JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
44, 1, pp. 75-90, Warsaw 2006

INTERACTIVE BUCKLING IN THIN-WALLED BEAM-COLUMNS WITH WIDTHWISE VARYING ORTHOTROPY

Tomasz Kubiak

Department of Strength of Materials and Structures, Technical University of Lodz
e-mail: tomek@p.lodz.pl

An analysis of local buckling of thin-walled beam-columns, taking account global precritical bending within the first order approximation, is presented in the paper. The problem of interactive buckling of the structure is solved by means of Byskov and Hutchinson's (1977) or Koiter's (1976) approximation theory. Beam-columns made from orthotropic plates with the main directions of orthotropy parallel to the wall edges characterised by a widthwise varying orthotropy coefficient $\eta_i = E_{yi}/E_{xi}$ are investigated. Beam-columns with open sections (i.e. channel sections), simply supported on the loaded edges, are analysed. The girders are subjected to loads which cause a uniform or linearly variable shortening of the edges.

Key words: interactive buckling, thin-walled structures, orthotropy

Notation

A	–	cosinusoid amplitude
$a_g, a_{gLL}, a_L, a_{ggg}$	–	coefficients of the non-linear equilibrium equation, where the subscript g denotes the global mode ($n = 1$), L refers to the local buckling mode ($n = 2$)
b_i	–	ith band (narrow plate) width
D_i, D_{1i}	–	plate stiffness of the ith band; ($D_i = E_i h_i^3/[12(1-\eta_i\nu_i^2)]$, $D_{1i} = G_i h_i^3/6$)
$E_i = E_{ix}$	–	lengthwise Young's modulus for the ith band of the girder wall
E_{iy}	–	widthwise Young's modulus for the ith band

G_i	–	modulus of elasticity (Kirchhoff's modulus) for the ith band
h_i	–	thickness of the ith band
i	–	number of the band, wall (subscript $i = 1, 2, \ldots$)
l	–	girder length
M_{ix}, M_{iy}, M_{ixy}	–	sectional bending moment of the ith band
$\overline{\boldsymbol{N}}$	–	force field
$\overline{N}_i^{(0)}$	–	force field of the zero state (prebuckling state)
$\overline{N}_i^{(n)}$	–	force field of the nth buckling mode
N_{ix}, N_{iy}, N_{ixy}	–	sectional membrane forces for the ith band
$\overline{\boldsymbol{U}}$	–	displacement field
$\overline{U}_i^{(0)}$	–	displacement field of the zero state (prebuckling state)
$\overline{U}_i^{(n)}$	–	displacement field of the nth buckling mode
u_i, v_i, w_i	–	middle surface displacements for the ith band
$u_i^{(0)}, v_i^{(0)}, w_i^{(0)}$	–	prebuckling displacement field for the ith band (zero state)
$u_i^{(1)}, v_i^{(1)}, w_i^{(1)}$	–	critical displacement field for the ith band (for the first order)
x_i, y_i, z_i	–	local Cartesian co-ordinate system for the ith band
$\beta_0 = 3.2292$	–	assumed constant value for the inversed coefficient of orthotropy
$\beta_i = 1/\eta_i$	–	inverse of the assumed coefficient of orthotropy, assumed in order to facilitate the analysis of data
$\varepsilon_{ix}, \varepsilon_{iy}$	–	relative strain along x_i, y_i
$\gamma_{ixy} = 2\varepsilon_{ixy}$	–	non-dilatational strain angle
κ	–	parameter of the external load distribution (ratio of the displacement of the upper part of the girder with respect to the bottom part)
$\eta_i = E_{iy}/E_{ix}$	–	coefficient of orthotropy of the ith plate (band)
λ	–	scalar load parameter
λ_g	–	critical value of λ (critical value of buckling) of the global mode
λ_L	–	critical value of λ (critical value of buckling) of the first local mode
λ_{min}	–	minimal critical value of λ (critical value of buckling)
λ_*	–	critical value of local buckling with global pre-critical bending taken into account

$\nu_i = \nu_{ixy}$	–	Poisson's ratio for the ith band in the x direction (the first subscript denotes a transverse direction, whereas the second one – the load direction)
ν_{iyx}	–	Poisson's ratio for the ith band in the y direction (the first subscript denotes a transverse direction, whereas the second one – the load direction)
φ	–	angle enclosed between the wall i and $i+1$
ξ	–	amplitude of the linear eigenvector of buckling (normalised with the equality condition between the maximum deflection and the thickness of the first plate h_1)
ξ_n	–	amplitude of the linear eigenvector of buckling for the nth buckling mode
ξ_L	–	amplitude of the linear eigenvector of buckling for the first local buckling mode
ξ_g	–	amplitude of the linear eigenvector of buckling for the global buckling mode
ξ_L^*	–	initial imperfection consistent with the first local buckling mode
ξ_g^*	–	initial imperfection consistent with the global buckling mode.

Moreover, the following notation has been used

$$(\cdot)_{,x} = \frac{\partial(\cdot)}{\partial x} \qquad\qquad (\cdot)_{,y} = \frac{\partial(\cdot)}{\partial y}$$

1. Introduction

Interactive buckling of isotropic and orthotropic thin-walled structures has been investigated in many works (e.g. Luongo and Pignataro, 1988; Maniewicz and Kołakowski, 1997). Results of these investigations show a possibility of building thin-walled structures that are light, safe and reliable.

As far as composites are concerned, their material properties can be freely modelled in selected directions or regions, thus it is possible to manufacture plates or girders with variable strength properties. An example of materials characterised by such properties are fibrous composites with properly distributed (concentrated or dispersed) fibres. Composite materials are most often modelled as orthotropic materials. In the wide literature devoted to stability problems, there is a lack of analysis of the influence of plate widthwise varying orthotropy on values of critical loads of coupled buckling of girders built of such plates.

In the present paper, the problem of local loss of stability, accounting for global pre-critical bending in the elastic range, is discussed. Thin-walled beam-columns with open sections built of homogeneous orthotropic plates with widthwise varying orthotropy are considered. The consideration of a particular variation in the orthotropy is carried out simply to demonstrate that such variations can be dealt with analytically, and to illustrate the influence of material properties.

2. Problem under consideration

Beam-columns with open sections and made of plates with widthwise varying orthotropy (Fig. 1) have been analysed. For discretized material properties varying widthwise, a model built of narrow longitudinal orthotropic bands has been assumed. Each band has constant material properties. The coefficient of orthotropy for individual bands (narrow plates) of the model varies according to the formula

$$\beta_i = \beta_0 + A\cos\frac{2\pi y_i}{b_i} \tag{2.1}$$

where $\beta_0 = 3.2292$, $A \in \langle -2, 2\rangle$ is the amplitude of the cosine wave, y_i is a coordinate defining the distance of the band from the one of the longitudinal edges; b_i is the plate width.

It has been assumed that the main axes of wall orthotropy are parallel with respect to the wall edges. For the ith orthotropic band, a complete strain tensor for thin plates has been assumed in the form

$$\begin{aligned}
\varepsilon_{ix} &= u_{i,x} + \frac{1}{2}(w_{i,x}^2 + u_{i,x}^2 + v_{i,x}^2)\\
\varepsilon_{iy} &= v_{i,y} + \frac{1}{2}(w_{i,y}^2 + u_{i,y}^2 + v_{i,y}^2)\\
2\varepsilon_{ixy} &= \gamma_{ixy} = u_{i,y} + v_{i,x} + w_{i,x}w_{i,y} + u_{i,x}u_{i,y} + v_{i,x}v_{i,y}
\end{aligned} \tag{2.2}$$

where: u_i, v_i, w_i are displacements parallel to the respective axes x_i, y_i, z_i of the local Cartesian system of co-ordinates, whose plane (x_i, y_i) coincides with the central area of the ith plate (ith band) before its buckling (Fig. 2).

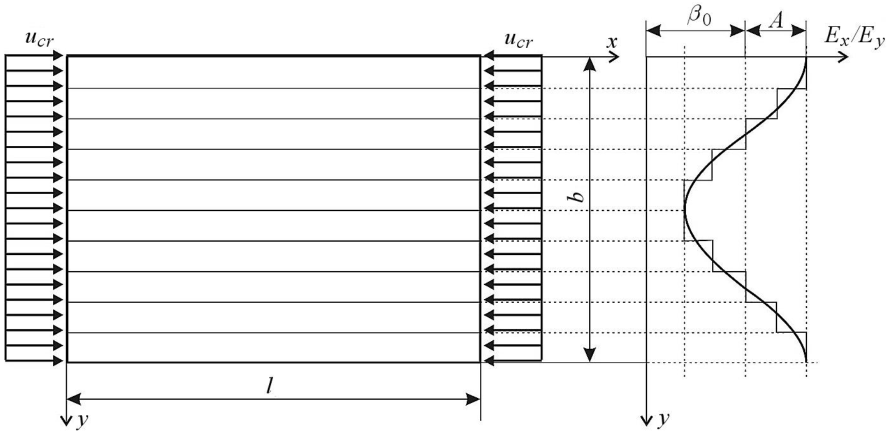

Fig. 1. The band model of a plate characterised by variable orthotropy

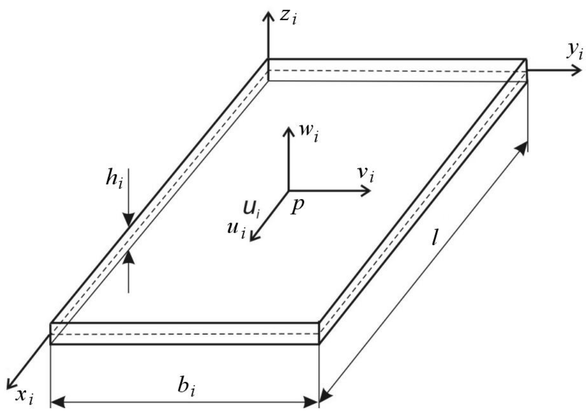

Fig. 2. Dimensions of the ith plate and the assumed local system of co-ordinates

Well known in the theory of orthotropic plates relations (e.g. Chandra and Raju, 1973; Królak, 1995) describe sectional forces and moments reduced to the middle surface of the ith plate (ith band)

$$
\begin{aligned}
N_{ix} &= \frac{E_i h_i}{1-\eta_i\nu_i^2}(\varepsilon_{ix}+\eta_i\nu_i\varepsilon_{iy}) && M_{ix} = D_i(\kappa_{ix}+\eta_i\nu_i\kappa_{iy}) \\
N_{iy} &= \frac{E_i h_i}{1-\eta_i\nu_i^2}(\eta_i\nu_i\varepsilon_{ix}+\eta_i\varepsilon_{iy}) && M_{iy} = \eta_i D_i(\nu_i\kappa_{ix}+\kappa_{iy}) \\
N_{ixy} &= N_{iyx} = G_i h_i\gamma_{ixy} = 2G_i h_i\varepsilon_{ixy} && M_{ixy} = D_{1i}\kappa_{ixy}
\end{aligned}
\tag{2.3}
$$

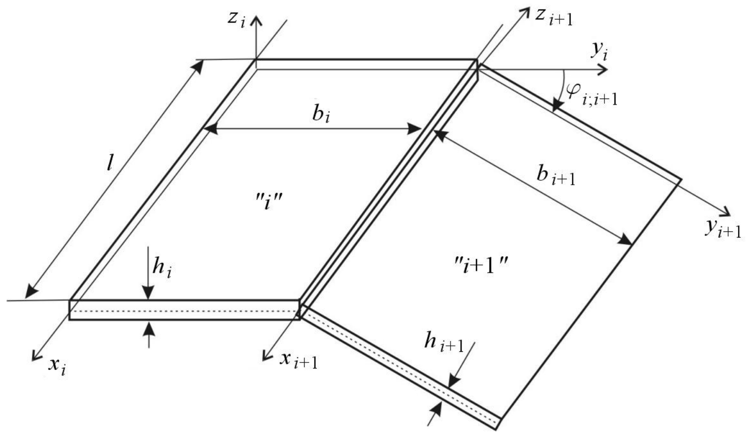

Fig. 3. Local co-ordinate systems of interactive walls (bands)

where

$$E_i \equiv E_{ix} \qquad \nu_i \equiv \nu_{ixy} \qquad \eta_i = \frac{E_{iy}}{E_{ix}} \tag{2.4}$$

According to the Maxwell-Betti theorem, Young's moduli and Poisson's ratios occurring in equations (2.3) have to fulfil the following relation

$$E_i\nu_{iyx} = E_{iy}\nu_i \tag{2.5}$$

The equation of equilibrium of thin-walled structures has been derived using a variational method (Kołakowski, 1993; Królak, 1995). The total potential energy variation for the ith plate (ith band) can be written as

$$\begin{aligned}
\delta\Pi_i &= \delta\int\limits_{S_i}(N_{ix}\varepsilon_{ix} + N_{iy}\varepsilon_{iy} + N_{ixy}\gamma_{ixy})\,dS_i + \\
&-\delta\int\limits_{S_i}(M_{ix}w_{i,xx} + M_{iy}w_{i,yy} + 2M_{ixy}w_{i,xy})\,dS_i - \int p_i(y_i)h_i\delta u_i\,dy_i + \\
&-\int p_i(x_i)h_i\delta v_i\,dx_i - \int \tau_{ixy}h_i\delta v_i\,dy_i - \int \tau_{ixy}h_i\delta u_i\,dx_i - \int q_i\delta w_i\,dS_i
\end{aligned} \tag{2.6}$$

where: $p_i(y)$, $p_i(x)$, τ_{ixy} are the external pre-critical loads of the plate, and q_i – transverse load.

The present paper deals with the stability problem, hence in the further part of the present analysis, the transverse load q_i will be neglected ($q_i = 0$).

Equation (2.6) indicates that the potential energy Π_i of the ith plate sector in the equilibrium state has a non-varying value in the class of permissible equations. It means that equation (2.6) for all permissible virtual displacements complying with the imposed constrains has to be satisfied.

In order to obtain the variation of potential energy for an orthotropic plate, strain tensor relation (2.2) has been substituted into equation (2.6). After grouping the components at respective variations, variational equations of equilibrium and boundary conditions have been obtained. Variational equations of equilibrium corresponding to equations (2.2) take the form

$$
\begin{aligned}
&\int\limits_S [N_{ix,x} + N_{ixy,y} + (N_{ix}u_{i,x})_{,x} + (N_{iy}u_{i,y})_{,y} + (N_{ixy}u_{i,x})_{,y} + \\
&\qquad +(N_{ixy}u_{i,y})_{,x}]\delta u_i \; dS = 0 \\
&\int\limits_S [N_{ixy,x} + N_{iy,y} + (N_{ix}v_{i,x})_{,x} + (N_{iy}v_{i,y})_{,y} + (N_{ixy}v_{i,x})_{,y} + \\
&\qquad +(N_{ixy}v_{i,y})_{,x}]\delta v_i \; dS = 0 \\
&\int\limits_S [(N_{ix}w_{i,x})_{,x} + (N_{iy}w_{i,y})_{,y} + (N_{ixy}w_{i,x})_{,y} + (N_{ixy}w_{i,y})_{,x} + \\
&\qquad +M_{ix,xx} + M_{iy,yy} + 2M_{ixy,xy}]\delta w_i \; dS = 0
\end{aligned}
\tag{2.7}
$$

and the boundary conditions are as follows

$$
\begin{aligned}
&\int\limits_y M_{ix}\delta w_{i,x} \; dy_i\Big|_{x_i=\mathrm{const}} = 0 \\
&\int\limits_x M_{iy}\delta w_{i,y} \; dx_i\Big|_{y_i=\mathrm{const}} = 0 \\
&2M_{ixy}\Big|_{x_i=\mathrm{const}\,;\; y_i=\mathrm{const}} \delta w_i = 0 \\
&\int\limits_x (N_{ixy} + N_{iy}u_{i,y} + N_{ixy}u_{i,x} - h_i\tau_{ixy})\delta u_i \; dx_i\Big|_{y_i=\mathrm{const}} = 0 \\
&\int\limits_y (N_{ix} + N_{ix}u_{i,x} + N_{ixy}u_{i,y} - h_ip_i(y))\delta u_i \; dy_i\Big|_{x_i=\mathrm{const}} = 0 \\
&\int\limits_x (N_{iy} + N_{iy}v_{i,y} + N_{ixy}v_{i,x} - h_ip_i(x))\delta v_i \; dx_i\Big|_{y_i=\mathrm{const}} = 0 \\
&\int\limits_y (N_{ixy} + N_{ix}v_{i,x} + N_{ixy}v_{i,y} - h_i\tau_{ixy})\delta v_i \; dy_i\Big|_{x_i=\mathrm{const}} = 0
\end{aligned}
\tag{2.8}
$$

$$\int_x (M_{iy,y} + 2M_{ixy,x} + N_{iy}w_{i,y} + N_{ixy}w_{i,x})\delta w_i \; dx_i\Big|_{y_i=\text{const}} = 0$$

$$\int_y (M_{ix,x} + 2M_{ixy,y} + N_{ix}w_{i,x} + N_{ixy}W_{i,y})\delta w_i \; dy_i\Big|_{x_i=\text{const}} = 0$$

Static interaction conditions at the longitudinal edges of neighbouring plates, which follow from (2.8) for $y=$ const, can be written as

$$\begin{aligned}
&u_{i+1}\Big|_{y_{i+1}=0} = u_i\Big|_{y_i=b_i}\\
&w_{i+1}\Big|_{y_{i+1}=0} = w_i\Big|_{y_i=b_i}\cos(\varphi_{i;i+1}) - v_i\Big|_{y_i=b_i}\sin(\varphi_{i;i+1})\\
&v_{i+1}\Big|_{y_{i+1}=0} = w_i\Big|_{y_i=b_i}\sin(\varphi_{i;i+1}) + v_i\Big|_{y_i=b_i}\cos(\varphi_{i;i+1})\\
&w_{i+1,y}\Big|_{y_{i+1}=0} = w_{i,y}\Big|_{y_i=b_i}\\
&M_{(i+1)y}\Big|_{y_{i+1}=0} = M_{iy}\Big|_{y_i=b_i}\\
&N^*_{(i+1)y}\Big|_{y_{i+1}=0} - N^*_{iy}\Big|_{y_i=b_i}\cos(\varphi_{i;i+1}) - Q^*_{iy}\Big|_{y_i=b_i}\sin(\varphi_{i;i+1}) = 0\\
&Q^*_{(i+1)y}\Big|_{y_{i+1}=0} + N^*_{iy}\Big|_{y_i=b_i}\sin(\varphi_{i;i+1}) - Q^*_{iy}\Big|_{y_i=b_i}\cos(\varphi_{i;i+1}) = 0\\
&N^*_{(i+1)xy}\Big|_{y_{i+1}=0} = N^*_{ixy}\Big|_{y_i=b_i}
\end{aligned} \tag{2.9}$$

where

$$\begin{aligned}
N^*_{iy} &= N_{iy} + N_{iy}v_{i,y} + N_{ixy}v_{i,x}\\
N^*_{ixy} &= N_{ixy} + N_{ixy}u_{i,x} + N_{iy}u_{i,y}\\
M_{iy} &= -\eta_i D_i(w_{i,yy} + \nu_i w_{i,xx})\\
Q^*_{iy} &= -\eta_i D_i w_{i,yyy} - (\nu_i\eta_i D_i + 2D_{1i})w_{i,xxy} + N_{iy}w_{i,y} + N_{ixy}w_{i,x}
\end{aligned} \tag{2.10}$$

The interactive (coupled) stability problem has been solved by means of Koiter's asymptotic method (Koiter, 1976). The fields of displacements $\overline{U}_i$ and the sectional forces $\overline{N}_i$ have been expanded into power series with respect to the parameter ξ_n, i.e. the linear eigenvector amplitude of buckling (normalised with the equality condition between the maximum deflection and the thickness of the first plate h_1)

$$
\begin{aligned}
\overline{\boldsymbol{U}}_i &= \lambda \overline{U}_i^{(0)} + \xi_n \overline{U}_i^{(n)} + \ldots \\
\overline{\boldsymbol{N}}_i &= \lambda \overline{N}_i^{(0)} + \xi_n \overline{N}_i^{(n)} + \ldots
\end{aligned}
\tag{2.11}
$$

for $n = 1, 2, \ldots, N$, where N is the number of coupled buckling modes, and λ is the load parameter.

In the investigations, only the first non-linear approximation, in which system characteristics depend only on eigenvectors, is taken into consideration. According to (2.11) and the number of coupled buckling modes $N = 2$, the displacement of the ith wall (band) has been assumed in the form

$$
\begin{aligned}
u_i &= \lambda u_i^{(0)} + \xi_1 u_i^{(1)} + \xi_2 u_i^{(2)} + \ldots \\
v_i &= \lambda v_i^{(0)} + \xi_1 v_i^{(1)} + \xi_2 v_i^{(2)} + \ldots \\
w_i &= \xi_1 w_i^{(1)} + \xi_2 w_i^{(2)} + \ldots
\end{aligned}
\tag{2.12}
$$

At the point where the load parameter λ reaches its maximum value for the imperfect structure with regard to the imperfection of the buckling mode with the amplitude ξ_n^* (secondary bifurcation or limit points), the Jacobian of the non-linear system of the equilibrium equation

$$
a_n\Big(1 - \frac{\lambda}{\lambda_n}\Big)\xi_r + a_{jkn}\xi_j\xi_k + \ldots = \frac{\lambda}{\lambda_n}a_n\xi_n^* \qquad n = 1, 2, \ldots, N \tag{2.13}
$$

is equal to zero. The expressions a_n and a_{jkn} in Eq. (2.13) are calculated by know from literature (Byskov and Hutchinson, 1977; Królak, 1995) formulas which only depend on the buckling modes. This fact is worth noticing as it reduces solutions to problems in the case when considerations can be limited to the first order non-linear approximation.

The non-linear equations of equilibrium are simplified within the first order approximation to a large extent in the case of interactions between two modes of buckling only. In a further part of this paper, $N = 2$ is assumed and equilibrium equations (2.13) have the form

$$
\begin{aligned}
a_g\Big(1 - \frac{\lambda}{\lambda_g}\Big)\xi_g + a_{ggg}\xi_g^2 + a_{gLL}\xi_L^2 &= a_g\xi_g^*\frac{\lambda}{\lambda_g} \\
a_L\Big(1 - \frac{\lambda}{\lambda_L}\Big)\xi_L + 2a_{gLL}\xi_g\xi_L &= a_L\xi_L^*\frac{\lambda}{\lambda_L}
\end{aligned}
\tag{2.14}
$$

where the subscript g denotes the global mode $(n = 1)$, L refers to the local buckling mode $(n = 2)$, ξ_g^* stands for the initial imperfection of the global character, and ξ_L^* indicates the initial imperfection consistent with the first local buckling mode.

If we assume that there are no local initial deflections $(\xi_L^* = 0)$, $\xi_g^* \neq 0$ and that the minimum value of the critical stress corresponds to the local buckling mode $(\lambda_{min} = \lambda_L)$, then equations (2.14) assume the form

$$
\begin{aligned}
&a_g\Big(1 - \frac{\lambda}{\lambda_g}\Big)\xi_g + a_{ggg}\xi_g^2 + a_{gLL}\xi_L^2 = a_g\xi_g^*\frac{\lambda}{\lambda_g} \\
&\xi_g\Big[a_L\Big(1 - \frac{\lambda}{\lambda_{\min}}\Big)\frac{\xi_L}{\xi_g} + 2a_{gLL}\xi_L\Big] = 0
\end{aligned}
\tag{2.15}
$$

The following notation has been introduced

$$
\Big(1 - \frac{\lambda}{\lambda_{\min}}\Big)\frac{1}{\xi_g} = \psi \tag{2.16}
$$

where ψ denotes the slope of straight line (2.16) being the post-critical equilibrium path that lies in the plane (λ, ξ_g).

In the pre-critical state $\xi_L = 0$, the global deflections, according to (2.13), are described by the relation

$$
\xi_g = \xi_g^*\frac{\lambda}{\lambda_g - \lambda} \tag{2.17}
$$

Then, equation $(2.15)_2$ takes the form corresponding to the eigenvalue problem

$$
\Big(\frac{2a_{gLL}}{a_L} + \psi\Big)\xi_L = 0 \tag{2.18}
$$

For the eigenvalue determined from (2.17), ψ can be obtained from the following equation

$$
\psi = -\frac{2a_{gLL}}{a_L} \tag{2.19}
$$

The coupled (interactive) buckling between the global and local mode starts when we obtain a non-zero solution $\xi_L \neq 0$. For ψ from equation (2.19), the eigenvector has been calculated with the accuracy up to the constant ρ which has been normalised with the condition $\sqrt{(\xi_L^0)^2} = 1$. Hence, equation $(2.14)_1$ can be written as

$$
\rho^2 = \frac{a_g}{a_{gLL}\xi_L^0}\Big[\xi_g^*\frac{\lambda}{\lambda_g} - \Big(1 - \frac{\lambda}{\lambda_g}\Big)\Big(1 - \frac{\lambda}{\lambda_L}\Big)\frac{1}{\psi} - \frac{a_{ggg}}{a_g}\Big(1 - \frac{\lambda}{\lambda_L}\Big)^2\frac{1}{\psi^2}\Big] \tag{2.20}
$$

The maximum value (the so called limit load carrying capacity) λ_* obtained within the first order non-linear approximation and corresponding to $\rho = 0$ (the intersection point of pre-critical path (2.17) with post-critical one (2.16)) can be called the critical value of the local buckling mode that accounts for global pre-critical bending (as $\xi_L^* = 0$, $\xi_g^* \neq 0$). This approach is similar to that found in Pignataro and Luongo (1987), Luongo and Pignataro (1988) and Manievicz and Kołakowski (1997), where, however, $a_{ggg} = 0$ was assumed.

For $\rho = 0$, equation (2.20) takes the form of a quadratic equation

$$\xi_g^* \frac{\lambda_*}{\lambda_g} + \Big(1 - \frac{\lambda_*}{\lambda_g}\Big)\Big(1 - \frac{\lambda_*}{\lambda_L}\Big)\frac{a_L}{2a_{gLL}} - \frac{a_{ggg}}{a_g}\Big(1 - \frac{\lambda_*}{\lambda_L}\Big)^2 \frac{a_L^2}{4a_{gLL}^2} = 0 \tag{2.21}$$

or

$$\lambda_*^2\Big[\frac{a_{ggg}}{a_g}\frac{1}{\lambda_L^2} - \frac{2a_{gLL}}{a_L}\frac{1}{\lambda_L\lambda_g}\Big] + \lambda_*\Big[\frac{\lambda_L + \lambda_g}{\lambda_L\lambda_g}\frac{2a_{gLL}}{a_L} - 2\frac{a_{ggg}}{a_g}\frac{1}{\lambda_L} - \xi_g^*\frac{4a_{gLL}^2}{a_L^2}\frac{1}{\lambda_g}\Big] + \frac{2a_{gLL}}{a_L} + \frac{a_{ggg}}{a_g} = 0 \tag{2.22}$$

The maximum value of the load λ_* determined on the basis of equation (2.21) is smaller than the critical value $\lambda_L = \lambda_{min}$. Thus the load λ_* can be interpreted as such that accounts for the influence of the load corresponding to global buckling ($\xi_g^* \neq 0$, $\xi_g \neq 0$) on the local load value. The usage of the term "critical value of local buckling load with global pre-critical bending taken into account" is probably too long, but it renders the main idea of the problem. The author suggests here making use of the term "reduced critical value of local buckling load" instead.

3. Results of calculations

The analysis presented is an expansion of the analysis taking into account the global pre-critical bending presented by Roorda (1988).

A beam-column with an open channel section and a channel section with edge stiffened flanges characterised by the following dimensions:

section width	–	$a = 50\,\mathrm{mm}$
section height	–	$b = 25\,\mathrm{mm}$
edge stiffener width	–	$c = 12.5\,\mathrm{mm}$
wall thickness	–	$h_a = h_b = h_c = 1\,\mathrm{mm}$
length	–	$l = 650\,\mathrm{mm}$

has been analysed.

In order to characterise the way in which the load is applied, a coefficient of edge shortening $\kappa = u_1/u_2$, where u_1, u_2 (Fig. 4) are values of displacements of the lower and upper plate of the girder under consideration for $x = 0$, l, has been introduced.

Some sample results of numerical calculations obtained on the basis of the analysis of interactive buckling of thin-walled beam-columns with channel cross-sections with boundary reinforcement (Fig. 1) within the first order non-linear approximation are presented below.

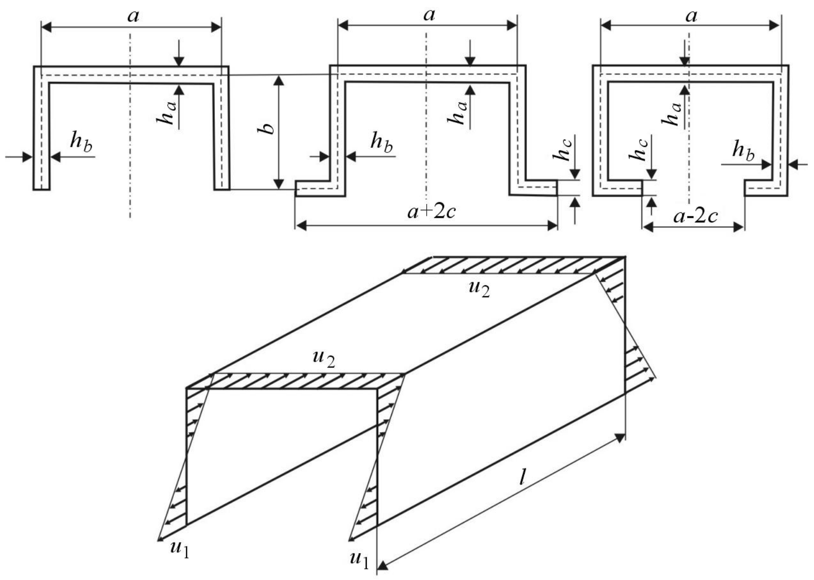

Fig. 4. Cross-sections of the considered beam-columns

As it is known, some initial global deflection ξ_g^*, which should not exceed 0.001 of the girder length, i.e. $\xi_g^* = 0.001/h$ according to European standards, is admissible for long girders.

Figures 5-7 present the influence of the parameter A on the limit load carrying capacity within the first order approximation λ_* (2.21) with respect to the critical value of the local symmetrical buckling mode λ_L of the beam-columns. The results are presented for beam- columns with channel sections and channel sections with reinforcement subjected to a load causing non-uniform shortening of the loaded edges $(\kappa = 0)$.

Two curves are shown on diagrams representing the ratio $\lambda_/\lambda_L$ vs. A, namely: a dashed curve for the admissible initial deflection equal to 0.65 mm $(\xi_g^* = 0.65)$ and a continuous one for the initial deflection higher than the admissible one and equal to 1 mm $(\xi_g^* = 1)$.

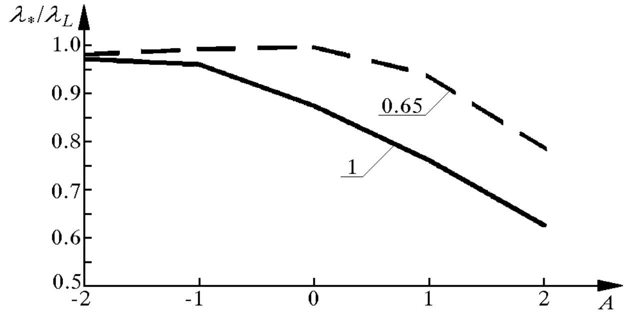

Fig. 5. Influence of the parameter A on the reduced critical value of local buckling of the beam-column with a channel section

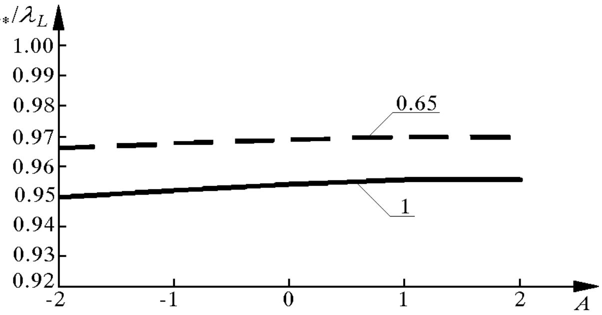

Fig. 6. Influence of the parameter A on the reduced critical value of local buckling of the beam-column with a channel section with inner reinforcement

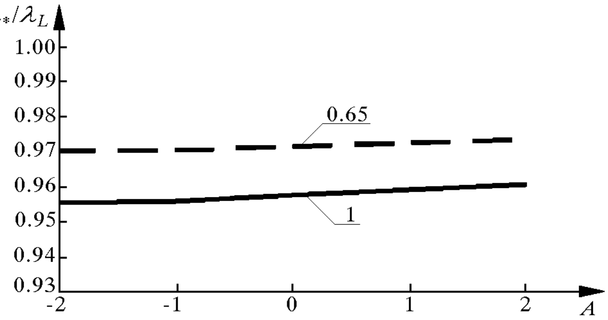

Fig. 7. Influence of the parameter A on the reduced critical value of local buckling of the beam-column with a channel section with outer reinforcement

It has been assumed that there are no local imperfections $\xi_L^* = 0$.

For a beam-column with a plain channel section at the assumed initial deflection $\xi_g^* = 0.65$ (Fig. 5), the reduced critical value of local buckling decreases with an increase in the parameter A, and it is lower than the local critical value by approx. 3% for $A = -2$ and by approx. 22% for $A = 2$, respectively. A sharp decrease in the value of λ_*/λ_L is caused by the fact that the coefficient a_{ggg} in (2.20), which increases abruptly with an increase in the parameter A (Fig. 8), is taken into consideration in the analysis of interactive buckling. It results from the fact that if there are no boundary reinforcements of the channel, the outer plates (flanges) have dominant impact on the stability, and the buckling mode corresponds to flexural-distorsional buckling.

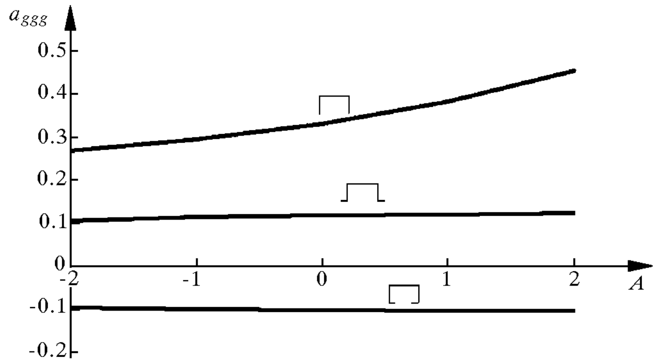

Fig. 8. Influence of the parameter A on the value of the coefficient a_{ggg} for thin-walled beam-columns with open sections characterised by the loading coefficient $\kappa = 0$

For the same initial deflection $\xi_g^* = 0.65$ in beams-columns with channel sections with inner (Fig. 6) and outer (Fig. 7) reinforcements, i.e. lipped channels with inwardly and outwardly turned lips, the reduced critical value of local buckling slightly increases with the parameter A in the range -2 to 2, and it is lower than the local critical value by approximately 3%.

4. Conclusions

The results of numerical calculations presented here show that variable material properties (the coefficient of orthotropy) exert an influence only in the case of a beam-column with a plain channel section. However, in the case of a beam-column with a lipped channel section, the variability in the coefficient of orthotropy along the wall width does not affect the reduced critical value of local buckling in the analysed structures by a great amount.

The presented method of modelling material properties allows one to select such a function describing the widthwise varying orthotropy that the local critical load of the structure, accounting for global imperfections, reaches its required value for a given structure.

It should also be noted that the presented method of investigating the interactive buckling of thin-walled structures is faster than the very popular recently finite element method. However, it has some disadvantage: in the presented method, the material properties and cross-section must be constant along the longitudinal axes of beam-columns; proffesional FEM software has better postprocessor.

References

1. BYSKOV E., HUTCHINSON J.W., 1977, Mode interaction in axially stiffened cylindrical shells, *AIAA*, **15**, 7, 941-948

2. CHANDRA R., RAJU B., 1973, Postbuckling analysis for rectangular orthotropic plates, *Int. J. Mech. Sci.*, **16**, 81-97

3. KOITER W.T., 1976, General theory of mode interaction in stiffened plate and shell structures; WTHD, Report 590, Delf, p.41

4. KOŁAKOWSKI Z., 1993, Interactive buckling of thin-walled beam-columns with open and close cross-sections, *Thin Walled Structures*, **15**, 159-183

5. KRÓLAK M. (ED.), 1995, *Stability, Postcritical Modes and Load Carrying Capacity of Flat Orthotropic Thin-Walled Structures*, Technical University of Łódź, Monographs (in Polish)

6. LUONGO A., PIGNATARO M., 1988, Multiple interaction and localization phenomena in the postbuckling of compressed thin-walled members, *AIAA Journal*, **26**, 11

7. MANIEVICZ A., KOŁAKOWSKI Z., 1997, Multiple interaction of buckling modes in thin-walled members and imperfection sensitivity, *Proc. 7th Symposium on Structure Stability*, Zakopane, 181-186

8. PIGNATARO M., LUONGO A., 1987, Asymmetric interactive buckling of thin-walled columns with initial imperfection, *Thin Walled Structures*, **3**, 365-386

9. ROORDA J., 1988, Buckling behaviour of thin-walled columns, *Can. J. Civ. Engng.*, **15**, 107-116

Wyboczenie interakcyjne cienkościennych belek-słupów o zmiennym współczynniku ortotropii wzdłuż szerokości ścian

Streszczenie

W pracy analizowano wyboczenie lokalne cienkościennych belek-słupów z uwzględnieniem globalnego dokrytycznego zginania w ramach przybliżenia pierwszego rzędu. Wyboczenie interakcyjne konstrukcji rozwiązano stosując aproksymacyjną teorię Byskova i Hutchinsona (1977) lub Koitera (1976). Badano belki-słupy zbudowane z płyt ortotropowych o głównych kierunkach ortotropii równoległych do krawędzi ścian charakteryzujących się zmiennym wzdłuż szerokości współczynniku ortotropii $\eta_i = E_{yi}/E_{xi}$. Analizowano belki-słupy o przekrojach otwartych (ceowym i ceowym ze wzmocnieniem), przegubowo podparte na obciążonych brzegach. Dźwigary poddano obciążeniom ppowodującym równomierne i liniowo zmienne zbliżenie brzegów.

Manuscript received May 18, 2005; accepted to print September 30, 2005