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PLASTIC INTERFACIAL SLIP OF PERIODIC SYSTEMS OF RIGID THIN INCLUSIONS UNDERGOING LONGITUDINAL SHEAR

Vasyl A. Kryven

Faculty of Control Measurements and Computer Systems, Electrical Ternopil State Technical University, Ternopil, Ukraine
e-mail: mmethod@tu.edu.te.ua

Georgiy T. Sulym

Faculty of Mechanical Engineering, Bialystok University of Technology, Poland
e-mail: sulym@pb.bialystok.pl

Myrosława I. Yavorska

Faculty of Control Measurements and Computer Systems, Electrical Ternopil State Technical University, Ternopil, Ukraine
e-mail: mmethod@tu.edu.te.ua

Plastic interfacial slip at the longitudinal shear of double periodic systems of thin rigid inclusions in elasto-plastic solids is investigated. Plastic deformations are considered to be localized in the thin layers on the inclusion-matrix boundary at the inclusion tips. The length of plastic layers and the rupture displacement value at the inclusions tips caused by plastic interfacial slip are determined. Particular cases of uniperiodical parallel or collinear inclusion systems are analyzed in detail.

Key words: longitudinal shear, periodic problem, thin inclusion, elasto-plastic solid, plastic slip

1. Introduction

While investigating mechanical properties of composite materials, the influence on stress and strain states of inclusions of most various forms and configurations should be given into account (Vanin, 1985). Especially, stress-deformed state investigations in bodies with periodic systems of inclusions represent considerable interest for the strength theory of composites, reinforced materials and for prediction and optimization of their deformation characteristics. For linear-elastic bodies, this problem was investigated in a lot of works (Beregnytsky *et al.*, 1983; Deliavsky *et al.*, 1998), but for elastic-plastic bodies, has been studied insufficiently.

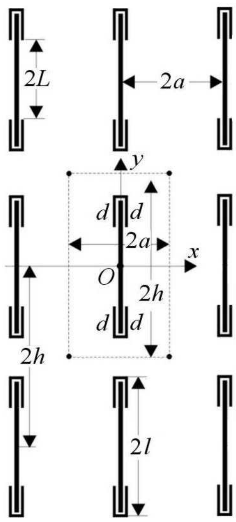

Fig. 1. Geometrical scheme of the problem

There are two approaches to plastic deformation analysis near stress concentrators. The first of them assumes that plastic deformations are located in a region with an unknown boundary (a continuum plastic zone). In the second ome, it is supposed that the plastic deformations are concentrated in some layers with almost vanished thickness (discrete linear plastic zones). Both kinds of zones are observed in experimental works. The localization of plastic deformations in thin layers was found independently with the shape of concentrators: for tension of thin (Leonow *et al.*, 1963) and thick (Kaminskij *et al.*, 1994) plates with cracks, for tension of plates with circular holes (Rabotnow and Stankiewicz, 1965) and for torsion of circular smooth shafts (Nadai, 1954). Plastic deformations are often located in plastic layers, called Lüders-Czernov's bands (Nadai, 1954; Sokołowskij, 1969) for materials with sharp transitions between the elastic and plastic states on the stress-strain curve with a well seen plastic flow zone. It should be emphasized that discrete linear plastic zones are not an absolute alternative for continuum zones. In that case, when the number of discrete plastic zones increases, it can be shown (Kryven, 1983) that in the limit a continuum zone is obtained.

The thin layered localizations of plastic zones on interfaces between inclusions and a matrix can lead to separate inclusions, which has great influence on the material strength. This phenomenon should be studied in detail.

2. Common case of the double-periodic problem

2.1. Problem formulation

Within the framework of antiplane deformation we consider a double-periodic problem connected with the plastic interfacial slip of thin rigid plate inclusions forming a rectangular lattice in an ideal elastic-plastic body with the Treska-St. Venant or Huber-Mises-Hencky yield condition

$$\tau_{yz}^2(x,y)+\tau_{xz}^2(x,y)=k^2$$

k is the yield limit of the material subject to shear. The inclusions occupy the sectors $x=2na$, $|y+2mh|\leqslant l$ $(n,m\in\mathbb{Z})$, where $2a$, $2h$ are distances between the inclusion centers in horizontal and vertical directions respectively.

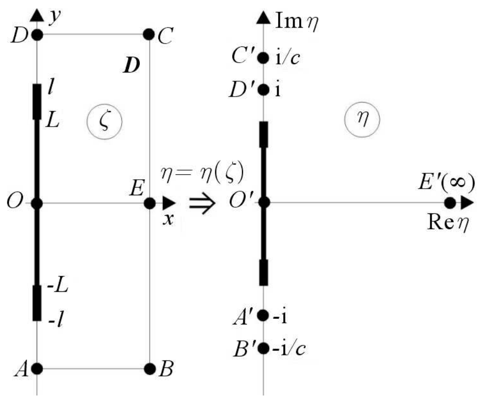

Fig. 2. Scheme of the conformal mapping

Consider now the case, when the non-zero component $w(x,y)$ of the displacement vector is symmetric with respect to straight-lines directed along the inclusions axis and asymmetric with respect to straight-lines passing across the inclusion centers and directed perpendicular to the inclusions.

We assume also that conditions of ideal soldering with the matrix are satisfied before loading, and the plasticity zones arising as a consequence of the loading are very thin layers adjoined to the inclusions and they initiate from the points of maximum stress concentrations. It means that we suppose that the borders of plasticity zones are thin layers and areas of inclusions $x=2na\pm 0$, $L\leqslant|y+2mh|\leqslant l$, where the length of the plasticity strips $l-L$ must be determined $(2L$ is the length of the inclusion ideal contact zone).

The stress field is determined by two non-zero stress tensor components $\tau_{xz}(x,y)$ and $\tau_{yz}(x,y)$, and according with Hooke's law we have $\tau_{xz}(x,y)=\mu\partial w/\partial x$, $\tau_{yz}(x,y)=\mu\partial w/\partial y$ $(\mu$ – shear modulus of the material). Taking into account geometrical symmetry of the problem, we assume that the body loading is such that on the boundary of a periodic rectangle $((2n-1)a;\ (2m-1)h)$, the shear stress $\tau_{xz}=0$. Let us denote the stress τ_{yz} in the tips of the periodic rectangle by τ_0 and determine the composite stress-deformed state considering τ_0 as given. A constant τ_0 determine the loading of the body.

According with the equilibrium equations and Hooke's law, the stress tensor components τ_{xz}, τ_{yz} in the elastic part of the body are described by an analytical function $\tau(\zeta)=\tau_{yz}+\mathrm{i}\tau_{xz}$ of the complex variable $\zeta=x+\mathrm{i}y$. Due to periodicity of the problem, it is sufficient to determine them in the rectangle $-a\leqslant x\leqslant a$, $-h\leqslant y\leqslant h$. The function $\tau(\zeta)$ is analytical in this rectangle with a notch along the segment $x=0$, $|y|\leqslant l$. Due to symmetry of the problem instead of the periodic rectangle, it is sufficient to consider the rectangle $ABCD$: $0\leqslant x\leqslant a$, $0\leqslant y\leqslant h$.

The following conditions must be satisfied on its boundary:

$$
\begin{aligned}
&|\tau(\zeta)|=k && (\zeta=\mathrm{i}y,\ L\leqslant|y|\leqslant l)\\
&\mathrm{Re}\tau(\zeta)=0 && (\zeta=\mathrm{i}y,\ |y|<L)\\
&\mathrm{Im}\tau(\zeta)=0 && ((\zeta=\mathrm{i}y,\ l<|y|\leqslant h)\vee(\zeta=x\pm\mathrm{i}h,\ 0\leqslant x\leqslant a)\vee\\
& && \vee(\zeta=a+\mathrm{i}y,\ |y|\leqslant h))
\end{aligned}
\tag{2.1}
$$

The first one is a plasticity condition $\tau_{yz}^2(\xi)+\tau_{xz}^2(\xi)=k^2$ on the region $\zeta=\mathrm{i}y$, $L\leqslant|y|\leqslant l$. The second condition is the ideal soldering condition $w(x,y)=0$ on the inclusion surface $\zeta=\mathrm{i}y$, $-L\leqslant y\leqslant L$ without plastic slip. The condition $w=0$ on the given interval is equivalent to $\partial w(x,y)/\partial y=0$. Due to Hook's law, it can be reduced to $\tau_{yz}(0,y)=0$ $(|y|\leqslant L)$. The last equation in (2.1) describes the condition $\tau_{xz}=0$ on the remained boundary of the region $ABCD$.

Equation (2.1) together with the condition

$$
\tau(a,h)=\tau_0 \tag{2.2}
$$

formulae the boundary value problem for the function $\tau(\zeta)$.

2.2. Solution to the problem

Boundary problem (2.1), (2.2) in the rectangular $ABCD$ for the function $\tau(\zeta)$ can be easily reduced to a problem in a half-plane for a new function $\tau_1(\eta)$ by mapping $\eta(\zeta)$ the rectangular to the half-plane $\mathrm{Re}\eta\geqslant 0$. We search a solution to problem (2.1), (2.2) as a composition of functions

$$
\tau(\zeta)=\tau_1\Big[-\mathrm{i}\,\mathrm{sn}\Big(\frac{\mathrm{i}K\zeta}{h},c\Big)\Big] \tag{2.3}
$$

where sn is a Jacobian elliptic function reversed to one given by the integral equality

$$\zeta = \frac{h}{K}\int\limits_0^{\eta} \frac{dt}{\sqrt{(1+t^2)(1+c^2t^2)}} \tag{2.4}$$

where K is the full elliptic integral of the first mode

$$K = \int\limits_0^{1} \frac{dt}{\sqrt{(t^2-1)(1-c^2t^2)}}$$

and $c\,(0 < c < 1)$ is the modular of the elliptic integral-solution of the equation

$$aK = hK'$$

and K' – full elliptic integral of the second mode

$$K' = \int\limits_1^{1/c} \frac{dt}{\sqrt{(t^2-1)(1-c^2t^2)}}$$

This gives a possibility of changing the boundary value problem for the function $\tau(\zeta)$ in the rectangle to the problem for the new unknown function $\tau_1(\zeta)$ in the half-plane (simpler for further calculations).

The function $\zeta(\eta)$ is a conformal mapping of the right half-plane $\mathrm{Re}\eta > 0$ into the rectangle $0 < \mathrm{Re}\zeta < a$, $-h < \mathrm{Im}\zeta < h$. The points 0, $-\mathrm{i}$, $-\mathrm{i}/c$, i/c, i correspond to $-\mathrm{i}h$, $a-\mathrm{i}h$, $a+\mathrm{i}h$, $\mathrm{i}h$, respectively. The infinitely remote point (E') of the plane corresponds to the point $\zeta = a$ (E). Therefore, it is easy to verify that the function $\tau_1(\eta)$ will satisfy the following boundary conditions in the half-plane $\mathrm{Re}\eta \geqslant 0$:

$$\begin{aligned}
&|\tau_1(\eta)| = k && \Big(\mathrm{Re}\eta = 0,\ \mathrm{sn}\,\frac{\mathrm{i}KL}{h} \leqslant |\mathrm{Im}\eta| \leqslant \mathrm{sn}\,\frac{\mathrm{i}Kl}{h}\Big) \\
&\mathrm{Re}\tau_1(\eta) = 0 && \Big(\mathrm{Re}\eta = 0,\ |\mathrm{Im}\eta| \leqslant \mathrm{sn}\,\frac{\mathrm{i}KL}{h}\Big) \\
&\mathrm{Im}\tau_1(\eta) = 0 && \Big(\mathrm{Re}\eta = 0,\ |\mathrm{Im}\eta| > \mathrm{sn}\,\frac{\mathrm{i}Kl}{h}\Big) \qquad \tau_1\Big(\frac{\mathrm{i}}{c}\Big) = \tau_0
\end{aligned} \tag{2.5}$$

For further simplification we will omit the modular at the argument of the Jacobian elliptic function sn.

The solution to boundary problem (2.5) can be received in the way given by Kryven (1979), by taking into account that according to conditions (2.5) the conformal mapping $\tau_1(\zeta)$ is known *a priori* because the half-plane $\mathrm{Re}\eta \geqslant 0$ is transformed into the half-circle $\mathrm{Re}\tau_1 > 0$, $|\tau_1| < k$ with the cut along the segment $\tau_0 < \mathrm{Re}\tau_1 < k$, $\mathrm{Im}\tau_1 = 0$. After direct construction of the mapping, we find the function $\tau(\zeta)$ by substituting $\tau_1(\eta)$ in (2.3)

$$\tau(\zeta) = k\frac{\mathrm{sn}\,\frac{Kl}{h}\sqrt{\mathrm{sn}^2\frac{\mathrm{i}K\zeta}{h} + \mathrm{sn}^2\frac{KL}{h}} - \mathrm{sn}\,\frac{KL}{h}\sqrt{\mathrm{sn}^2\frac{\mathrm{i}K\zeta}{h} + \mathrm{sn}^2\frac{Kl}{h}}}{\mathrm{sn}\,\frac{\mathrm{i}K\zeta}{h}\sqrt{\mathrm{sn}^2\frac{Kl}{h} - \mathrm{sn}^2\frac{KL}{h}}} \tag{2.6}$$

Let us find the length of the plastic strips by taking into account the third relation of condition (2.5). From the limit case in (2.6) at $\zeta \to a + \mathrm{i}h$, after some calculations we receive

$$\mathrm{sn}\,\frac{KL}{h} = \frac{k^2 - \tau_{01}^2}{k^2 + \tau_{01}^2}\,\mathrm{sn}\,\frac{Kl}{h}$$

Here we have

$$\tau_{01} = \frac{\sqrt{p + k^2 q} - \sqrt{p - k^2 q}}{\sqrt{2q}} \qquad p = k^4 + \tau_0^4 - 2k^2c^2\tau_0^2\,\mathrm{sn}^2\frac{Kl}{h}$$

$$q = 2\tau_0^2\Big(1 - c^2\,\mathrm{sn}^2\frac{Kl}{h}\Big)$$

The quantity τ_{01} corresponds to the stress component τ_{yz} in the point $(a, 0)$ of the rectangular boundary.

To obtain the plasticity strip length $d = l - L$ we determine L from the function reversed to sn by integral (2.4).

The maximal displacement jump $[\![w]\!] = g$ is achieved in the inclusion tips, and it is given by the following formula

$$g = \frac{k}{\mu}\int_L^l \tau_{yz}(+0, y)\,dy \tag{2.7}$$

where μ denotes the shear modulus of the body material. We can find the component τ_{yz} from formula (2.6)

$$\tau_{yz}(0, y) = k\frac{\mathrm{sn}\,\frac{Kl}{h}\sqrt{\mathrm{sn}^2\frac{Ky}{h} - \mathrm{sn}^2\frac{KL}{h}}}{\mathrm{sn}\,\frac{Ky}{h}\sqrt{\mathrm{sn}^2\frac{Kl}{h} - \mathrm{sn}^2\frac{KL}{h}}} \qquad L < |y| < l$$

By substituting the variable $\mathrm{sn}^2(Ky/h) = t$ into integral (2.7), we have

$$g = \frac{kh}{2\mu K}\frac{\mathrm{sn}\,\frac{Kl}{h}}{\sqrt{\mathrm{sn}^2\frac{Kl}{h} - \mathrm{sn}^2\frac{KL}{h}}}\int_{\mathrm{sn}^2\frac{KL}{h}}^{\mathrm{sn}^2\frac{Kl}{h}}\sqrt{\frac{t - \mathrm{sn}^2\frac{KL}{h}}{t(1-t)(1-c^2t^2)}}\,dt \tag{2.8}$$

Let us look at particular cases of the slip zone evolution problem for periodic systems of inclusions.

3. The case of a collinear system of rigid thin equidistant inclusions in the same plane

A collinear inclusion system is a consequence of the double-periodic systemone when distance between the centers of inclusions in the horizontal direction trends to infinity: $a \to \infty$. In this case, we have

$$\operatorname{sn}\frac{\mathrm{i}K\zeta}{h} \to \sinh\frac{\pi\zeta}{2h} \qquad \operatorname{sn}\frac{Kl}{h} \to \sin\frac{\pi l}{2h} \qquad \operatorname{sn}\frac{KL}{h} \to \sin\frac{\pi L}{2h}$$

Therefore, at $a \to \infty$ we receive from (2.6)

$$\tau(\zeta) = k\frac{\sin\frac{\pi l}{2a}\sqrt{\sinh^2\frac{\pi\zeta}{2h} - \sin^2\frac{\pi L}{2h}} - \sin\frac{\pi L}{2a}\sqrt{\sinh^2\frac{\pi\zeta}{2h} - \sin^2\frac{\pi l}{2h}}}{\sinh\frac{\pi\zeta}{2h}\sqrt{\sin^2\frac{\pi l}{2h} - \sin^2\frac{\pi L}{2h}}}$$

Due to the conditions $\tau_{yz}^{\infty} = \tau_0$, $\tau_{xz}^{\infty} = 0$, we have at infinity

$$\tau_0 = k\frac{\sin\frac{\pi l}{2h} - \sin\frac{\pi L}{2a}}{\sqrt{\sin^2\frac{\pi l}{2h} - \sin^2\frac{\pi L}{2h}}}$$

The dependence of the plastic strips length on the load is given by

$$d = l - \frac{2h}{\pi}\arcsin\Big(\frac{k^2 - \tau_0^2}{k^2 + \tau_0^2}\sin\frac{\pi l}{2h}\Big) \tag{3.1}$$

Displacement jumps in the inclusion tips can also be expressed in a closed form as a function of the load

$$\begin{aligned} g &= \frac{2kh}{\pi\mu}\frac{\sin\frac{\pi l}{2h}}{\sqrt{\sin^2\frac{\pi l}{2h} - \sin^2\frac{\pi L}{2h}}}[f_2(l) - f_2(L)] \\ f_2(y) &= \sin\frac{\pi L}{2h}\arctan\left[\sin\frac{\pi L}{2h}\tan\left(\arcsin\frac{\cos\frac{\pi y}{2h}}{\cos\frac{\pi L}{2h}}\right)\right] - \arcsin\frac{\cos\frac{\pi y}{2h}}{\cos\frac{\pi L}{2h}} \end{aligned} \tag{3.2}$$

4. The case of a complanar system of rigid thin equidistant inclusions

A periodic system of parallel inclusions is a consequence of the double-periodic system when the distance between the inclusion centers in the vertical direction trends to infinity: $h \to \infty$. Here we have

$$\operatorname{sn}\frac{\mathrm{i}K\zeta}{h} \to \tan\frac{\pi\zeta}{2a} \qquad \operatorname{sn}\frac{Kl}{h} \to \tanh\frac{\pi l}{2a} \qquad \operatorname{sn}\frac{KL}{h} \to \tanh\frac{\pi L}{2a}$$

Thus from (2.6) we obtain:

$$\tau(\zeta) = k\frac{\tanh\frac{\pi l}{2a}\sqrt{\tan^2\frac{\pi\zeta}{2a}+\tanh^2\frac{\pi l}{2a}} - \tanh\frac{\pi L}{2a}\sqrt{\tan^2\frac{\pi\zeta}{2a}+\tanh^2\frac{\pi l}{2a}}}{\tan\frac{\pi\zeta}{2a}\sqrt{\tanh^2\frac{\pi l}{2a}-\tanh^2\frac{\pi L}{2a}}}$$

Since $\lim_{\zeta\to\infty}\tau(\zeta) = \tau_0$, $\lim_{y\to\infty}\tan(\mathrm{i}\pi y/2a) = \mathrm{i}$, and due to the last formula, we find

$$\tau_0 = k\frac{\tanh\frac{\pi l}{2a}\sqrt{1-\tanh^2\frac{\pi L}{2a}} - \tanh\frac{\pi L}{2a}\sqrt{1-\tanh^2\frac{\pi l}{2a}}}{\sqrt{\tanh^2\frac{\pi l}{2a}-\tanh^2\frac{\pi L}{2a}}} \qquad \begin{aligned}\tau_{yz}^{\infty} &= \tau_0\\ \tau_{xz}^{\infty} &= 0\end{aligned}$$

and

$$d = l - \frac{2a}{\pi}\operatorname{arcsinh}\Big(\frac{k^2-\tau_0^2}{k^2+\tau_0^2}\sinh\frac{\pi l}{2a}\Big) \tag{4.1}$$

In this case, the displacement jump in the inclusion tips can be expressed in a closed form from integral (2.8)

$$\begin{gathered}g = \frac{ka}{\mu\pi}\frac{\tanh\frac{\pi l}{2a}\tanh\frac{\pi L}{2a}}{\cosh^2\frac{\pi L}{2a}\sqrt{\tanh^2\frac{\pi l}{2a}-\tanh^2\frac{\pi L}{2a}}}[f_1(l)-f_1(L)]\\ f_1(y) = \frac{1}{2\sinh\frac{\pi L}{2a}}\ln\left|\frac{1-\cosh\frac{\pi L}{2a}\sqrt{\tanh^2\frac{\pi y}{2a}-\tanh^2\frac{\pi L}{2a}}}{1+\cosh\frac{\pi L}{2a}\sqrt{\tanh^2\frac{\pi y}{2a}-\tanh^2\frac{\pi L}{2a}}}\right| - \arcsin\frac{\tanh\frac{\pi L}{2a}}{\tanh\frac{\pi y}{2a}}\end{gathered} \tag{4.2}$$

5. Results and discussion

The dependence of the plastic strips length and the displacement jump on the load τ_0/k is shown in Fig. 3. In the case of collinear inclusions (lines 1-3) this length decreases as the distance $2h$ between the inclusions increases. The jump value of the displacement g in the inclusion tips changes in the same way.

If $h\to\infty$, we arrive at the case of a single inclusion (Vytvytsky and Kryven, 1979). Thus, from (3.1), we have

$$d = \frac{2\tau_0^2 l}{k^2+\tau_0^2} \qquad g = \frac{kl}{\mu}\Big(1 - \frac{k^2-\tau_0^2}{k^2+\tau_0^2}\arccos\frac{k^2-\tau_0^2}{k^2+\tau_0^2}\Big) \tag{5.1}$$

That is a dependence of d on τ_0 for one inclusion. As a matter of fact, it does not differ from the dependence described by curve 3 in Fig. 3. It means that in the case of inclusions periodically situated on the same plane, the

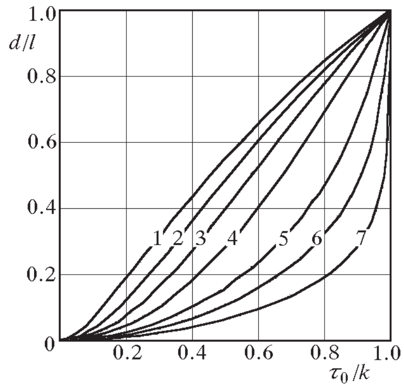

Fig. 3. Length of plastic strips versus load; 1, 2, 3 – $l/h = 0.9$, 07, 02;
3, 4, 5, 6, 7 – $l/a = 0.1$, 1, 2, 3, 5

solution for $l/h \leqslant 0.2$ does not differ from the solution corresponding to a single inclusion any longer.

In the case of a periodic system of coplanar inclusions for $a \to \infty$, we also obtain expression (5.1) from (3.1) for a single inclusion. The plastic strips length decreases again together with the decrease of distances $2a$ between the inclusions (lines 4-7 in Fig. 3). Moreover, the obtained results for the coplanar inclusions system, as a mater of fact, do not differ from those obtained from formula (5.1) for a single already inclusion at $l/a \leqslant 0.1$.

Solution (2.6) to the two-periodic problem can be treated as an approximate one for the case of plastic interfacial slip of a rigid thin inclusion $x = 0$, $|y| \leqslant l$ in the rectangular $|x| \leqslant a$, $|y| \leqslant h$ deformed by a constant shear stress $\tau_{yz} = \tau_0$ along the sides $y = \pm h$, $|x| \leqslant a$. Accuracy of such an approximation can be determined as a measure of nonhomogeneity of the stress τ_{yz} on horizontal sides of the periodic rectangular (in both cases, the stresses $\tau_{xz} = 0$).

From formula (2.6), it follows that the stress component τ_{yz} monotonically decrease on the periodic rectangular side from the point $(0, h)$ to the point (a, h), while the second stress component equals zero. The stress component τ_{yz} attains its maximum value on this rectangle side

$$\tau_{01} = k\frac{\operatorname{sn}\frac{Kl}{h}\operatorname{cn}\frac{KL}{h} - \operatorname{sn}\frac{KL}{h}\operatorname{cn}\frac{Kl}{h}}{\sqrt{\operatorname{sn}^2\frac{Kl}{h} - \operatorname{sn}^2\frac{KL}{h}}} \qquad \operatorname{cn}^2 x = 1 - \operatorname{sn}^2 x$$

at the point $(0, h)$, and the minimum value τ_0 at the point (a, h).

Thus, the measure of nonhomogeneity of the stress τ_{yz} on the horizontal sides of the periodical rectangle for the double-periodic problem can be defined as

$$\varTheta = \max_{\tau_0 \in (0,k)} \frac{\tau_{01} - \tau_0}{\tau_{01}} = \max_{L \in (0,l)} \frac{\tau_{01} - \tau_0}{\tau_{01}} \tag{5.2}$$

The regions (between lines and the axis $l/h = 0$) in the space of geometrical parameters for which, according to the solution to the double-periodic problem, the stress τ_{yz} on the horizontal sides of the periodical rectangle can be treated as constant with an accuracy Θ are given in Fig. 4.

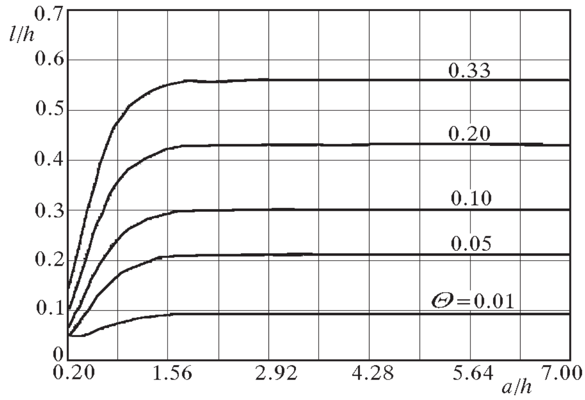

Fig. 4. Regions of geometric parameters in which the solution to the double-periodic problem gives an approximation of the solution corresponding to the periodical rectangle with the relative error not exceeding Θ

The closed-form solutions to problems under consideration enable one to determine the effective shear modulus in composites reinforced by rigid bands in the same way as it was done for the elastic problem by Ju and Chen Tsung-Muh (1994a,b), Porohovsky *et al.* (1998), and finally to formulate conditions of composite rupture on the basis of known deformation or energetic criteria.

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Plastyczny międzyfazowy poślizg okresowego układu sztywnych cienkich inkluzji podczas wzdłużnego ścinania

Streszczenie

W ramach antypłaskiego stanu odkształcenia przeanalizowano plastyczny poślizg na granice kontaktu dwuokresowego układu cienkich sztywnych inkluzji z ośrodkiem sprężysto-plastycznym podczas ścinania. Założono, że odkształcenia plastyczne znajdują się w cienkich warstwach na granicy inkluzji w otoczeniu ich końców. Wyznaczono długość warstw plastycznych oraz wartość skoku przemieszczenia spowodowanego plastycznym prześlizganiem. Dokładnie rozpatrzono również szczególne przypadki jednookresowych zagadnień dla inkluzji w jednej i w równoległych płaszczyznach.

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