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TURBULENT HEAT TRANSFER IN THIN LIQUID FILMS AT LOW AND HIGH HEAT FLUXES

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In the paper presented are considerations of turbulent heat transfer in thin liquid films at low and high heat fluxes. Postulated have been simple models of heat transfer for laminar and turbulent liquid films formed by impinging jets and exposed to nucleate boiling, namely under high heat fluxes, as well as without nucleate boiling, at low heat fluxes, as a simplified case. Turbulence in such case is strongly modified and difficult to be modelled. Turbulence model due to Prandtl has been applied where, in the case of high heat fluxes, the mixing length is strongly modified. In the case of high heat fluxes, incorporated into the model is a blowing velocity, which models the transverse transport of momentum caused by departing bubbles. Calculated have been the velocity and temperature distributions in the liquid film, which enabled determination of the corresponding heat transfer coefficient and the Nusselt number.

Key words: blowing velocity, impinging jet, nucleate boiling

Notations

c_p	–	specific heat
δ	–	boundary layer thickness
h	–	film thickness
λ	–	thermal conductivity
ν	–	kinematic viscosity
ρ	–	density
q	–	heat flux density
q_{vap}	–	heat flux due to evaporation of film
Q	–	volumetric flow rate
T	–	temperature

τ	–	shear stress
ϑ, w	–	radial and vertical velocity components
r, z	–	radial and vertical co-ordinates
u_τ	–	friction velocity, $u_\tau = \sqrt{\tau_{w0}/\rho}$

Subscripts

tr	–	border between laminar and turbulent sublayers
i	–	interface
l	–	liquid
m	–	mean value
w	–	wall
0	–	initial value, nozzle outlet

Superscripts

$+$	–	non-dimensional

1. Introduction

Surface cooling by means of thin liquid films has many practical applications. The demand for effective cooling schemes increases the need for the development of appliances incorporating high-heat flux convective heat transfer mechanisms such as change of phase. Apart from the areas, where up to date the efficiency of cooling was highly desirable, just to mention metallurgy and nuclear power, there is rapidly developing a new branch of applications in microelectronics, where seemingly steady trend of achieving ever larger scales of circuit integration is straining the capabilities of existing high-heat flux technologies. In such case there are numerous limiting restrictions such as available space, choice of coolant, local environmental conditions and maximum allowable surface temperatures. In relation to the range of plate temperatures we may face different modes of convective heat transfer, namely forced convection, nucleate boiling, transition boiling or film boiling. Liquid film evaporation can significantly increase the rate of heat removal from the solid surface and renders this kind of heat transfer very efficient. Heat transfer rates are even greater in the case of nucleate boiling. In the paper considered has been a problem of nucleation in the film formed by impinging jet on a hot plate, as well as a simplified case when nucleate boiling is not present. In case of high heat fluxes, temperature of the plate exceeds the saturation temperature, which causes nucleation of bubbles. Several works can be found, where similar topic is investigated, for example Buyevich and Mankevich (1996), Deb and Yao (1989), Liu and Wang (2001), Mikielewicz and Mikielewicz (2005), Webb and Ma (1996), Wolf *et al.* (1994). Despite numerous studies regarding that topic, there is still a lack of complete understanding of the phenomenon as well as theoretical models, which would enable calculation of hydrodynamics and heat transfer in liquid films produced by impinging liquid jets in the nucleate boiling regime.

In the paper the issues of turbulence in modelling of thin films featuring nucleate boiling have been addressed in a greater detail. That issue is by no means simple, since the presence of generated bubbles significantly modifies velocity field and hence the turbulence. The theory known from single phase flows has been applied to the cases where strong modifications of velocity field are found. Modelling of turbulence in such cases is a very challenging task. Standard models of turbulence are usually perceived as not capable, without serious modifications, of revealing the phenomena occuring in the flow. In the paper turbulence is modelled using the Prandtl mixing length model, which resolves the problem quite well. The model of the thin liquid film formed as a result of jet impingement, scrutinised in the paper, is a simple model of boiling heat transfer within such liquid films just outside the stagnation region. The case with nucleate boiling is regarded as a case where large rates of heat are present. In such a model considered is a change of film thickness due to inertia forces and friction. The transverse component of bubble velocity, in the study, is regarded as constant and has been modelled by means of the theory of "blowing". The case taken into consideration consists of two layers, i.e. laminar sublayer and turbulent core in the bulk of the flow. The model is based on the consecutive solution of the conservation equation of mass, momentum and energy. The obtained solutions are approximate, but in the analytical form, which enables further qualitative examination of solutions. The developed model forms an extension of the earlier models by the authors (Mikielewicz and Mikielewicz, 1999, 2001, 2002), where the laminar and turbulent films have been considered and the solutions were obtained by means of the "thin layer theory". These cases will also be presented here, derived on the basis of simplification of the model featuring the nucleate boiling. In the present work, according to procedures developed for respective models, calculated have been shear stresses, velocity and temperature distributions in the liquid film, which enabled also determination of the heat transfer coefficient and Nusselt number, corresponding to the relevant cases.

2. Governing equations

Figure 1 shows a schematic diagram of the case under scrutiny applicable to both cases of high and low heat fluxes respectively. The following non-dimensional quantities have been introduced into the analysis

$$\begin{aligned} &\vartheta^+ = \frac{\vartheta}{u_\tau} && z^+ = \frac{zu_\tau}{\nu} && r^+ = \frac{ru_\tau}{\nu} && \delta^+ = \frac{\delta u_\tau}{\nu} \\ &T^+ = \frac{T}{T_0} && \tau^+ = \frac{\tau}{\tau_w} && \tau_0^+ = \frac{\tau_w}{\tau_{w0}} && q^+ = \frac{q}{q_w} \end{aligned} \tag{2.1}$$

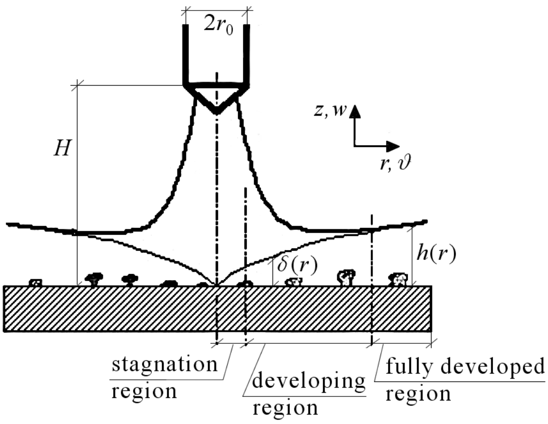

Fig. 1. Schematic diagram of a single phase jet

Equations describing the behaviour of non-compressible two-dimensional turbulent liquid films written in the boundary layer approximation in cylindrical co-ordinates yield (Deb and Yao, 1989; Mikielewicz and Mikielewicz, 2005):
— continuity equation

$$\frac{\partial(r^+\vartheta^+)}{\partial r^+} = 0 \tag{2.2}$$

— momentum equation

$$\vartheta^+\frac{\partial\vartheta^+}{\partial r^+} + \frac{w_0}{u_\tau}\frac{\partial\vartheta^+}{\partial z^+} = \tau_0^+\frac{\partial\tau^+}{\partial z^+} \tag{2.3}$$

— energy equation

$$\frac{\partial(\vartheta^+T^+)}{\partial r^+} + \frac{w_0}{u_\tau}\frac{\partial T^+}{\partial z^+} = \frac{q_w}{\rho c_p T_0 u_\tau}\frac{\partial q^+}{\partial z^+} \tag{2.4}$$

In the above equations the transverse velocity component, w, is assumed constant, namely $w \approx w_0 = \text{const}$, which corresponds to implementation of the so-called "blowing velocity". In the discussed case it models the contribution from nucleate boiling. Value of that velocity can be estimated from the rate of evaporation, i.e. $w_0 = q_{vap}/h_{fg}\rho_v$. The total heat flux consists of two components, namely the heat flux due to convection in the liquid and the heat flux due to evaporation

$$q = q_l + q_{vap} \tag{2.5}$$

The heat flux due to evaporation in considered case is very small compared to the heat flux through liquid, however bubbles significantly distort the velocity profile modifying in such a way the turbulence. In the expression (2.5) the radiative heat flux contribution could also be considered, but in the present study that contribution has been omitted due to the fact that in cases to be studied, there are no significant surface temperatures involved.

The momentum and energy equations enable determination of general forms of the shear stress and heat flux distributions in the liquid film

$$\begin{aligned}
\tau^+ &= 1 + \frac{1}{\tau_0^+}\Big(\int\limits_0^{z^+} \vartheta^+ \frac{\partial \vartheta^+}{\partial r^+}\, dz^+ + \frac{w_0 \vartheta^+}{u_\tau}\Big) \\
q^+ &= 1 + \frac{\rho c_p T_0 u_\tau}{q_w} \int\limits_0^{z^+} \frac{\partial(\vartheta^+ T^+)}{\partial r^+}\, dz^+ + \frac{w_0}{u_\tau}(1 - T_w^+)
\end{aligned} \tag{2.6}$$

These distributions are valid both for the laminar and the turbulent flow conditions. The boundary conditions for the considered case yield: $z = 0 \Rightarrow \tau^+ = \tau_0^+$ and $q^+ = 1$. The challenge now is in appropriate resolution of the shear stress and heat flux in respective sublayers.

In the model presented below, thermal-hydraulic analysis of the phenomena in high heat fluxes case followed by the low heat flux case just outside the stagnation region is presented.

2.1. Hydrodynamics of thin liquid films at high heat fluxes

2.1.1. Developing film

The boundary layer in the developing region grows as presented in Fig. 1. The analysis below pertains to the case where the boundary layer, δ^+, is smaller than the liquid film thickness, h^+. Such situation is present within the distance of approximately eight nozzle radii from the stagnation point. In such a case the problem needs to be considered in two zones, in the region of boundary layer and beyond, in the region of undisturbed liquid between the boundary layer border and the film border. As the first approximation the velocity profile for a turbulent boundary layer flow on a flat plate has been assumed

$$\vartheta^+ = \frac{\vartheta_0}{u_\tau}\Big(\frac{z^+}{\delta^+}\Big)^{\frac{1}{m}} \tag{2.7}$$

Such profile satisfies the conditions of zero velocity at the wall and a value of undisturbed velocity at the border of the boundary layer, ϑ_0. Beyond the boundary layer the velocity does not vary and is equal to ϑ_0. The flow in the boundary layer will be analysed in two regions, namely in the laminar and turbulent sublayers. In the case of high heat fluxes, the border of the laminar boundary layer is assumed to fall at $z^+ = 11.6$. That fact should be properly recognised as bubbles strongly modify the velocity field and still the usual assumptions for the division of boundary layer are acceptable. Substitution of (2.7) into the momentum equation gives a relation describing the shear stress distribution in the laminar sublayer

$$\tau^+ = 1 + \frac{1}{\tau_0^+}\Big[a(r^+)(z^+)^{\frac{2}{m}+1} + \frac{w_0\vartheta_0}{u_\tau^2}\Big(\frac{z^+}{\delta^+}\Big)^{\frac{1}{m}}\Big] \tag{2.8}$$

where

$$a(r^+) = -\frac{\vartheta_0^2\delta^{-\frac{m+2}{m}}}{(m+2)u_\tau^2}\frac{d\delta^+}{dr^+}$$

Solving (2.8) in the laminar sublayer, where $\tau^+ = d\vartheta^+/dz^+$, we obtain velocity distribution in that region, i.e. $z^+ \leqslant 11.6$

$$\vartheta^+ = z^+ + \frac{ma(r^+)}{2(m+1)}(z^+)^{\frac{2(m+1)}{m}} + \frac{w_0\vartheta_0}{u_\tau^2}\frac{m(\delta^+)^{\frac{1}{m}}}{m+1}(z^+)^{\frac{m+1}{m}} \tag{2.9}$$

In the turbulent sublayer $(11.6 < z^+ \leqslant \delta^+)$, the Prandtl mixing length model has been applied

$$\tau^+ = \kappa^2(z^+)^2\Big(\frac{d\vartheta^+}{dz^+}\Big)^2 \tag{2.10}$$

The resulting differential equation has the following form

$$\frac{d\vartheta^+}{dz^+} = \frac{1}{\kappa z^+}\sqrt{1 + \frac{1}{\tau_0^+}\Big[a(r^+)(z^+)^{\frac{2}{m}+1} + \frac{w_0\vartheta_0}{u_\tau^2}\Big(\frac{z^+}{\delta^+}\Big)^{\frac{1}{m}}\Big]} \tag{2.11}$$

Knowledge of laminar and turbulent velocity profile distributions enables determination of the boundary layer thickness, which can be determined from the condition that at the border of boundary layer, the following condition must hold

$$\frac{d\vartheta^+}{dz^+}\Big|_{z^+=\delta^+} = 0 \tag{2.12}$$

This condition enables determination of the change of the liquid film thickness as a function of the nozzle outlet velocity ϑ_0

$$\frac{d\delta^+}{dr^+} = (2+m)\frac{u_\tau^2}{\vartheta_0^2}\Big(\tau_0^+ + \frac{w_0\vartheta_0}{u_\tau^2}\Big) \tag{2.13}$$

By integration of (2.13), using the boundary condition that for $r^+ = 0 \Rightarrow \delta^+ \approx 0 \Rightarrow C_2 = 0$ we obtain the ratio of thickness of the boundary layer with respect to the radius. Next, from the mass balance equation, we perform the liquid film flow rate balance

$$Q^+ = \frac{Qu_\tau}{2\pi\nu^2} = r^+\Big[\int\limits_0^{\delta^+} \vartheta^+\, dz^+ + \frac{\vartheta_0}{u_\tau}(h^+ - \delta^+)\Big] \tag{2.14}$$

Relation (2.14) enables determination of the liquid film thickness at any radius. The velocity profile integral is to be determined from integration of the respective velocity profiles in laminar and turbulent sublayers

$$\int\limits_0^{\delta^+} \vartheta^+\, dz = \int\limits_0^{11.6} \vartheta_l^+\, dz^+ + \int\limits_{11.6}^{\delta^+} \vartheta_t^+\, dz^+ \tag{2.15}$$

Expression (2.14) permits to derive the limiting value of radius, r_{gr}^+, where the boundary layer will reach the film thickness, i.e. $h^+ = \delta^+$

$$r_{gr}^+ = \frac{Q^+}{\int\limits_0^{\delta^+} \vartheta^+\, dz^+} \tag{2.16}$$

2.1.2. Fully developed film

The region we are about to consider now extends from the point where the boundary layer has reached the film thickness. The following analysis is similar to the one presented above and therefore only the most important points will be highlighted. In the considered case the thickness of the film consists of two sublayers, namely the laminar and the turbulent ones and the analysis will be carried out in both sublayers respectively. Similarly as in the latter case, the velocity profile corresponding to turbulent flow in the boundary layer flow on a flat plate has been assumed but in the present case, instead of the boundary layer thickness, the film thickness is applied in its distribution

$$\vartheta^+ = \frac{\vartheta_0}{u_\tau}\Big(\frac{z^+}{h^+}\Big)^{\frac{1}{m}} \tag{2.17}$$

In the case of high heat fluxes the border of the laminar boundary layer is assumed to fall at $z^+ = 11.6$. Substitution of (2.17) into the momentum equation gives a relation describing the shear stress distribution across the film thickness

$$\tau^+ = 1 + \frac{1}{\tau_0^+}\Big[a(r^+)(z^+)^{\frac{2}{m}+1} + \frac{w_0\vartheta_0}{u_\tau^2}\Big(\frac{z^+}{h^+}\Big)^{\frac{1}{m}}\Big] \tag{2.18}$$

where

$$a(r^+) = -\frac{\vartheta_0^2 (h^+)^{-\frac{m+2}{m}}}{(m+2)u_\tau^2} \frac{dh^+}{dr^+}$$

Solving (2.18) in the laminar sublayer, where $\tau^+ = d\vartheta^+/dz^+$, we obtain the distribution of velocity in that region, i.e. $z^+ \leqslant 11.6$, where nucleation of bubbles is again modelled by means of the blowing velocity

$$\vartheta^+ = z^+ + \frac{ma(r^+)}{2(m+1)} (z^+)^{\frac{2(m+1)}{m}} + \frac{w_0 \vartheta_0}{u_\tau^2} \frac{m(h^+)^{\frac{1}{m}}}{m+1} (z^+)^{\frac{m+1}{m}} \tag{2.19}$$

In the turbulent sublayer $(11.6 < z^+ \leqslant \delta^+)$, the Prandtl mixing length model has been applied. The resulting differential equation has the following form

$$\frac{d\vartheta^+}{dz^+} = \frac{1}{\kappa z^+} \sqrt{1 + \frac{1}{\tau_0^+} \Big[a(r^+)(z^+)^{\frac{2}{m}+1} + \frac{w_0 \vartheta_0}{u_\tau^2} \Big(\frac{z^+}{h^+} \Big)^{\frac{1}{m}} \Big]} \tag{2.20}$$

Next, using the mass balance equation, we obtain the liquid film flow rate balance

$$Q^+ = \frac{Q u_\tau}{2\pi\nu^2} = r^+ \Big(\int\limits_0^{h^+} \vartheta^+ \, dz^+ \Big) \tag{2.21}$$

The solution of relation (2.21) enables determination of the liquid film thickness at any radius in the fully developed case of spreading film.

2.2. Hydrodynamics of thin liquid films at low heat fluxes

In the case to be considered below, the term of low heat fluxes corresponds to the case where no bubble nucleation is present. This means that the blowing velocity is equal to zero, $w_0 = 0$. Equations (2.2)-(2.4) describing the problem in Section 2.1 are valid also in the present case, but they assume a slightly simpler form. Let's first consider the flow in the developing range of the film.

2.2.1. Analysis of the developing film

The boundary layer in the developing region, in the case of low heat fluxes, develops also as presented in Fig. 1. The analysis below pertains to the case when the boundary layer δ is smaller than the liquid film thickness. In such a case the problem needs to be considered in two zones, namely in the region within the boundary layer and beyond, in the region of undisturbed liquid between the boundary layer border and the film border. The region within the boundary layer consists of laminar and turbulent sublayers. The velocity profile typical of a turbulent boundary layer flow on a flat plate has been assumed as the first approximation. In such case, a form presented by Equation (2.7) has been used. Such profile obeys the condition of zero velocity at the wall and a value of undisturbed velocity at the border of the boundary layer, ϑ_0. Beyond the boundary layer the velocity does not vary and is equal to ϑ_0. The flow in the boundary layer can be analysed in two regions: in the laminar and turbulent sublayers. The border of the laminar boundary layer is assumed conventionally to be $z^+ = 11.6$. Note that here we are not dealing with the presence of bubbles. Substitution of (2.7) into the momentum equation gives the expression for the distribution of shear stress

$$\tau^+ = 1 + a(\delta^+) z^{\frac{2}{m}+1} \tag{2.22}$$

where

$$a(\delta^+) = -\frac{\vartheta_0^2 (\delta^+)^{-\frac{m+2}{m}}}{2(m+2)u_\tau^2} \frac{d\delta^+}{dr^+}$$

Solving (2.22) in the laminar sublayer, where $\tau^+ = d\vartheta^+/dz^+$, we obtain the velocity distribution in that region, i.e. $z^+ < 11.6$

$$\vartheta_l^+ = z^+ - \frac{ma(\delta^+)}{2(m+1)} (z^+)^{\frac{2(m+1)}{m}} \tag{2.23}$$

In the turbulent sublayer, the Prandtl mixing length model can again be applied. Neglecting the blowing velocity in the equation of motion allows us also to obtain the simple analytical form of velocity distribution in that region. The resulting differential equation has been solved using the following approximation

$$\sqrt{1 + a(r^+)(z^+)^{\frac{m+2}{m}}} \approx 1 + \frac{1}{2} a(r^+)(z^+)^{\frac{m+2}{m}} \tag{2.24}$$

Application of (2.24) enables to obtain an analytical solution to velocity distribution in the turbulent flow regime in the form

$$\vartheta_t^+ = \frac{1}{\kappa}\Big[\ln\frac{z^+}{11.6} - \frac{1}{2}\frac{m}{m+2} a(\delta^+)\Big((z^+)^{\frac{m+2}{m}} - 11.6^{\frac{m+2}{m}}\Big)\Big] \tag{2.25}$$

Knowledge of laminar and turbulent velocity profiles enables determination of the boundary layer thickness, which can be determined from the condition (2.14). This condition expresses the change of the liquid film thickness δ^+ as a function of the nozzle outlet velocity ϑ_0. The result is similar to relation (2.16), if "blowing velocity" w_0 would be assumed to be zero. Integration of (2.14) using the boundary condition that for $r^+ = 0 \Rightarrow \delta^+ \approx 0 \Rightarrow C_2 = 0$, yields the ratio of thickness of the boundary layer with respect to the radius. Similarly as in the case of high heat flux, we can find the location where the boundary layer meets the film thickness, i.e. $h^+ = \delta^+$, namely relation (2.16).

2.2.2. Fully developed film flow

Similar task has been considered in Mikielewicz and Mikielewicz (2002, 2004). The solution is sought for in two sublayers where turbulent velocity profile (2.17) has been assumed. The final distributions of velocity in laminar and turbulent sublayer give forms similar to Equations (2.19) and (2.20) where the blowing velocity has been assumed to equal 0.

3. Heat transfer during the film development

Solution of the heat transfer problem is based on the analysis of energy equation (2.4). As a partial differential equation, it is difficult to be solved in other than a numerical manner. However, using the assumptions presented below it is possible to develop analytical relations describing the temperature field in the liquid film.

In the case of the approximate analysis of developing film, we should consider two cases:

1. The motion is laminar in the entire film, i.e. $\delta^+ < z^+_{tr}$.
2. The mixed motion where the boundary layer thickness $\delta^+ > z^+_{tr}$. In such a case in the region $0 < z^+ < z^+_{tr}$ there exists the laminar flow and in the region $11.6 < z^+ < z^+_{tr}$ – the turbulent flow.

In the case when the boundary layer thickness is greater than the laminar sublayer thickness, we can assume that the undisturbed temperature T_0 extends from the film border to the border of the boundary layer, i.e. $T_\delta = T_0$. For that reason, the total temperature drop in the liquid film in the boundary layer consists of two drops, in the laminar and turbulent sublayers. In the case when the boundary layer thickness is smaller than the laminar sublayer thickness, we are dealing only with one laminar temperature distribution in the boundary layer. In the present paper, a boundary layer is considered where temperature drop exists both in the laminar and turbulent sublayers.

First term of the energy equation (2.4) will be modelled in the present work as a mean product of instantaneous velocity and temperature, namely

$$\frac{\partial(\vartheta^+ T^+)}{\partial r^+} \cong \frac{\partial(\vartheta^+ T^+)_m}{\partial r^+} \tag{3.1}$$

Next, let us define the mean film temperature in the form

$$T_m = \frac{\int\limits_0^h \vartheta T\, dz}{\int\limits_0^h \vartheta\, dz} = \frac{2\pi r}{Q}\int\limits_0^h \vartheta T\, dz \tag{3.2}$$

The denominator in (3.2) can be determined from the continuity equation in the jet. The integral appearing in the denominator is the sought term in the first term of energy equation. Then, following the conversion of Equation (3.2) into the non-dimensional variables, we arrive at the relation for the mean temperature in the film

$$T_m^+ = \frac{r^+}{Q^+} \int\limits_0^{h^+} (\vartheta^+ T^+)\; dz^+ \tag{3.3}$$

In order to apply the assumption (3.1) into the energy equation (2.4) we need the define the mean product of velocity and temperature $(\vartheta^+ T^+)_m$. That can be obtained in the following form

$$(\vartheta^+ T^+)_m = \frac{1}{h^+} \int\limits_0^{h^+} (\vartheta^+ T^+)\; dz^+ \tag{3.4}$$

Using the definition of mean temperature (3.3) we can determine its radial distribution from single integration of the energy equation (2.4) in the limit from 0 to the film thickness h^+

$$Q^+ \frac{\partial}{\partial r}\Big(\frac{T_m^+}{r^+}\Big) = \frac{q_w}{\rho c_p T_0 u_\tau}(q_i^+ - q_w^+) + \frac{w_0}{u_\tau}(T_w^+ - 1) \tag{3.5}$$

where q_w^+ is the density of heat flux at the wall and q_i^+ is the heat flux density at the liquid-gas interface. Integrating (3.5) at the boundary condition $r = 0$, $T_m = T_0$, we obtain the mean temperature distribution in the film dependent on the radial co-ordinate

$$T_m^+ = \frac{1}{Q^+}\Big[\frac{q_w}{\rho c_p T_0 u_\tau}(q_i^+ - q_w^+) + \frac{w}{u_\tau}(T_w^+ - 1)\Big](r^+)(r^+ - r_0^+) + \frac{r^+}{r_0^+} \tag{3.6}$$

In order to use the assumption (3.1) we explore the relation (3.4). Then the mean product of velocity and temperature yields

$$\frac{\partial}{\partial r^+}(\vartheta^+ T^+)_m = Q^+ \frac{\partial}{\partial r^+}\Big(\frac{T_m^+}{r^+ h^+}\Big) \tag{3.7}$$

The energy equation (2.4) is now transformed to the form:

$$f(r^+) + \frac{w_0}{u_\tau}\frac{\partial T^+}{\partial z^+} = \frac{1}{\rho c_p \nu}\frac{\partial}{\partial z^+}\Big(\lambda_{\mathit{eff}} \frac{\partial T^+}{\partial z^+}\Big) \tag{3.8}$$

where

$$f(r^+) = Q^+ \frac{\partial}{\partial r^+}\Big(\frac{T_m^+}{r^+ h^+}\Big)$$

The first term in (3.8) can only vary in the radial direction, hence is not dependent the z direction, thanks to the assumption (3.1). In (3.8) λ_{eff} is the effective thermal conductivity, which consist of molecular and turbulent components. Some attention will now be devoted to the way in which effective thermal conductivity has been determined. In the turbulent flow molecular effects can be neglected and the effective thermal conductivity yields

$$\lambda_{\mathit{eff}} = \frac{c_p \mu_t}{\sigma_t} \tag{3.9}$$

In (3.7) σ_t represents the turbulent Prandtl number and in the present study we have assumed a constant value of $\sigma_t = 0.85$ (Mikielewicz and Ihnatowicz, 1996). Considerations will, in the first instance, be conducted for the case in which the turbulent dynamic viscosity μ_t will be determined from one of the simplest models, namely the Prandtl mixing length model. In the case of single phase flow models, that model plays a very useful role as a tool enabling to obtain analytical solutions in the case of velocity field for simple flow cases, such as flows in boundary layers or in tubes. Additionally, the model is very simple in application and effective enough in numerous cases. In the case considered here, the velocity profile does not attain maximum at any other location than the core of the flow. If that would be the case then the model would not be applicable in turbulence modelling. In authors' opinion, only in cases when simple models fail to reveal appropriate behaviour of the phenomenon, the more complex models should be considered or more complex models are capable of leading to more accurate calculations of turbulence. The turbulent dynamic viscosity, in the case of Prandtl mixing length model, has the form

$$\frac{\mu_t}{\mu} = (l_m^+)^2 \frac{d\vartheta^+}{dz^+} = \kappa^2 (z^+)^2 \frac{d\vartheta^+}{dz^+} \tag{3.10}$$

where l_m represents the mixing length. In calculations carried out in the present work it turned out that a classical definition of the mixing length is appropriate. That enabled us to obtain realistic velocity and temperature profiles.

In solving the Equation (3.8) it should be borne in mind that T_m^+ and h^+ are known from solutions describing distributions of film thickness, either in a developing flow (2.14), or in the fully developed case, relation (2.20) and mean temperature (3.6). A good assumption is also constant value of the film thickness. In order to obtain analytical form of temperature distribution in the film, we will evaluate the mean value of turbulent conductivity. Such procedure is fully justified since the film is rather thin and no significant variations of turbulent conductivity are expected

$$\overline{\lambda}_{\mathit{eff}} = \frac{1}{\delta}\int\limits_0^\delta \lambda_{\mathit{eff}}\; dz = \frac{\rho c_p \kappa^2 \vartheta_0 \nu}{(2m+1)\sigma_t u_\tau}\delta^+ \tag{3.11}$$

Determination of the mean value of turbulent thermal conductivity allows to linearise Equation (3.8). The solution procedure of (3.8) will first require to obtain the solution of a homogeneous equation and subseqently, the solution of the non-homogeneous equation. The general solution to the homogeneous Equation (3.8) yields

$$T^+ = \Big(\frac{w_0}{u_\tau}\frac{\nu}{a_t}\Big)^{-1} C\exp\Big(\frac{w_0}{u_\tau}\frac{\nu}{a_t}z^+\Big) \tag{3.12}$$

where $a_t = \lambda_{\mathit{eff}}/(\rho c_p)$. Subsequently, we seek a unique solution to Equation (3.8) in the linear form. The final solution to temperature distribution yields

$$T^+ = \Big(\frac{w_0}{u_\tau}\frac{\nu}{a_t}\Big)^{-1} C\exp\Big(\frac{w_0}{u_\tau}\frac{\nu}{a_t}z^+\Big) + D - \Big(\frac{w_0}{u_\tau}\Big)^{-1} f(r^+)z^+ \tag{3.13}$$

The constants C and D can be established from the boundary conditions

$$\begin{aligned} &\text{for}\quad z^+ = 0 \qquad T^+ = T_w^+\\ &\text{for}\quad z^+ = h^+ \quad \frac{\partial T^+}{\partial z^+} = -\frac{q_i\nu}{\lambda_{\mathit{eff}} T_0 u_\tau} \end{aligned} \tag{3.14}$$

Then the constant C has the form

$$C = \Big(\frac{w_0}{u_\tau}\Big)^{-1} f(r^+) - \frac{q_i\nu}{\lambda_{\mathit{eff}} u_\tau T_0}\exp\Big(-\frac{w_0}{u_\tau}\frac{\nu}{a_t}h^+\Big) \tag{3.15}$$

The constant D yields

$$D = T_w^+ - C\Big(\frac{w_0}{u_\tau}\frac{\nu}{a_t}\Big)^{-1} \tag{3.16}$$

The convective heat transfer coefficient α, as well as the Nusselt number, are finally defined in the following way

$$\alpha = \frac{q_w}{T_m - T_w} = \frac{q_w}{T_0(T_m^+ - T_w^+)} \qquad\qquad Nu = \frac{\alpha r}{\lambda} \tag{3.17}$$

4. Results

Calculations have been made using the Mathcad11 software package. The following input has been assumed: $Q^+ = 1.169 \cdot 10^7$, $u_\tau = 0.037\,\mathrm{m/s}$, $q_i/q_w = 0.8$, $T_0 = 20°\mathrm{C}$, properties of the film correspond to water properties at $20°\mathrm{C}$, the nozzle diameter 1 mm. Thickness of the boundary layer in the considered example corresponds to about $\delta^+ \approx 15$ whereas when nucleation is present, then $\delta^+ \approx 5$. Calculated have been the distributions of the turbulent viscosity, velocity and temperature profiles in both the fully developed and developing regions, together with the calculations of heat transfer coefficient in both regions for low heat fluxes, without bubble generation as well as for high heat fluxes where nucleation is present. The results have been presented in Fig. 2 to Fig. 5. It seems that proper qualitative trends are depicted by the postulated model.

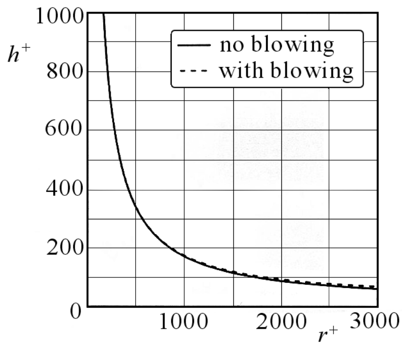

Fig. 2. Non-dimensional liquid film thickness distribution in the developing region

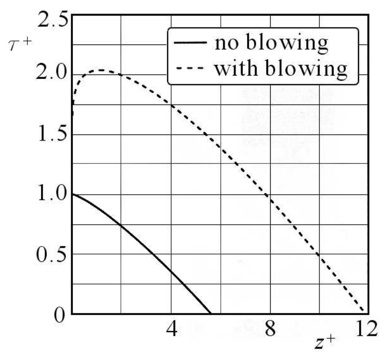

Fig. 3. Distribution of shear stress in the developing region in case of presence of blowing and without it

Examination of the obtained results shows that the film thickness has not been changed very much by the presence of nucleation, Fig. 2. More pronounced changes are seen in Fig. 3, where, as expected, presence of bubbles strongly modifies the shear stress distribution. It should also be noted that the boundary layer thicknesses are different in cases of presence of nucleation and without it. The heat transfer coefficient (Fig. 4) assumes relatively high values, which are however typical for such cases, where these exceed the value of 10000. Although the predicted heat transfer coefficients are very high, we can observe some 20% enhancement of the heat transfer coefficient for the case when nucleation is present. That happens despite a very small thickness of the liquid film, which in the present case was about 0.4 mm. Finally, temperature distributions are shown in Fig. 5.

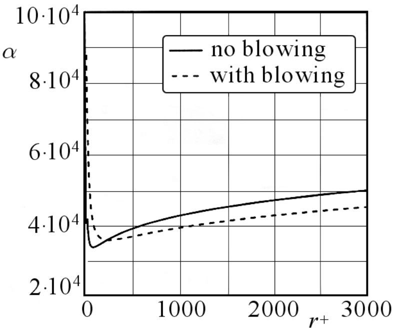

Fig. 4. Distribution of heat transfer coefficient in the developing region in case of presence of nucleation and without it

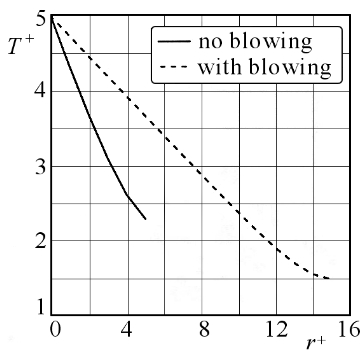

Fig. 5. Distribution of temperature in the developing region in case of presence of nucleation and without it

5. Conclusions

Formulated has been a simple model of single-phase jet impinging on a surface and forming a thin film on that surface. The model consists of conservation equations of mass, momentum and energy. For such case the heat transfer coefficients have been determined. Approximate analytical solutions have been obtained in the developing and fully developed regions, allowing for a wide discussion of the obtained results.

The model incorporates the so-called "blowing velocity" to model the momentum transfer across the film. The heat flux consists of two components, namely the convective heat flux through the liquid and the evaporative heat flux. The latter component is approximately equal only to 1% for considered case of the former one and such a share in the heat balance is usually negligible. However, it changes the velocity profile to a significant extent. The boundary layer, due to presence of the vapour bubbles, is very distorted, what means also that the turbulence is also very vigorous as compared with single phase films. In the present work the mixing length theory was applied to enable calculation of the appropriate velocity. For the velocity profile determined in such a way, the heat transfer coefficients have been determined allowing for a wide discussion of the obtained results. Values of the latter agree qualitatively with the heat transfer coefficients found in experiments for other fluids, what confirms the presence of intense heat transfer under nucleate boiling regime.

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Turbulentna wymiana ciepła w cienkich filmach cieczowych przy małych i dużych strumieniach ciepła

Streszczenie

W pracy przedstawiono rozwiązanie turbulentnej wymiany ciepła w cienkich filmach cieczowych przy małych i dużych strumieniach ciepła. Zaproponowano proste modele wymiany ciepła dla przypadków laminarnego i turbulentnego filmu cieczowego wytworzonego uderzającą strugą w warunkach dużych i małych strumieni cieplnych. W takich przypadkach turbulencja jest silnie modyfikowana i z tego względu trudna do modelowania. W pracy zastosowano model drogi mieszania Prandtla i w przypadku dużych strumieni cieplnych droga mieszania jest szczególnie mocno zmodyfikowana. W przypadku dużych strumieni ciepła wprowadzono do modelu tzw. prędkość wzdmuchu, która modeluje wymianę pędu w kierunku poprzecznym do przepływu wywołaną odrywającymi się od ścianki pęcherzykami. Wyznaczono rozkłady prędkości i temperatury w filmie cieczowym, które umożliwiły wyliczenie współczynnika przejmowania ciepła i liczby Nusselta.

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