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EXPERIMENTAL VERIFICATION OF FATIGUE LOADING NONPROPORTIONALITY MODEL

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The paper deals with the experimental verification of a model of the fatigue loading nonproportionality. The assumption of the proposed model is that the loading nonproportionality degree depends on: (a) modules of vectors of rotating stresses, (b) angular distances of these stress vectors measured in relation to the critical plane. The performed experiment involved modeling of the nonproportional fatigue loading through programmed changes of principal axes positions. Different positions of the principal axes were obtained by using two loading blocks: fully reversed torsion and complex loading, that is to say, tension-compression and torsion. During the tests, the influence of the range of the orientation angle of principle axes on fatigue life was examined. The obtained results allowed one to confirm the thesis that the nonproportionality degree depends also on the angular distance in which the stress vector acts in relation to the critical plane.

Key words: multiaxial fatigue criteria, nonproportional loading, loading nonproportionality measure

1. Introduction

Nowadays, one of the most important fatigue issues consist in the assessment of fatigue behavior and damage phenomena occurring in conditions of the so called nonproportional loading. We can talk about such loading if during a fatigue cycle the principal axes change their location. Compared to the proportional loading, nonproportionality of loading can cause intensification of the fatigue damage cumulation process. In effect, fatigue life and fatigue strength are significantly impaired. Because of common character of this phenomenon and its heavy influence on fatigue behavior, this kind of loading should be taken into consideration in calculation models.

In the authors previous works (Skibicki and Sempruch, 2001, 2002, 2004; Skibicki, 2004a,b), a fatigue strength criterion was proposed and presented, whereas fatigue life criterion can be found in Skibicki (2004b, 2006). Both criteria respect the phenomenon of fatigue loading nonproportionality and are used for biaxial sinusoidal nonproportioanl loadings in situations of the nonzero mean values cycle.

These criteria are based on the critical plane approach, since it is assumed that also in conditions of nonproportional loading the fatigue damage cumulation process results mainly from shear and normal stresses acting on a given plane. Hence, in the formula for the equivalent amplitude of nonproportional stresses

$$\tau_{eq(a)}^{np} = \tau_{eq(a)}\Big(1 + \frac{t_{-1}}{b_{-1}}H^3\Big) \leqslant t_{-1} \tag{1.1}$$

there occurs equivalent stress amplitude $\tau_{eq(a)}$ connected with the critical plane

$$\tau_{eq(a)} = (\tau_a + c_1\sigma_a + c_2\sigma_m) \tag{1.2}$$

where t_{-1} – fatigue limit in torsion, b_{-1} – fatigue limit in bending, and

$$c_1 = 1.9\frac{t_{-1}}{b_{-1}} - 1 \qquad\qquad c_2 = 0.5\frac{b_{-1}}{R_m} \tag{1.3}$$

R_m – tensile strength, τ_a – shear stress amplitude on the critical plane, σ_a, σ_m – amplitude and mean value of normal stress on the critical plane consistent with the shear stress amplitude τ_a action moment, t_{-1}/b_{-1} measure of the material sensitivity with respect to loading nonproportionality (in the form of a quotient of fatigue limit in torsion to fatigue limit in bending), H – loading nonproportionality measure. In the accepted solutions, the critical plane is determined by the action course of the maximum shear stress in the cycle.

In nonproportional loading conditions, the prediction of fatigue life and strength on the basis of equivalent stress based only on the critical plane approach could be burdened with an error. The mathematical model has to account also for nonproportionality influence. The description of the loading nonproportionality effect in equation (1.1), is based on a nonproportionality term containing two functions: measure of the material sensitivity to loading nonproportionality t_{-1}/b_{-1}, and a function defining the loading nonproportionality degree called the loading nonproportionality measure H.

An commonly accepted assumption is that the bigger is the part of stresses acting beyond the critical plane in the damage cumulation process the higher is the nonproportionality degree.

Two further assumptions characterizing this part have been formulated, i.e.:

(a) it is directly proportional to modules of stresses acting beyond the critical plane,

(b) it depends on their angular distance in such a way that the vectors acting in a larger angular distance in relation to the critical plane increase the loading nonproportionality degree more than the vectors of the same module acting within a smaller angular distance.

The nonproportionality measure H is considered here by both criteria proposed by the author and respects these theses.

Assumption (a) was accounted for through application of a filling factor defined as the ratio of the loading path field of reduced stress to the field of a circle circumscribed about the loading path (Fig. 1). For a proportional loading, the value of such a defined measure is equal to zero, whereas for a nonproportional loading of the highest degree its value is one. In the second case, the rotating vector of equivalent stress does not change its module, so the loading path is a circle.

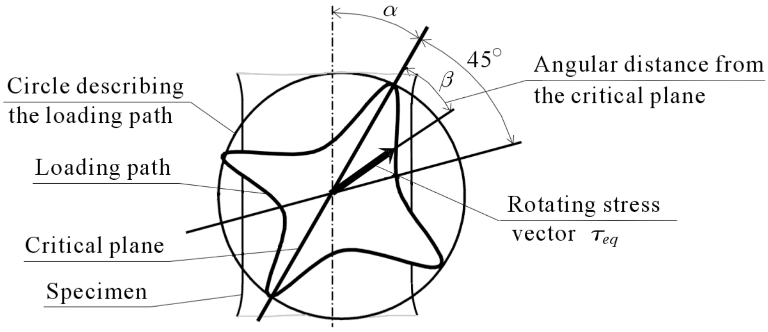

Fig. 1. Loading nonproportionality

One of the first ideas of quantification of the degree of nonproportionality was a rotation factor proposed by Kanazawa *et al.* (1979). This factor is based on the interaction of slip on different planes and is defined as the ratio of the shear strain at 45° in the maximum shear strain range direction to the maximum shear strain range. At present, there exist many solutions which describe the nonproportionality degree through the loading path of rotating vectors (Itoh *et al.*, 1997; Chen, 1996; Duprat, 1997; Morel *et al.*, 1997; Borodii and Strizhalo, 2000). Some of them account for it by means of some characteristic dimensions of loading paths (Duprat, 1997; Morel *et al.*, 1997). There are criteria for which the created hodograph field is the basis of the nonproportionality measure formulation similiarly to the hereby proposed solution (Chen, 1996). In all these cases, the nonproportionality measure is characterized only by modules of vectors acting beyond the critical plane. Thus, a simplification is usually accepted that all the rotating stress vectors have the same fatigue effect (they influence the fatigue damage cumulation process in the same manner) regardless of their location.

However, it is worth noting that there exist criteria for a general nonproportional loading which, for different reasons, e.g. application for insensitive materials, do not take into account the influence of nonproportionality, like for example Łagoda *et al.* (1999), Papadopoulos (1995), Macha (1989), Dang Van *et al.* (1989). There also exist criteria predicting fatigue properties in nonproportional loading conditions by means of introducing nominal loading quantities to the criterion, like a phase shift φ. That idea was used in the case of Lee (1985) and Lee and Chiang (1991) criteria. However, such solutions are restricted to a particular loading case.

In the solution proposed by the author, the mathematical description of thesis (b) assumes a form of the weight function W. Its value in the critical plane is zero, and for the course most distant from the critical plane (course turned by 45°) the value is one (Fig. 2). Multiplying the modules of rotating vectors by W their parts are being differentiated. The stresses more distant from the critical plane correspond to a greater weight, thus their effect in the process of fatigue damage cumulation is bigger.

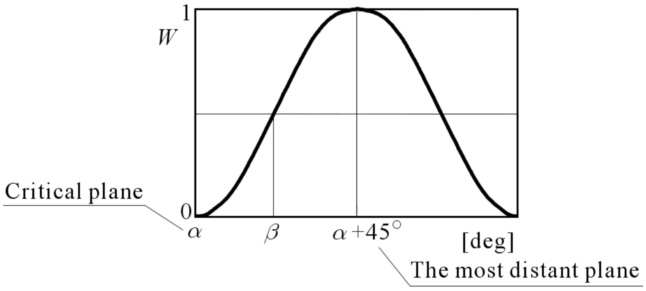

Fig. 2. Weight function

Thanks to the weight function, the accuracy of obtained calculation results increases.

The influence of the vector position in the conditions of loading nonproportionality on fatigue properties has also been noticed by other authors. Experimental works in this field were carried out by Sonsino *et al.* (2004) and Yousefi-Hashtyani (2004). These researchers claim that the nonproportionality degree depends on the changing angle of the principal axes position. For small principal axes rotation angles, the influence of nonproportionality on fatigue properties is smaller than in the case when the pricipal axes rotation angle is bigger.

An attempt of accounting for the influence of vector position can be found in Itohs criterion (Itoh *et al.*, 1997). For the description of nonproportionality, the author proposes an integral of the main strain vector projection onto the direction perpendicular to the critical one. Thus, it assigns to vectors different weights depending on their particular positions.

Contrary to the commonly accepted concept of nonproportinality description by means of stress modules (or indirectly by description of loading path characteristics), attempts to introduce a quantity depending on their positions into the description are very rare. Therefore, it is not difficult to notice that thesis (b) needs to be verified.

In the paper, an experimental work whose aim was verification of the assumption that the loading proportionality degree depends both on modules of stresses acting beyond the critical plane and on angular distances of these stresses vectors measured in relation to the critical plane is discussed.

2. Testing methodology

The tests require creation of nonproportional loading conditions so that there would be a possibility of controlling the rotation angle value of the principal axes position. In these tests, the controlled changes of the position of principal axes were realized by means of a two-block loading program. Block I consisted of fully reversed torsion and block II consisted of fully reversed cycles with biaxal torsion and compression.

In literature, experimental works based on a similar approach can be found. In many papers, the effect of the main direction changes was examined (Lee and Lee, 1997; Wheelhouse *et al.*, 2001; Morel, 2000; Bonacuse and Kalluri, 2001) sometimes referring to the character of those examinations directly as loading nonproportionality modeling (Wheelhouse *et al.*, 2001). Those experiments involved realization of loading blocks of different types: torsion with push-pull (Lee and Lee, 1997; Wheelhouse *et al.*, 2001; Bonacuse and Kalluri, 2001), torsion-bending (Morel, 2000).

In order to prove the influence of the vectors of angular distances on fatigue properties, the principal axes rotation range was controlled in successive fatigue tests. This effect was obtained through the change of the principal axes position in block II and with unchanged position of the axes in block I (torsion). The assumed quantity of the principal axes position in block II was reached by establishing proper values of the shear-to-normal stress amplitude ratio.

In order to examine only the effect of the principal axes position change, the equivalent stress values for both blocks remained unchanged. Their concrete values resulted from the stress level accepted for a given test. Because of the applicability range of the proposed criteria those were the levels of stresses corresponding to the high cycle regime.

The analysed result of carried out tests was the fatigue life.

In connection with the fact that equivalent stresses used in the authors proposed criteria are of shear character, the achieved results were compared with the fatigue life curve in torsion. It was accepted that the verified assumption would be confirmed if along with the increase of the principal axes angle between blocks, the degree of modelled nonproportionality would rise, loading would be more damaging, and the obtained fatigue lives decrease.

3. Testing conditions

The tests were carried out on the Instron 8874 biaxial hydraulic mechanical testing machine realising tension-compression in the range ±25 kN and torsion in the range ±100 Nm with the possibility of phase shift.

For the tests, steel X5CrNi18-10 ($R_m = 740\,\mathrm{MPa}$, $R_e = 620\,\mathrm{MPa}$, $A_5 = 52\%$) was used. As proved in the analysis carried out by Skibicki (2006), this material is very sensitive to loading nonproportionality. For this type of steel, a nonproportional loading is much more destructive than a proportional loading. Thus, the use of this steel for examining the effect of modelled loading nonproportionality was expected to bring the best results.

Geometrical features of the test sample are presented in Fig. 3. The choice of the sample was made according to the standard ASTM E 2207-02 defining fatigue test conditions for axial loading and torsion. Deviations from the standard recommendations resulted from technical parameters of the test stand.

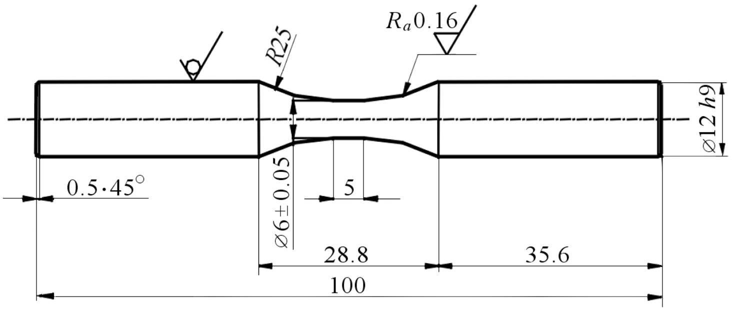

Fig. 3. Geometrical characteristics of the tested sample

4. Experiment and analysis of achieved results

In the first stage of tests, the fatigue life curve in torsion was determined (Fig. 4). In relation to that curve, further results of fatigue tests were compared.

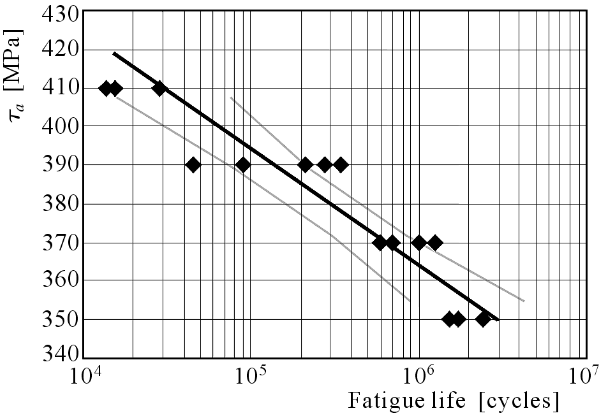

Fig. 4. Fatigue life curve in torsion

Due to the fact that the realized research program concerned biaxial loadings it was necessary to use the equivalent stress formula. For this purpose, equation (1.2) was used. Further, experimental verification of this stress was carried out. For this purpose, two fatigue tests were performed: torsion with tension with push-pull. The value of the stress amplitudes was selected so that the tests could be carried out for two levels of the equivalent stress amplitude (370 and 390 MPa) and for 3 shear-to-normal stress amplitude ratios (thereby for three different positions of the maximum shear stress vector).

Ratios of shear-to-normal stress amplitudes were established in such a way that the angles describing the maximum shear vector position in relation to the direction of the maximal shear stress vector in the block of pure torsion had the following values: 7.5°, 15° and 22.5° (Fig. 5). In order to make the fatigue damage cumulation and crack growth process course for all fatigue tests similar to the basic torsion, cases of biaxial loading with torsion prevailing were chosen. For the extreme case of the angle equal 22.5°, the amplitude ratio $\tau_{(a)xy}/\sigma_{(a)x}$ was 0.5.

For both levels of the equivalent stress and all three levels of maximum tangent vector, the obtained fatigue lives are consistent with the prediction made on the basis of the torsion curve (Fig. 6).

Furthermore, there was chosen a level of stress for carrying out the main tests. Because of the applicability range of the proposed criteria it was a stress level of 370 MPa corresponding to the high cycle range of fatigue life.

The main tests included 3 variants of programmed fatigue tests. Each variant consisted of a block of $5 \cdot 10^3$ cycles of reversed torsion and a successive block of biaxial loading of the same length. These variants were different in respect of the maximum shear stress position in relation to reversed torsion.

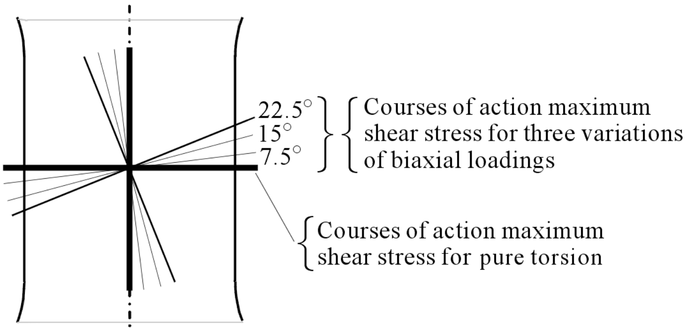

Fig. 5. Positions of maximum shear stress vectors for biaxial tests

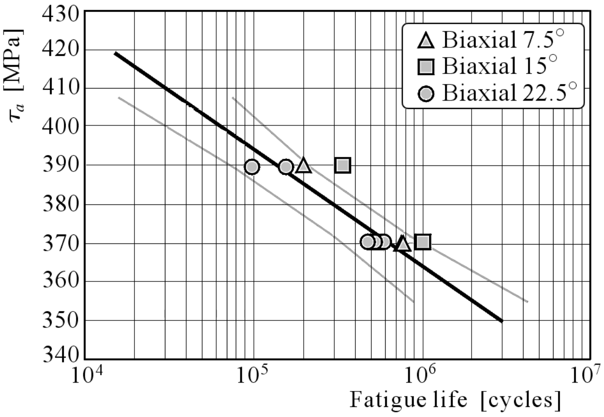

Fig. 6. Fatigue lives obtained in biaxial tests for three different positions of maximum shear vector: triangle – 7.5° (3 tests), square – 15° (3 tests) and circle – 22.5° (5 tests)

Values of the angles were the same as those established for equivalent stress verification tests, i.e. 7.5°, 15° and 22.5°.

On a diagram of fatigue life (Fig. 7), the obtained numbers of cycles have been marked.

For each variant, mean values have been calculated from tests. The results are presented in Fig. 8.

Black color is used to mark the base fatigue life obtained for the case of torsion.

Gray color marks the mean value of fatigue life obtained under biaxial loading for three different maximum shear stress vector positions. They correspond to the value of reversed torsion which means that the formula for the equivalent stress properly determines its value.

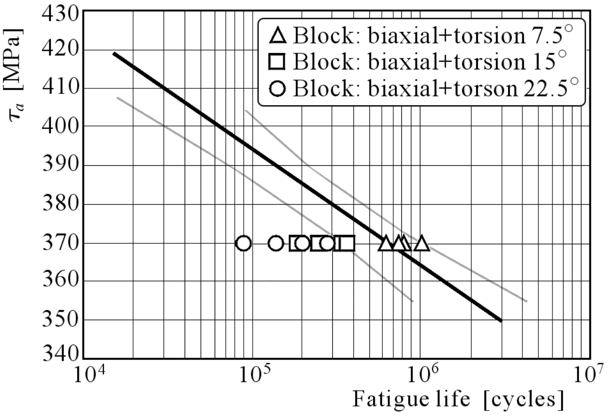

Fig. 7. Fatigue lives obtained in block loading tests for three different positions of maximum shear vector: triangle – 7.5° (4 tests), square – 15° (4 tests) and circle – 22.5° (4 tests)

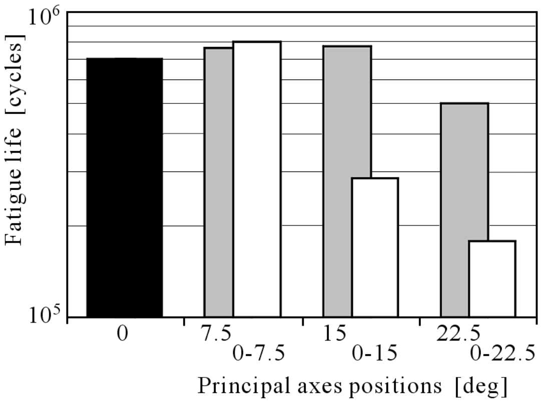

Fig. 8. Mean values set up in the whole research program

White color is used to mark the mean values of fatigue life obtained in result of programmed loadings, i.e. with principal axes positions changing between the blocks.

No influence of the changes of principal axes positions on the fatigue life has been observed for angle 7.5°. This proves that the block character of loading does not change the fatigue life and does not affect the results of tests.

For angles 15° and 22.5°, the influence of principal axes position changes between blocks is visible and rises along with increase of the angle value.

5. Conclusions

- Fatigue lives obtained in the programmed tests depend on the principal axes rotation angle variability range between the loading blocks. It has been observed that along with the rise of principal axes rotation angles the obtained lives were getting smaller.
- The obtained results can serve as a basis for formulation of a conclusion that the nonproportionality degree is determined both by the modules of stresses acting in result of the principal axes rotation beyond the critical plane, and their positions in relation to the critical plane direction. It can be concluded that the contribution of these stresses in the process of fatigue damage cumulation under nonproportional loading depends on the angular distance from the critical plane.
- The weight function is an important element of the proposed nonproportioanlity measure and its existence in the criterion notation, (1.1), is justified.

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Eksperymentalna weryfikacja modelu nieproporcjonalności obciążenia zmęczeniowego

Streszczenie

W artykule przedstawiono weryfikację eksperymentalną modelu nieproporcjonalności obciążenia zmęczeniowego, który był podstawą sformułowanych przez autora wieloosiowych kryteriów zmęczeniowych. W modelu tym zakłada się, że w warunkach obciążeń nieproporcjonalnych można wyróżnić płaszczyznę krytyczną. Jednocześnie, obrót osi głównych powodować może intensyfikację procesu kumulacji uszkodzeń zmęczeniowych. Decyduje o tym stopień nieproporcjonalności obciążenia. W przyjętym modelu założono, że o stopniu nieproporcjonalności obciążenia decydują: (a) moduły wektorów obracających się naprężeń, (b) kątowe odległości wektorów tych naprężeń mierzone w stosunku do płaszczyzny krytycznej. Zrealizowany eksperyment polegał na modelowaniu zmęczeniowego obciążenia nieproporcjonalnego poprzez programowane zmiany położenia osi głównych. Różne położenia osi głównych uzyskiwano realizując dwu blokowy program obciążeń: wahadłowego skręcania i obciążenia złożonego, w tym przypadku rozciągania ze skręcaniem. Badano wpływ zmian położenia osi głównych w trakcie próby uzyskiwaną na trwałość zmęczeniową. Otrzymane wyniki dają podstawę do przyjęcia tezy, że stopień nieproporcjonalności zależy nie tylko od modułów obracających się naprężeń, ale jest zależny także od odległości kątowej, w jakiej wektor naprężenia działa w stosunku do płaszczyzny krytycznej.

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