rho h_0}{D_0} R^4 \omega^2 = 0 \tag{3.17}$$

Remark: for $m = 3$, $\nu = 1/3$, we obtain $A_1 = 1/24$, $B_1 = 1/120$ (Jaroszewicz and Zoryj, 2005). Formulas (3.10), (3.11) are in accordance with the values quoted in the above remark.

4. The simplest lower estimator for the basic frequency

All roots of the frequency equation of the problem defined by (2.2), (2.3) ought to be real, positive and in most cases, simple numbers according to the general characteristics of the frequency spectra of elastic systems (Vasylenko, 1992). Therefore, we obtain the simplest estimator of the basic (the smallest) frequency from (3.17)

$$\omega_1 > \gamma(m) \frac{1}{R^2} \sqrt{\frac{D_0}{\rho h_0}} \tag{4.1}$$

or

$$\omega_1 > \gamma(m) \frac{h_0}{R^2} \sqrt{\frac{E}{12\rho(1-\nu^2)}} \tag{4.2}$$

where

$$\gamma(m) = \frac{1}{\sqrt{a_1}} \tag{4.3}$$

Examples:

1. For the constant thickness plate $(m = 0)$, the coefficient $a_1 = 1/96$ (see Jaroszewicz *et al.*, 2004). So it follows that $\gamma(0) = \sqrt{96} \approx 9.798$, which consists 95.92% of the exact value 10.214 (3.196^2) (Vasylenko, 1992).

2. For the diaphragm $m = 3$, $\nu = 1/3$ (linearly variable thickness) $a_1 = 1/60$ (see Jaroszewicz and Zoryj, 2005). So, $\gamma(3)\big|_{\nu=1/3} = \sqrt{60} \approx 7.746$, which consists 88.56% of the exact value 8.747 (Conway, 1958b).

Examples of constant thickness plates $(m = 0)$ and the law of linearly variable thickness $(m = 3)$ are presented in this paper as the standard solutions.

For cases (3.5) using (3.10) and (3.11), we transform (3.15) to the following form

$$a_1 = \frac{2}{ab(b+1)(b+2)} \tag{4.4}$$

hence for $m = 3$ with $a = 2$, $b = 3$, on the base of equations (3.10), (3.15), a_1 equals $1/60$.

Taking into consideration relationships (3.11), it is possible to transform formula (4.4) into

$$a_1 = \frac{3^4}{(6-m)(6+m)(9+m)(12+m)} \tag{4.5}$$

On the base of expression (4.5) for $m > 2$ and $\nu = 1/m$, we formulate the conclusion that the simplest lower estimator can be applied when $a_1 > 0$ under the necessary condition that $m < 6$. The term a_1 has negative values for $m > 6$ and it is indefinite for $m = 6$. These two cases $(m > 6$ and $m = 6)$ cannot be accepted. It should be pointed out that construction materials having $\nu \leqslant 1/6$ do not exist in reality. The border value of $\nu \leqslant 1/6$ corresponds to a concrete plate. However, manufacturing a concrete plate for $m > 6$ is technologically impossible and, therefore, such a case has only theoretical value.

We introduce a change of the coefficient $\gamma(m)$ for $m > 3$ applying (4.5) $(\nu = 1/m)$. The results of calculations are shown in Fig. 1. Precise values of $\gamma(m)$ are displayed in Table 1.

The results of calculations of the base frequency for selected materials and values of m are presented in Table 2.

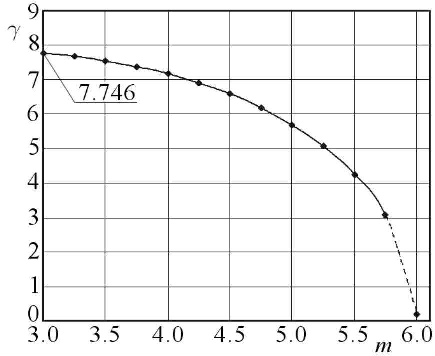


Fig. 1. The curve showing the influence of the plate thickness index on the simplest estimator of the basic frequency coefficient

Table 1. Results of calculations of the variable-rigidity plate for $m = 3.25$ to $m = 5.999$

m	3.25	3.5	3.75	4	4.25	4.5
γ	7.659451488	7.537201975	7.374735166	7.166451332	6.905149243	6.581223292
m	4.75	5	5.25	5.5	5.75	5.999
γ	6.181238257	5.685155024	5.060184657	4.244186290	3.081354161	0.199979495

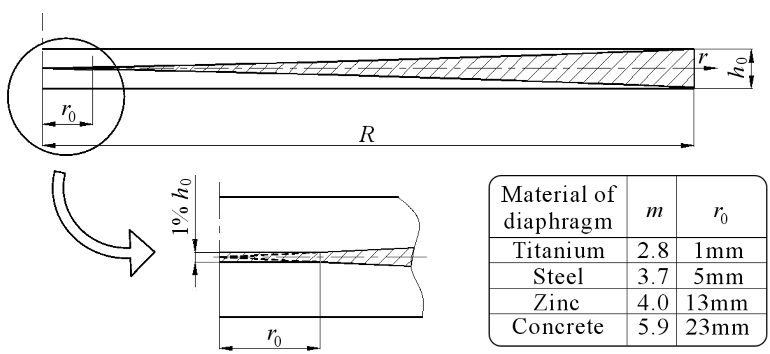
Table 2. Results of calculations of the base frequency coefficient

No.	Coefficient m	Poisson's ratio of plate material ν	First term of characteristic series a_1	Simplest estimator γ	Value of exact solution
1	0	arbitrary from values 0-0.5	$\frac{1}{96}$	9.798	10.214 (Conway, 1958a)
2	3	$\frac{1}{3}$	$\frac{1}{60}$	7.746	8.730 (Conway, 1958a)
3	4	$\frac{1}{4}$	$2\frac{4}{3}\frac{10}{3}\frac{13}{3}\frac{16}{3}$	7.166	Exact value is unknown
4	5	$\frac{1}{5}$	$2\frac{2}{3}\frac{11}{3}\frac{14}{3}\frac{17}{3}$	5.685	
5	5.9	0.169	0.255	1.979	
6	5.99	0.16(7)	2.505	0.632	

In the case of the constant thickness plate $m = 0$ (No. 1), the coefficient of natural frequency $\gamma(0)$ is not dependent on ν, which has been already described.

5. FEM verification of the results of calculations of the base frequency simplest estimators

The Solid Works software was used in the modeling of diaphragm type plates made of the selected materials: titanium, zinc, steel, concrete, which satisfy the condition of the following relation between the thickness index m and Poisson's ratio $\nu = 1/m$. Calculations were performed for thin orthotropic circular plates attached on perimeter. The plates were characterized by exponentially changing thickness and the following main dimensions: radius $R = 250\,\mathrm{mm}$, thickness in the area of attachment $h_0 = 10\,\mathrm{mm}$. A half of the radial section of the plate is presented in Fig. 2. Shaping the geometry of a plate with exponentially changing thickness (described by formula (2.1)) requires selection of the minimum thickness in the centre of symmetry equal to 0.1 mm, which amounts to 1% of the plate thickness in the area of attachment. The values of the radius r_0 for which the thickness is in reality lower than 0.1 mm are displayed in Fig. 2. Dimensions of applied FEM elements allow the smallest thickness to be 0.1 mm on the plate surface limited by the radius r_0 in relation to the symmetry axis. The case of concrete considered in a given point in time has only a theoretical value.



Material of diaphragm	m	r_0
Titanium	2.8	1mm
Steel	3.7	5mm
Zinc	4.0	13mm
Concrete	5.9	23mm

Fig. 2. Lengthwise section along the radial coordinate r of the plate characterised by geometrical dimensions r_0, R, h_0

The mesh with changing size of elements was generated automatically. The numerical analysis was conducted using the Cosmos Work software. Two solvers: the Direct sparse solver and the FFEPlus were applied to perform the numerical analysis. The FEM analysis gives convergent results with analytical calculations by means of the Cauchy function method. The first five modes and corresponding natural frequencies were investigated. The results of the analysis are presented in Fig 3.

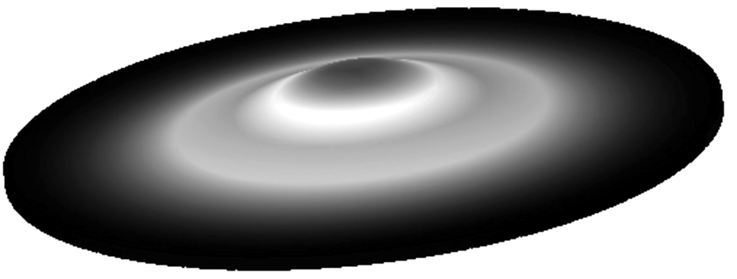


Fig. 3. The base mode of natural vibrations of a titanium diaphragm (330.19 Hz, scale of deformation: 0.025)

The differences between the results of calculations using FEM and the Cauchy function method are set together in Table 3.

Table 3. The base frequency of the plate with nonlinearly variable thickness $h(r \to 0) = 0.01\,\mathrm{mm}$

Material	ν	Coefficient m	D_0 [Nm]	Mass [kg]	Simplest estimator γ	Frequency of base estimator f [Hz]	Frequency by FEM analysis f [Hz]	Difference Δ [%]
Titanium	0.36	2.78	9778	6.093	7.795	288.67	330.19	12.6
Steel	0.27	3.7	18560	9.834	7.412	283.12	347.15	18.4
Zinc	0.25	4.0	10531	8.410	7.166	219.61	267.35	17.9
Concrete	0.17	5.9	1716	1.983	1.979	46.26	211.76	Not acceptable

The results of calculations of the base frequency using the simplest estimator, presented in Table 3, are accepted for titanium, steel and zinc, whereas for concrete they are not acceptable due to the fact that the difference exceeds 18%. It is worth to notice that the case of the concrete plate is only theoretical because such a structure cannot be manufactured in practice. The results of numerical analysis show that the simplest estimator can be used within $2.78 \leqslant m \leqslant 4$.

6. Conclusions

Deriving the above mentioned formulas for the Cauchy functions as well as the fundamental systems of the function operator $L_0[u]$, it allows one to study the convergence problem (velocity of convergence) of solutions to equation $(2.2)_1$ in the form of power series with respect to frequencies depending on the parameters m and ν.

Having the influence functions of the operator $L_0[u]$, we can determine corresponding solutions such as (3.12)-(3.17) and use them for any given and physically justified parameters m and ν $(2 < m < \infty;\ \nu = 1/m)$, when the exact solutions are unknown. On the basis of the quoted solutions, simple engineering formulas for the frequencies estimators of circular plates, which are characterised by variable parameter distribution, can be derived, and limitations on their application can be identified. The simplest lower estimator calculated using the first element of series (4.4) allows ont to observe with considerable credibility the effect of the material constants: Young's modulus E, Poisson's ratio ν, density ρ on frequencies of axi-symmetric vibrations of circular plates. Thickness or rigidity of the tested plates change along the radius according to an exponential function.

The coefficient of the base frequency $\gamma(m) = 1/sqrta_1$ can be determined for $3 < m < 6$ (Fig. 1), because $\gamma(m) \to 0$ for $m \to 6$ it corresponds to $a = 4-2m/3 \to 0$, $1/\sqrt{a_1} \to 0$. In this case, the simplest lower estimator gives strongly underestimated values, therefore it cannot be even used in approximate calculations. However, the simplest lower estimators can be applied to preliminary engineering calculations for constant and variable-thickness plates, when $0 \leqslant m \leqslant 4$.

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Ograniczenia w stosowaniu najprostszych dolnych estymatorów częstości podstawowej w badaniu drgań własnych płyt kołowych typu diafragma

Streszczenie

W pracy omówiono efekt zastosowania najprostszych estymatorów Bernštejna-Kieropiana do obliczania podstawowej częstości osiowosymetrycznych drgań własnych płyt o zmiennej grubości typu diafragma. Przeanalizowano i uwypuklono istotną rolę operatorów równań Eulera czwartego rzędu oraz ich fundamentalnej funkcji w osiowosymetrycznych zagadnieniach drgań płyt kołowych o potęgowo zmiennej grubości. Zbadano zależność rozwiązania od wartości wskaźnika zmiany grubości płyty i liczby Poissona. Weryfikacja modelu diafragm o potęgowo zmiennej grubości przy pomocy MES potwierdziła możliwość zastosowania najprostszego estymatora w przypadku metali: tytanu, stali, cynku. Porównanie wyników obliczeń najprostszych estymatorów z dołu częstości podstawowej uzyskanych przy wykorzystaniu zaproponowanej metody oraz przy zastosowaniu znanych z literatury rozwiązań ścisłych dla szczególnych przypadków zmiany grubości płyty potwierdziło wysoką dokładność proponowanej metody.

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