JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
43, 3, pp. 523-538, Warsaw 2005

TIME-OPTIMAL CONTROL OF HYDRAULIC MANIPULATORS WITH PATH CONSTRAINTS

Paweł Hołobut

Institute of Fundamental Technological Research PAS, Warsaw
e-mail: pholob@ippt.gov.pl

A method of optimization intended to speed up motions of non-redundant hydraulic manipulators along prescribed paths of their end-effectors is presented. A parametric path of the end-effector of a non-redundant manipulator determines the corresponding path in the manipulator joint-space. The optimization problem therefore reduces to finding the optimum distribution of the parameter of the path in time. The proposed optimization scheme is based on discretization of the distribution of the parameter into a fixed number of points, and finding their optimum locations by methods of constrained nonlinear programming. Incompressibility of the hydraulic fluid is assumed throughout for greater effectiveness of the procedure. Results of sample optimizations performed on a three-link hydraulic excavator are presented.

Key words: hydraulic manipulators, time-optimal control, trajectory optimization

1. Introduction

Hydraulic manipulators are used in many branches of industry to perform tasks requiring high force capacity on a part of a machine. These include, among others, ground works and transportation of objects having large masses. Frequently, such tasks can be operationally summarized as moving an end-effector along pre-defined paths in the machine work space. In many cases, speeding up the movement of the end-effector does not adversely affect the quality with which the task is executed (for example in excavators, where there is usually no risk of damage to transported objects). In such cases, it is often advantageous to increase the velocity of movement along the given trajectory to minimize the duration of the working cycle of the machine.

Optimization aiming at speeding up the movement of non-redundant manipulators along given paths was thoroughly studied in the last decades. This, however, was done for general manipulators, driven by generalized motor forces (with no respect to dynamics of actuators). Bobrow *et al.* (1985) formulated the problem as one dimensional in terms of the path parameter, and treated equations of motion of a manipulator as constraints in the optimization problem. Optimal control was then found by directly searching for switching points in control forces. Shin and McKay (1986) as well as Singh and Leu (1987) rewrote equations of motion of a manipulator in a discrete form, and found optimal controls by methods of dynamic programming. Finally, in papers by McCarthy and Bobrow (1992) and Shiller and Lu (1992), the general structure of resultant optimal control laws was presented. It was shown that in time-optimal motion along a path, at least one of the actuators is saturated (its force reaches imposed bounds).

In this work, a method of optimizing movements of non-redundant hydraulic manipulators in terms of their operational speed is presented, taking into account dynamics of hydraulic actuation systems. The method consists in transforming the problem into a form suitable for solution by existing numerical routines of constrained nonlinear programming. Such approach allows inclusion of many sorts of constraints in the optimization problem.

2. Mathematical model of hydraulic manipulators

Hydraulic manipulators are open chains of links connected by prismatic or revolute joints, and actuated by hydraulic motors - either linear pistons or rotational motors. The mathematical model of hydraulic manipulators consists therefore of two groups of equations: equations of motion of the chain of links under action of hydraulic motors forces, and equations of motion of the hydraulic power system.

For simplicity it is assumed that links are rigid, joints are precisely manufactured, and the base of a manipulator is fixed to a chosen inertial frame. In the case where the above assumptions hold, the configuration of an n-joint manipulator can be unambiguously described by n generalized coordinates, which can be chosen as piston displacements, joint rotations or any other set of independent variables. Dynamics of a chain of links is given by the Lagrange equations of motion (Spong and Vidyasagar, 1997):

$$\mathbf{D}(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{h}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \mathbf{Q}(\boldsymbol{x})\boldsymbol{s} \tag{2.1}$$

where $\boldsymbol{x}$ is an n-vector of generalized coordinates; $\mathbf{D}(\boldsymbol{x})$ is an $n \times n$ inertia matrix; $\boldsymbol{h}(\boldsymbol{x}, \dot{\boldsymbol{x}})$ is an n-vector of centrifugal, Coriolis, and gravity forces; $\boldsymbol{s}$ is an n-vector of piston forces; and $\mathbf{Q}(\boldsymbol{x})$ is an $n \times n$ transformation matrix converting the piston forces into generalized ones in the adopted system of generalized coordinates.

The equations of motion of the hydraulic system are derived on the assumption that each cylinder has its independent fluid supply, which means that each cylinder has its own pump or that the common pump can service all cylinders of a machine simultaneously. A schematic of the hydraulic system for a single cylinder is shown in Fig. 1 (the same type of hydraulic system is assumed for all cylinders of the manipulator). The system consists of a pump, a reservoir, a proportional servovalve, and a cylinder. The pump supplies the hydraulic fluid at a constant pressure P_P. Excess of the fluid is diverted to the reservoir whose pressure is P_R (usually atmospheric pressure). Fluid flow in the system is governed by a proportional servovalve, which is the control unit in the system. The servovalve spool displacement u from its neutral position (at which there is no flow through the valve) is the control signal to the hydraulic system, and as such - to the entire hydraulic manipulator.

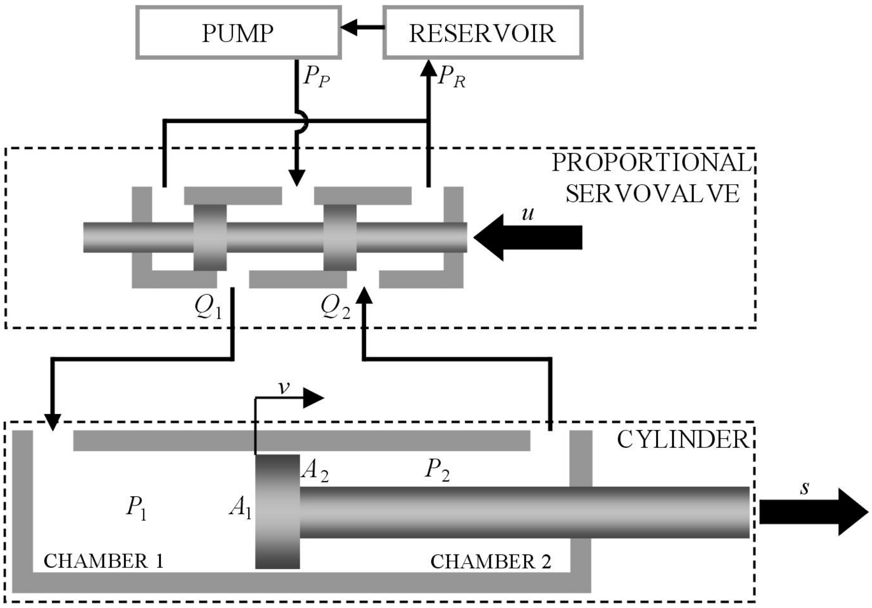

Fig. 1. A schematic of the hydraulic system

Basic equations describing dynamics of the hydraulic system are given in Tomczyk (1999), Walters (2000). The relation between fluid pressures and the piston force is

$$s = A_1 P_1 - A_2 P_2 \tag{2.2}$$

where s is the force exerted by the piston; A_1 and A_2 are areas of both sides of the piston; P_1 and P_2 are fluid pressures in chambers 1 and 2 of the cylinder, respectively. The equations of fluid flow through the servovalve are

$$\begin{aligned} Q_1 &= \begin{cases} C_{1P}\sqrt{P_P - P_1}\,u & \text{when} \quad u \geqslant 0 \\ C_{1R}\sqrt{P_1 - P_R}\,u & \text{when} \quad u < 0 \end{cases} \\ Q_2 &= \begin{cases} C_{2R}\sqrt{P_2 - P_R}\,u & \text{when} \quad u \geqslant 0 \\ C_{2P}\sqrt{P_P - P_2}\,u & \text{when} \quad u < 0 \end{cases} \end{aligned} \tag{2.3}$$

where Q_1 and Q_2 are fluid flow intensities into chamber 1 and out of chamber 2, respectively; and C_{1P}, C_{1R}, C_{2P} and C_{2R} are valve constants (flow coefficients). The above equations are all the relations needed in the developments to follow.

3. Formulation of the optimization problem

It is supposed that the end-effector of a manipulator has to move along a given parametric trajectory in the task space, which may consist of its endpoint positions and/or orientations with respect to the world coordinate system. Since the configuration of the end-effector is uniquely determined by the manipulator configuration $\boldsymbol{x}$ in its joint space, a prescribed path of the end-effector can be most generally expressed as a set of equality constraints $\boldsymbol{\Omega}(\boldsymbol{x}, p) = \mathbf{0}$ on $\boldsymbol{x}$, where $p:\ 0 \leqslant p \leqslant 1$ is a scalar parameter. The manipulator moves along a given trajectory of the end-effector if its successive configurations $\boldsymbol{x}$ satisfy $\boldsymbol{\Omega}(\boldsymbol{x}, p) = \mathbf{0}$ for successive values of p. It is here assumed that the manipulator is non-redundant relative to the given path. It means that path constraints imposed on the end-effector determine uniquely (or to a finite number of possibilities) the corresponding configurations of the manipulator in the joint space. In other words, for every $0 \leqslant p \leqslant 1$ there is a unique vector $\boldsymbol{x}(p)$ (or a finite number of such vectors) such that $\boldsymbol{\Omega}(\boldsymbol{x}(p), p) = \mathbf{0}$. Thus, to the given parametric path of the end-effector, there corresponds a unique continuous parametric path $\boldsymbol{x}(p)$ in the manipulator joint space (or a finite number of such paths), which satisfies $\boldsymbol{\Omega}(\boldsymbol{x}(p),p) = \mathbf{0}$ on the interval $0 \leqslant p \leqslant 1$. Therefore, from now on it will be assumed that the parametric path in the joint space is directly given as $\boldsymbol{x}_r(p)$, $0 \leqslant p \leqslant 1$ instead of constraints on configurations of the end-effector. The purpose of optimization is to minimize the time in which the manipulator passes the given trajectory under imposed constraints.

Let $\boldsymbol{x}(t)$ be a configuration of a manipulator at the time t. If the manipulator passes the trajectory $\boldsymbol{x}_r(p)$ in time T, to successive moments of time $0 = t_0 < t_1 < \ldots < t_k = T$ there correspond values of the parameter $0 = p_0, p_1, \ldots, p_k = 1$ such that $\boldsymbol{x}(t_i) = \boldsymbol{x}_r(p_i)$, $i = 0, 1, \ldots, k$. It leads to a distribution of the parameter in time $p(t): [0,T] \to [0,1]$, where T is the time at which the end of the trajectory is reached. For the purposes of optimization it can be assumed, without much loss of generality, that $p(t)$ is an increasing function of t on the interval $(0,1)$. This means that during time-optimal motion along the given trajectory, the manipulator does not stop or move back along the trajectory. Thus, the problem of optimization for speeding up the movement along the given parametric trajectory in the joint space reduces to finding the optimal distribution of the parameter $p(t)$, which is an increasing function of time.

Before stating the general optimization problem, it is necessary to discuss boundary conditions imposed on the solution. If compressibility of the hydraulic fluid was significant during time-optimal motion, the following boundary conditions would have to be given

$$\begin{array}{llll} p(0) = 0 & p(T) = 1 & \dot{p}(0) = \dot{p}_0 & \dot{p}(T) = \dot{p}_T \\ \ddot{p}(0) = \ddot{p}_0 & \ddot{p}(T) = \ddot{p}_T & \boldsymbol{P}_1(0) = \boldsymbol{P}_{10} & \end{array} \tag{3.1}$$

where $\boldsymbol{P}_1(0)$ is an n-vector of fluid pressures P_1 in all hydraulic cylinders of the manipulator at the beginning of motion (other forms of conditions on pressures are, of course, also possible), and $\dot{p}_0$, $\dot{p}_T$, $\ddot{p}_0$, $\ddot{p}_T$, $\boldsymbol{P}_{10}$ are supplied values. Here, however, for efficiency of computation, incompressibility of the hydraulic fluid is assumed, which reduces the boundary conditions to

$$p(0) = 0 \qquad p(T) = 1 \qquad \dot{p}(0) = \dot{p}_0 \qquad \dot{p}(T) = \dot{p}_T \tag{3.2}$$

Conditions on pressures and accelerations are no longer necessary because the joint space trajectory uniquely determines corresponding pressures in cylinder chambers at every instant, which will be demonstrated later, and pressures can change instantaneously.

A hydraulic manipulator is subjected to a number of physical limitations during its motion, which can be expressed as inequality constraints on control inputs and state variables of the system. In general, they can be written as

$$\Phi_i(\boldsymbol{u},\boldsymbol{x},\dot{\boldsymbol{x}},\boldsymbol{P}_1,\boldsymbol{P}_2) \leqslant 0 \qquad i=1,2,\dots,m \tag{3.3}$$

where $\boldsymbol{u}$, $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are n-vectors of u, P_1 and P_2, respectively, for all cylinders. The most typical constraints are constant bounds on control inputs which, for a single cylinder, take the form: $u - u_{max} \leqslant 0$ and $u_{min} - u \leqslant 0$, where u_{max} and u_{min} are constant limiting values. In the remainder of the paper, by the term *values of constraints* for the given $\boldsymbol{u}$, $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$, $\boldsymbol{P}_1$, $\boldsymbol{P}_2$ values $\Phi_i(\boldsymbol{u},\boldsymbol{x},\dot{\boldsymbol{x}},\boldsymbol{P}_1,\boldsymbol{P}_2)$, $i=1,2,\dots,m$ will be meant.

Usually, optimal control problems are formulated as problems of searching for such control inputs, which make a system move in the optimal way. Here, an inverse formulation will be utilized. Since to every optimal control there corresponds a unique optimal trajectory of a system in its state space (a trajectory along which the system moves under action of the given optimal control), the problem can be posed as a direct search for the optimal trajectory. The optimal control inputs corresponding to the movement of the system along the chosen optimal trajectory can be found from equations of motion of the system. Since finding the optimal trajectory is here tantamount to finding the optimal distribution of the parameter in time, the considered optimization problem can be stated in the following way.

Let equations of motion (2.1)-(2.3) of a hydraulic manipulator be given. Let $\boldsymbol{x}_r(p)$, $0 \leqslant p \leqslant 1$ be a given twice differentiable joint space trajectory along which the machine is supposed to move. Find a twice differentiable distribution of the parameter in time $p(t):\ [0,T] \to [0,1]$, satisfying boundary conditions (3.2), and such that on the trajectory $\boldsymbol{x}(t) = \boldsymbol{x}_r(p(t))$, $0 \leqslant t \leqslant T$ all control inputs and state variables of the system satisfy constraints (3.3), and T is the smallest possible.

To iteratively search for the optimal $p(t)$, a method of computing the performance index T and values of constraints (3.3) for any given $p(t)$ must be available. The value of T is given directly as $T = p^{-1}(1)$. Values of constraints can be computed as shown below.

Step 1. Compute the joint space trajectory and its time derivatives corresponding to the chosen $p(t)$

$$\begin{aligned}
\boldsymbol{x}(t) &= \boldsymbol{x}_r(p(t)) \\
\dot{\boldsymbol{x}}(t) &= \frac{d}{dt}\boldsymbol{x}_r(p(t)) = \frac{d\boldsymbol{x}_r}{dp}(p(t))\dot{p}(t) \\
\ddot{\boldsymbol{x}}(t) &= \frac{d}{dt}\dot{\boldsymbol{x}}_r(p(t)) = \frac{d\boldsymbol{x}_r}{dp}(p(t))\ddot{p}(t) + \frac{d^2\boldsymbol{x}_r}{dp^2}(p(t))\dot{p}(t)^2
\end{aligned} \tag{3.4}$$

Step 2. Compute piston forces $\boldsymbol{s}(t)$ from equations (2.1) corresponding to the movement of the system along the joint space trajectory $\boldsymbol{x}(t)$, and corresponding piston velocities $\boldsymbol{v}(t)$

$$
\begin{aligned}
\boldsymbol{s}(t) &= \mathbf{Q}(\boldsymbol{x}(t))^{-1}[\mathbf{D}(\boldsymbol{x}(t))\ddot{\boldsymbol{x}}(t) + \boldsymbol{h}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t))] \\
\boldsymbol{v}(t) &= \mathbf{Q}(\boldsymbol{x}(t))^{\top}\dot{\boldsymbol{x}}(t)
\end{aligned}
\tag{3.5}
$$

Step 3. Compute, on the basis of $\boldsymbol{v}(t)$ and $\boldsymbol{s}(t)$, distributions of control inputs $\boldsymbol{u}(t)$ and fluid pressures $\boldsymbol{P}_1(t)$ and $\boldsymbol{P}_2(t)$ during motion. If compressibility of the hydraulic fluid was included in the analysis, computation of fluid pressures and control inputs would require numerical integration of those values from the initial conditions on pressures over the entire time interval $[0, T]$. Here, to ease the computational burden at each stage of optimization, incompressibility of the hydraulic fluid is assumed. It allows the calculation of $\boldsymbol{u}(t)$, $\boldsymbol{P}_1(t)$ and $\boldsymbol{P}_2(t)$ without the need of integration, which is shown below.

Let the ith hydraulic cylinder of the manipulator be considered. Let $s(t)$ and $v(t)$ denote its piston force and velocity at the time t, respectively (they are ith components of the previously calculated vectors $\boldsymbol{s}(t)$ and $\boldsymbol{v}(t)$). The index t will from now on be omitted from the equations. It may be observed that if the fluid is incompressible, the same volume of the fluid flows into a given cylinder chamber at any time as flows out of it. Thus, neglecting possible leakage between cylinder chambers and elasticity of hydraulic lines, one may write (Fig. 1): $Q_1 = A_1 v$ and $Q_2 = A_2 v$. By using equations (2.3), this can be stated as

$$
\begin{aligned}
&\left.\begin{aligned}
C_{1P}\sqrt{P_P - P_1}\,u &= A_1 v \\
C_{2R}\sqrt{P_2 - P_R}\,u &= A_2 v
\end{aligned}\right\} \qquad \text{when} \quad u \geqslant 0 \\
&\left.\begin{aligned}
C_{1R}\sqrt{P_1 - P_R}\,u &= A_1 v \\
C_{2P}\sqrt{P_P - P_2}\,u &= A_2 v
\end{aligned}\right\} \qquad \text{when} \quad u < 0
\end{aligned}
\tag{3.6}
$$

By using proportions in the above pairs of equations, and noting that in the considered case, the sign of u always agrees with that of v, one obtains

$$
\begin{aligned}
A_2 C_{1P}\sqrt{P_P - P_1} &= A_1 C_{2R}\sqrt{P_2 - P_R} \qquad \text{when} \quad v \geqslant 0 \\
A_2 C_{1R}\sqrt{P_1 - P_R} &= A_1 C_{2P}\sqrt{P_P - P_2} \qquad \text{when} \quad v < 0
\end{aligned}
\tag{3.7}
$$

Equations (3.7) are the first set of relations between pressures in opposite cylinder chambers. The choice between the equations is done on the basis of the known sign of v. Since s was calculated in step 2, equation (2.2) gives another relation between fluid pressures. By combining equations (2.2) and (3.7), one obtains explicit formulae for fluid pressures in cylinder chambers, in the form

$$\left.\begin{aligned} P_1 &= \frac{A_2(W_2^+ P_P + W_1^+ P_R) + W_1^+ s}{W_1^+ A_1 + W_2^+ A_2} \\ P_2 &= \frac{A_1(W_2^+ P_P + W_1^+ P_R) - W_2^+ s}{W_1^+ A_1 + W_2^+ A_2} \end{aligned}\right\} \quad \text{when} \quad v \geqslant 0$$

$$\left.\begin{aligned} P_1 &= \frac{A_2(W_1^- P_P + W_2^- P_R) + W_1^- s}{W_1^- A_1 + W_2^- A_2} \\ P_2 &= \frac{A_1(W_1^- P_P + W_2^- P_R) - W_2^- s}{W_1^- A_1 + W_2^- A_2} \end{aligned}\right\} \quad \text{when} \quad v < 0 \tag{3.8}$$

where $W_1^+ = A_1^2 C_{2R}^2$, $W_2^+ = A_2^2 C_{1P}^2$, $W_1^- = A_1^2 C_{2P}^2$ and $W_2^- = A_2^2 C_{1R}^2$. Thus, at any given moment t, chamber pressures can be calculated on the basis of the given piston force s and piston velocity v. The value of the control signal u can now be computed from equations (3.6) after averaging, as

$$u = \begin{cases} \dfrac{(A_1 + A_2)v}{C_{1P}\sqrt{P_P - P_1} + C_{2R}\sqrt{P_2 - P_R}} & \text{when} \quad v \geqslant 0 \\ \dfrac{(A_1 + A_2)v}{C_{1R}\sqrt{P_1 - P_R} + C_{2P}\sqrt{P_P - P_2}} & \text{when} \quad v < 0 \end{cases} \tag{3.9}$$

Step 4. On the basis of $\boldsymbol{x}(t)$, $\dot{\boldsymbol{x}}(t)$, $\boldsymbol{P}_1(t)$, $\boldsymbol{P}_2(t)$ and $\boldsymbol{u}(t)$, calculate values of constraints (3.3) at any required t in the interval $[0, T]$.

4. Approximate method of solution

To obtain an effective method of solution, the original continuous optimization problem is converted into a finite-dimensional optimization problem. After proper formulation, the new problem can readily be solved by existing numerical methods of constrained nonlinear programming, implementations of which are available in many numerical packages. In this way, approximate solutions to the original continuous problem can be found.

The process of discretization exploits the assumption that the distribution of the parameter $p(t)$ is increasing in $(0, T)$. Therefore, to each $p^* \in [0, 1]$ there corresponds a unique $t^* \in [0, T]$ such that $p(t^*) = p^*$. In particular, if the interval $[0,1]$ is divided into k equal segments of the length $1/k$ by points $p_i = i/k$, $i = 0,1,\dots,k$, then to every p_i there corresponds a unique time t_i such that $p(t_i) = p_i$. Thus, the continuous distribution $p(t)$ of the parameter can be approximated by a set of points $\{t_i, p_i\}$, $i = 0,1,\dots,k$ (Fig. 2). In general, the distances in time between successive points $\Delta t_i = t_i - t_{i-1}$, $i = 1,2,\dots,k$ are uneven. Besides, values of Δt_i unambiguously determine the set of points $\{t_i, p_i\}$, since p_i are fixed and known beforehand. The time intervals Δt_i, $i = 1,2,\dots,k$ are therefore assumed as optimization variables. Thus, the original problem of searching for the optimal $p(t)$ is transformed into an approximately equivalent k-dimensional problem of searching for the optimal Δt_i, $i = 1,2,\dots,k$.

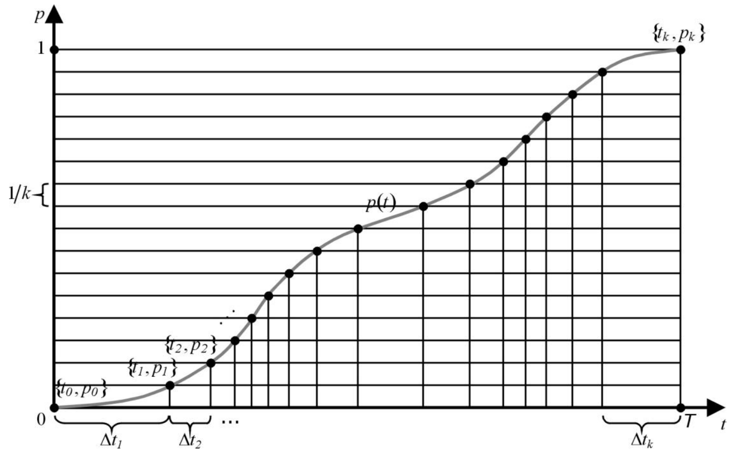

Fig. 2. Parameter distribution as a sequence of points

Numerical procedures capable of solving multidimensional constrained optimization problems of the discussed type and size, like Sequential Quadratic Programming (Gill *et al.*, 1981), are available in many numerical packages. Therefore, the discussed problem can be solved once it is presented in a form required by such a routine. A numerical optimization procedure searches for such a k-vector $\boldsymbol{z}$, which minimizes the given cost function $f(\boldsymbol{z})$, under given constraints $\Psi_i(\boldsymbol{z}) \leqslant 0$, $i = 1,2,\dots,q$. Thus, to employ the procedure, it is sufficient to specify $\boldsymbol{z}$, $f(\boldsymbol{z})$, and $\Psi_i(\boldsymbol{z})$, $i = 1,2,\dots,q$, in terms of the problem here considered. According to previous discussions: $\boldsymbol{z} := [\Delta t_1, \Delta t_2, \dots, \Delta t_k]$. The definition of the cost function is straightforward: $f(\boldsymbol{z}) := T = \sum_{i=1}^{k} \Delta t_i$. The constraint functions $\Psi_i(\boldsymbol{z})$, $i = 1,2,\dots,q$ represent values of constraints (3.3) at all instants t_i, $i = 0,1,\dots,k$; therefore $q = m(k+1)$. Calculation of $\Psi_i(\boldsymbol{z})$, $i = 1,2,\dots,q$ for a given $\boldsymbol{z} = [\Delta t_1, \Delta t_2, \dots, \Delta t_k]$ involves steps given below (following Section 3).

Step 1. Compute values of the joint space trajectory $\boldsymbol{x}(t)$ and its time derivatives at points $\{t_i, p_i\}$, $i = 0, 1, \dots, k$ using formulae (3.4)

$$
\begin{aligned}
&\boldsymbol{x}(t_i) = \boldsymbol{x}_r(p_i) \qquad\qquad \dot{\boldsymbol{x}}(t_i) = \frac{d\boldsymbol{x}_r}{dp}(p_i)\dot{p}_i \\
&\ddot{\boldsymbol{x}}(t_i) = \frac{d\boldsymbol{x}_r}{dp}(p_i)\ddot{p}_i + \frac{d^2\boldsymbol{x}_r}{dp^2}(p_i)\dot{p}_i^2
\end{aligned}
\tag{4.1}
$$

where

$$
p_i = \frac{i}{k} \qquad \dot{p}_i := \frac{p_{i+1} - p_i}{\Delta t_{i+1}} = \frac{1}{k\Delta t_{i+1}} \qquad \ddot{p}_i := \frac{\dot{p}_{i+1} - \dot{p}_i}{\Delta t_{i+1}}
$$

with adjustments for supplied boundary conditions at $i = 0$ and $i = k$. Alternative difference schemes can naturally be used as well.

Step 2. Compute, from equations of motion (2.1), vectors of piston forces at instants t_i as $\boldsymbol{s}(t_i) = \mathbf{D}(\boldsymbol{x}(t_i))\ddot{\boldsymbol{x}}(t_i) + \boldsymbol{h}(\boldsymbol{x}(t_i), \dot{\boldsymbol{x}}(t_i))$, and piston velocities as $\boldsymbol{v}(t_i) = \mathbf{Q}(\boldsymbol{x}(t_i))^\top \dot{\boldsymbol{x}}(t_i)$; $i = 0, 1, \dots, k$.

Step 3. From $\boldsymbol{s}(t_i)$ and $\boldsymbol{v}(t_i)$, $i = 0, 1, \dots, k$, compute vectors of fluid pressures $\boldsymbol{P}_1(t_i)$, $\boldsymbol{P}_2(t_i)$ and control inputs $\boldsymbol{u}(t_i)$, $i = 0, 1, \dots, k$ using equations (3.8) and (3.9), respectively.

Step 4. Calculate values of the constraints $\Psi_i(\boldsymbol{z})$, $i = 1, 2, \dots, q$ as

$$
\Psi_{mi+j}(\boldsymbol{z}) = \Phi_j(\boldsymbol{u}(t_i), \boldsymbol{x}(t_i), \dot{\boldsymbol{x}}(t_i), \boldsymbol{P}_1(t_i), \boldsymbol{P}_2(t_i)) \qquad \begin{aligned} &i = 0, 1, \dots, k \\ &j = 1, 2, \dots, m \end{aligned}
\tag{4.2}
$$

where Φ_j are the constraint functions in (3.3).

In this way, all the necessary information for the numerical optimization procedure has been specified. The procedure starts the search from the supplied initial value $\boldsymbol{z} = \boldsymbol{z}_0$, for which the constraints are satisfied: $\Psi_i(\boldsymbol{z}_0) \leqslant 0$, $i = 1, 2, \dots, q$.

Comments on the proposed approach:

- The algorithm may find a local minimum. Therefore, it is advisable to verify optimality of the obtained solution by restarting the algorithm with a different initial value $\boldsymbol{z}_0$.
- If the number k of time intervals Δt_i (dimension of the vector $\boldsymbol{z}$) is sufficiently large, so that the corresponding points $\{t_i, p_i\}$, $i = 0, 1, \dots, k$

can faithfully represent the target optimal distribution $p(t)$, any particular choice of k does not significantly influence the solution. Therefore, k should be chosen large enough to capture the expected variability of the optimal $p(t)$, and simultaneously as small as possible to reduce the amount of computation.

- Since the obtained optimal solution has the form of a sequence of points ($\boldsymbol{z} = [\Delta t_1, \Delta t_2, \ldots, \Delta t_k] \Leftrightarrow \{t_i, p_i\};\ i = 0, 1, \ldots, k$), interpolation has to be performed to turn them into practically usable data. Because of arbitrariness of such interpolation (and due to the use of a simplified model of manipulator dynamics), the application-oriented optimization should be performed under tightened constraints, leaving room for a closed-loop control scheme to correct the resultant errors.

5. Numerical example

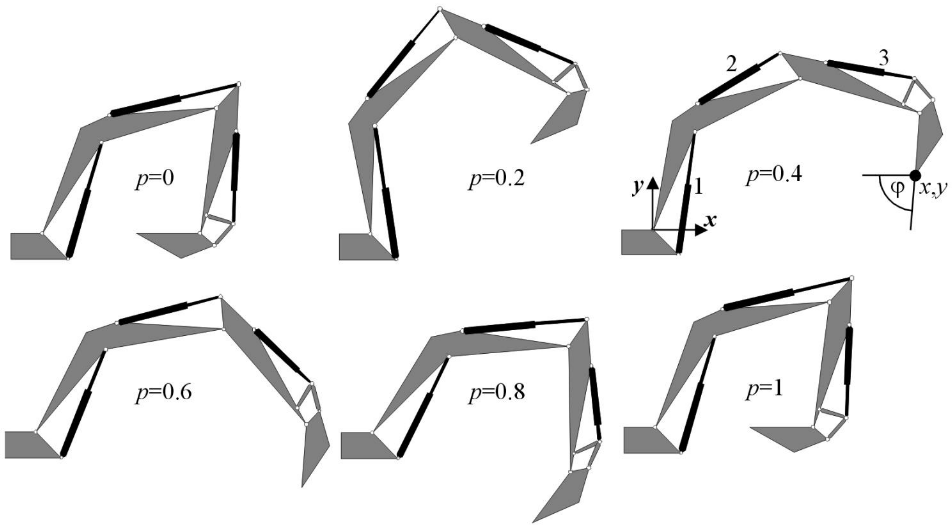

Fig. 3. Stages of motion along the path

Test optimizations are performed on a three-link excavator, shown in Fig. 3. The end-effector path, for $0 \leqslant p \leqslant 1$, is given by its positions and orientations in the coordinate system xy as: $x(p) = 3 - 1.5\cos(2\pi p)$, $y(p) = 1.5\sin(2\pi p)$, $\varphi(p) = \pi\sin(\pi p)/2$. The manipulator begins and ends its motion at rest. Discretization into $k = 100$ intervals is used. In the presented figures, u_1, u_2, u_3; v_1, v_2, v_3; P_{11}, P_{12}, P_{13}; and P_{21}, P_{22}, P_{23} denote control inputs u; piston velocities v; chamber pressures P_1; and chamber pressures P_2 for cylinders 1, 2 and 3, respectively.

- Results for constraints on control inputs $-7\,\mathrm{mm} \leqslant u_1, u_2, u_3 \leqslant 7\,\mathrm{mm}$ (Fig. 4 - Fig. 6).

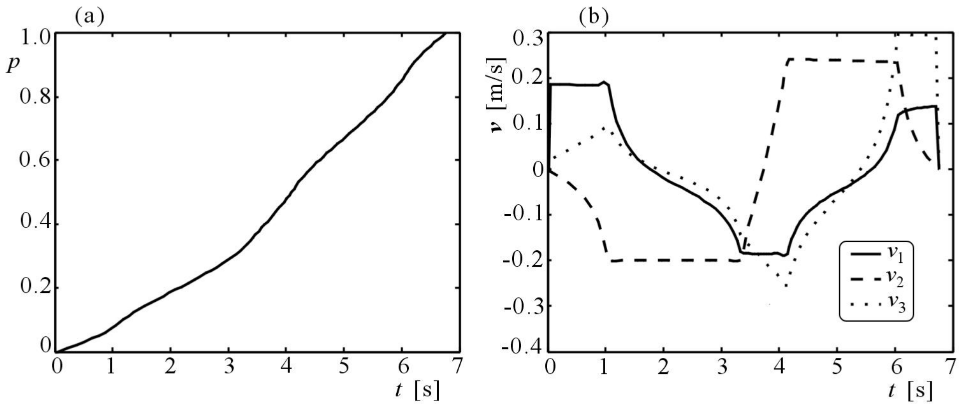

Fig. 4. (a) Parameter distribution; (b) piston velocities

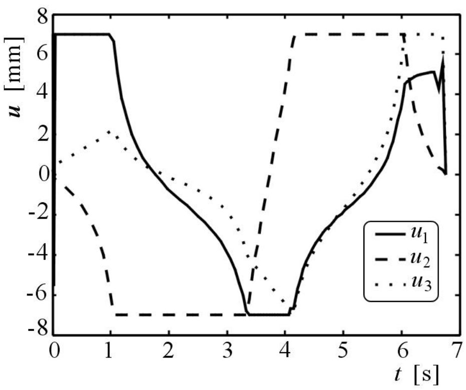

Fig. 5. Control signals

The obtained results are consistent with theoretical results concerning the structure of optimal control signals in the sense that at any moment of motion, at least one of the control signals is saturated (reaches imposed control bounds).

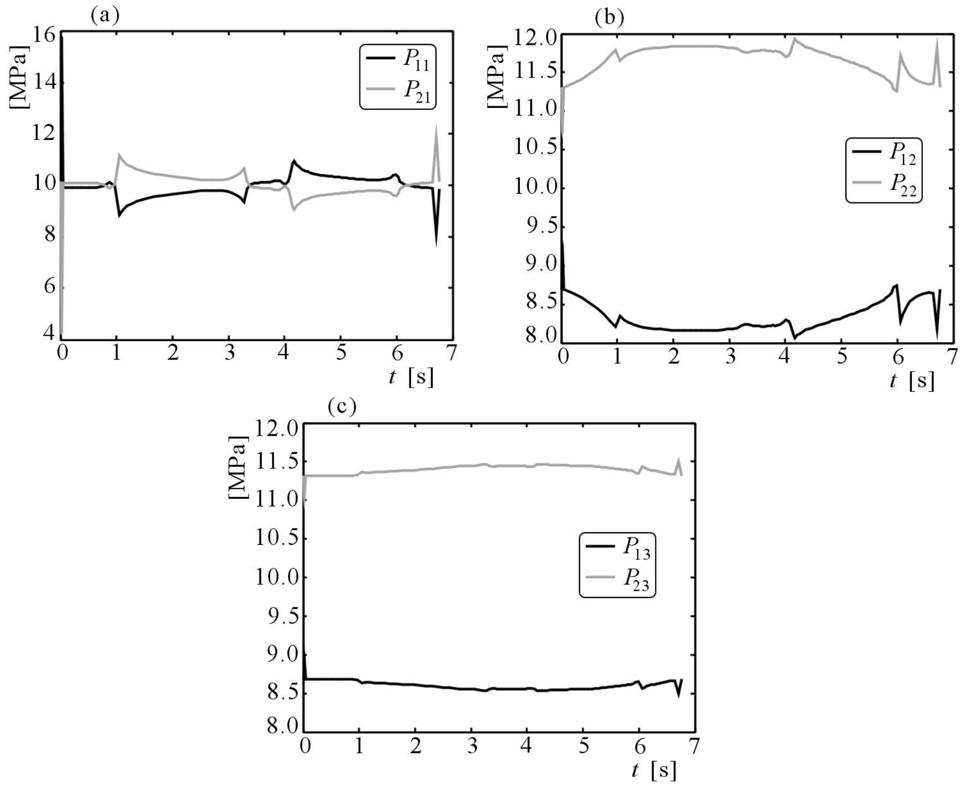

Fig. 6. Pressures in cylinder 1, 2, 3, respectively

- Results for constraints on control inputs $-7\,\mathrm{mm} \leqslant u_1, u_2, u_3 \leqslant 7\,\mathrm{mm}$ and constraints on maximum piston velocities $-0.2\,\mathrm{m/s} \leqslant v_1, v_2, v_3 \leqslant 0.15\,\mathrm{m/s}$ (Fig. 7 - Fig. 9).

Throughout the entire time of motion, either one of the control signals or one of the piston velocities is saturated.

6. Conclusions

A method of optimization of speed of movements of hydraulic manipulators along prescribed parametric paths was presented. It was built on the fact that in non-redundant manipulators the considered control problem can be effectively treated as one dimensional - a search for the optimal distribution

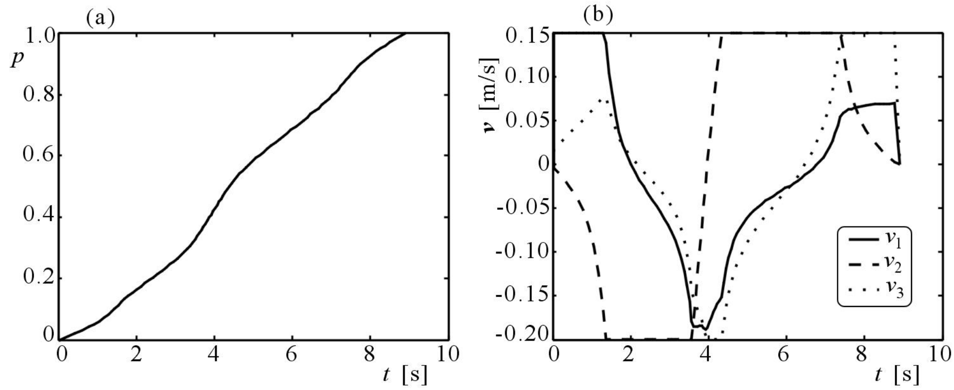

Fig. 7. (a) Parameter distribution; (b) piston velocities

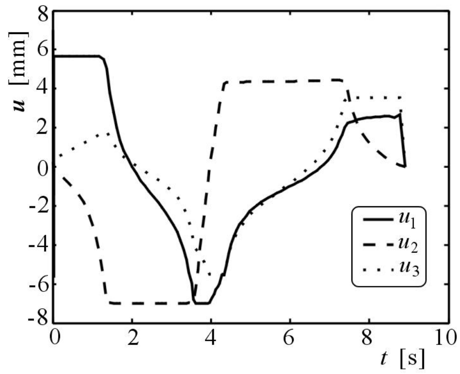

Fig. 8. Control signals

of the scalar parameter in time. The main features of the proposed method can be summarized as follows:

- The optimization is performed directly on the distribution of the parameter. Corresponding control signals are computed in the second place, from the chosen distribution of the parameter and equations of motion of the machine.
- Efficiency of the method rests on the assumption that the hydraulic fluid is incompressible.
- The original optimization problem is converted into an approximately equivalent finite dimensional problem. The distribution of the parameter being optimized is treated as a sequence of points.
- Optimization is executed by methods of constrained nonlinear programming.

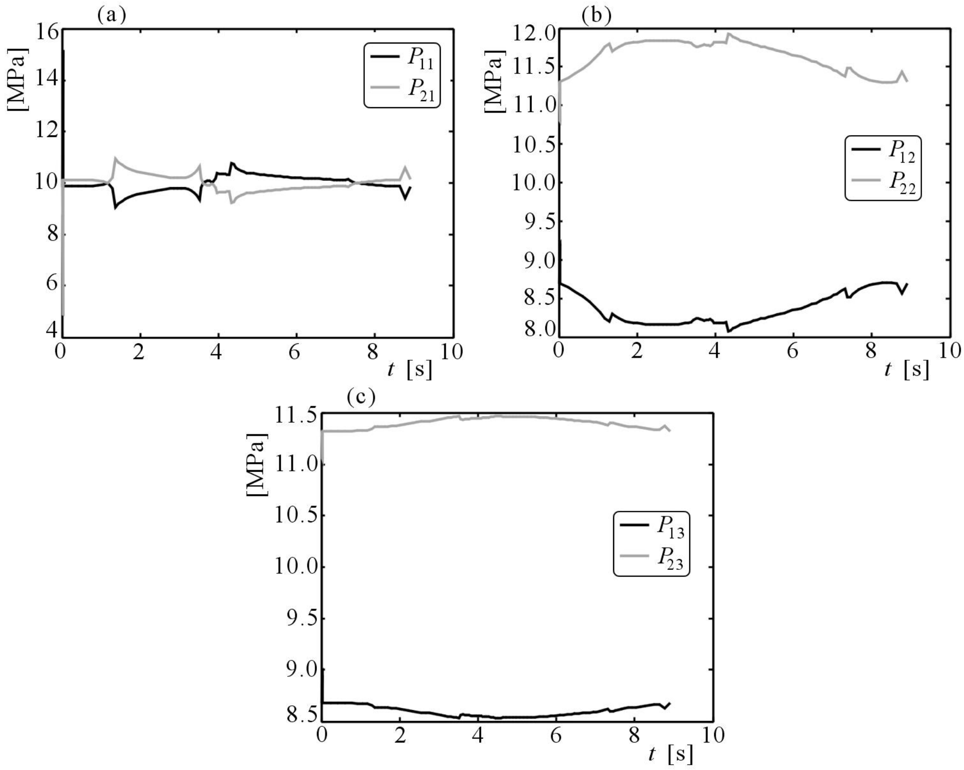

Fig. 9. Pressures in cylinder 1, 2, 3, respectively

The proposed method was tested to reach reliable, approximate solutions fast. One of the main advantages of the proposed formulation is its flexibility as it can handle many sorts of constraints in a uniform manner. That feature is not present in many other algorithms proposed in the literature for solving similar problems. However, the method is not easily extended to other optimization problems. In particular, it seems ill-suited to solving general time-optimal problems without path constraints.

Acknowledgement

This work was supported by the State Committee for Scientific Research of Poland (KBN) under grant No. 5T07A 002 23 for the years 2002-2004.

References

1. Bobrow J.E., Dubowsky S., Gibson J.S., 1985, Time-optimal control of robotic manipulators, *International Journal of Robotics Research*, **4**, 3, 3-17

2. GillL P.E., Murray W., Wright M.H., 1981, *Practical Optimization*, Academic Press, London

3. McCarthy J.M., Bobrow J.E., 1992, The number of saturated actuators and constraint forces during time-optimal movement of a general robotic system, *IEEE Transactions on Robotics and Automation*, **8**, 3, 407-409

4. Shiller Z., Lu H.H., 1992, Computation of path constrained time optimal motions with dynamic singularities, *ASME Journal of Dynamic Systems, Measurement, and Control*, **114**, 34-40

5. Shin K.G., McKay N.D., 1986, A dynamic programming approach to trajectory planning of robotic manipulators, *IEEE Transactions on Automatic Control*, **31**, 6

6. Singh S., Leu M.C., 1987, Optimal trajectory generation for robotic manipulators using dynamic programming, *ASME Journal of Dynamic Systems, Measurement, and Control*, **109**, 88-96

7. Spong M.W., Vidyasagar M., 1997, *Dynamika i sterowanie robotów*, Wydawnictwa Naukowo-Techniczne, Warszawa

8. Tomczyk J., 1999, *Modele dynamiczne elementów i układów napędów hydrostatycznych*, Wydawnictwa Naukowo-Techniczne, Warszawa

9. Walters R.B., 2000, *Hydraulic and Electro-Hydraulic Control Systems*, 2nd edition, Kluwer Academic Publishers, Dodrecht

Sterowanie minimalno-czasowe manipulatorów hydraulicznych po zadanej ścieżce

Streszczenie

Praca przedstawia metodę optymalizacji minimalno-czasowej ruchów manipulatorów hydraulicznych, po zadanej ścieżce członu roboczego. Zakładane jest, że ścieżka członu roboczego jednoznacznie określa odpowiadającą jej ścieżkę manipulatora w zmiennych uogólnionych. Optymalizacja sprowadza się wówczas do znalezienia optymalnego rozkładu parametru ścieżki w czasie. Proponowana metoda optymalizacji polega na przybliżeniu ciągłego rozkładu parametru zbiorem punktów, a następnie znalezieniu ich optymalnych położeń metodami programowania nieliniowego z ograniczeniami. Zakładana jest przy tym nieściśliwość cieczy hydraulicznej, w celu przyspieszenia obliczeń. Załączone są wyniki przykładowych optymalizacji, wykonanych na modelu trójczłonowej koparki hydraulicznej.

Manuscript received December 28, 2004; accepted for print May 5, 2005